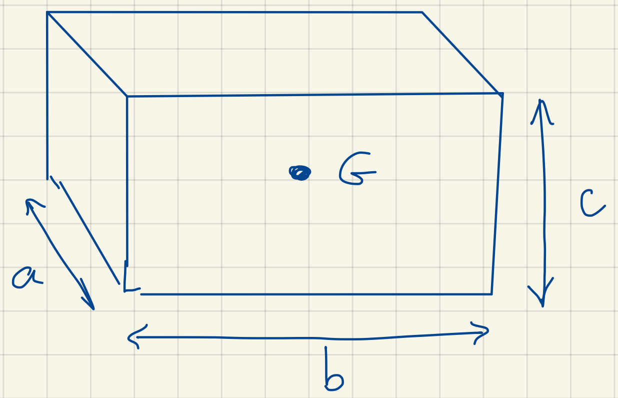
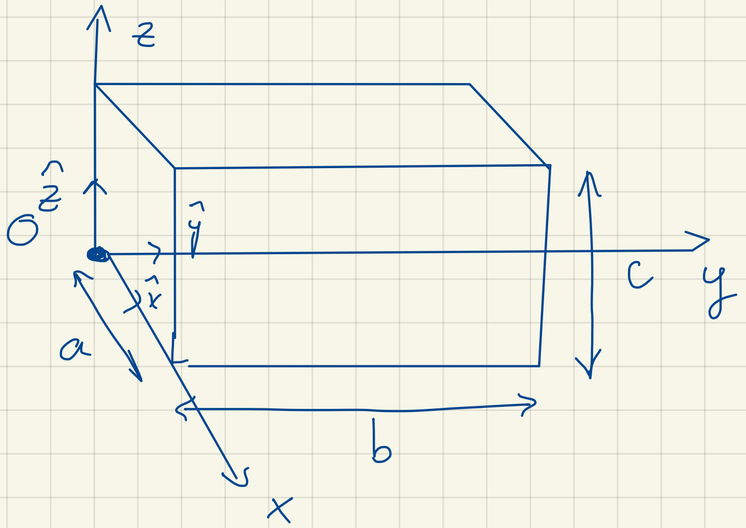
Inertie à trou

vidéo explicative.

Ph. Müllhaupt

12/1/21





$$\underline{\underline{I}}_G = \begin{bmatrix} \frac{1}{12} m (b^2+c^2) & \boxed{0} & 0 \\ 0 & \frac{1}{12} m (a^2+c^2) & 0 \\ 0 & 0 & \boxed{\frac{1}{12} m (a^2+b^2)} \end{bmatrix}$$

$$\underline{\underline{I}}_O = \begin{bmatrix} m \frac{b^2+c^2}{3} & \boxed{-\frac{mab}{4}} & -\frac{mac}{4} \\ -\frac{mab}{4} & m \frac{a^2+c^2}{3} & -\frac{mbc}{4} \\ -\frac{mac}{4} & -\frac{mbc}{4} & \boxed{m \frac{a^2+b^2}{3}} \end{bmatrix}$$

$$\overrightarrow{OG} = \begin{bmatrix} a/2 \\ b/2 \\ c/2 \end{bmatrix}_{(O, \hat{x}, \hat{y}, \hat{z})}$$

<u>Steiner généralisé</u>

$$\underline{I_O} = \underline{I_G} + \underline{m} \begin{bmatrix} \gamma_y^2 + \gamma_z^2 & \boxed{-\gamma_x \gamma_y} & -\gamma_x \gamma_z \\ -\gamma_y \gamma_x & \gamma_x^2 + \gamma_z^2 & -\gamma_y \gamma_z \\ -\gamma_x \gamma_z & -\gamma_y \gamma_z & \boxed{\gamma_x^2 + \gamma_y^2} \end{bmatrix}$$

$$\gamma_x^2 + \gamma_y^2 = \frac{a^2+b^2}{4}$$

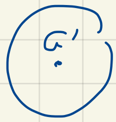
$$\boxed{\frac{1}{12} m \left(a^2 + b^2\right)} + m \frac{1}{4} a^2 + b^2 =$$

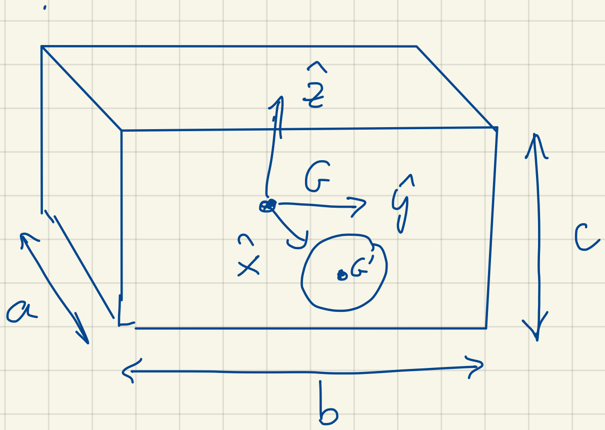
$$\left(\frac{1}{12} + \frac{3}{12}\right) m \left(a^2 + b^2\right) = m \frac{4}{12} \left(a^2 + b^2\right) = \frac{m}{3} \left(a^2 + b^2\right)$$

$$-m \gamma_x \gamma_y = -m \frac{a}{2} \frac{b}{2} = \boxed{-m \frac{ab}{4}}$$

Inertie à trou   sphère (boule pleine)

$$\underline{I}_{G'} = \frac{2}{5} m_s R^2$$





$$\underline{I}_{G'} = \begin{bmatrix} \frac{2}{5} m_s R^2 & 0 & 0 \\ 0 & \frac{2}{5} m_s R^2 & 0 \\ 0 & 0 & \frac{2}{5} m_s R^2 \end{bmatrix}$$

$$= \frac{2}{5} m_s R^2 \, \underline{\underline{I}}$$

↑ matrice identité

$$\overrightarrow{G'G} = \begin{bmatrix} \gamma_x \\ \gamma_y \\ \gamma_z \end{bmatrix}_{(G,\hat{x},\hat{y},\hat{z})}$$

$$\underline{I}_G = \begin{bmatrix} \frac{1}{12} m (b^2+c^2) & 0 & 0 \\ 0 & \frac{1}{12} m (a^2+c^2) & 0 \\ 0 & 0 & \frac{1}{12} m (a^2+b^2) \end{bmatrix}$$

$$\boxed{\underline{I}_{G,\,\text{complet}} = \underline{I}_G - \underline{I}_{G,\,\text{sphère}}}$$

$$\underline{I}_{G\ \text{sphère}} = \underline{I}_{G'} + m_s \begin{bmatrix} \gamma_y^2+\gamma_z^2 & -\gamma_x\gamma_y & -\gamma_x\gamma_z \\ -\gamma_y\gamma_x & \gamma_x^2+\gamma_z^2 & -\gamma_y\gamma_z \\ -\gamma_x\gamma_z & -\gamma_y\gamma_z & \gamma_x^2+\gamma_y^2 \end{bmatrix}$$

$\Longrightarrow$ c'est tout !