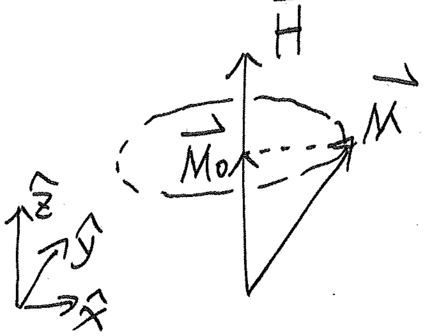
Equation of Motion

Landau-Lifshitz equation describing magnetization dynamics:

$$\frac{\partial \vec{M}}{\partial t} = -\gamma \vec{M} \times \mu_0 \vec{H}$$



γ: gyromagnetic ratio
μ_0 : vacuum permeability

Here the magnitude of $\vec{M}$ is conserved:

$$\frac{\partial \vec{M}^2}{\partial t} = 0$$

When dissipation is considered, the equation of motion is modified into the Landau-Lifshitz-Gilbert (LLG) equation:

$$\frac{\partial \vec{M}}{\partial t} = -\gamma \vec{M} \times \mu_0 \vec{H} + \frac{\alpha}{M} \vec{M} \times \frac{\partial \vec{M}}{\partial t}$$

α: Gilbert damping coefficient

Considering both the static and dynamic parts:

$$\vec{M} = \vec{M_0} + \vec{m}, \quad \vec{H} = \vec{H_0} + \vec{h}$$

where $\vec{m}$ and $\vec{h}$ are the dynamic components.

Then the equation of motion can be linearized as:

$$i\omega\vec{m} = -(\gamma\vec{m}\times\mu_0\vec{H_0} + \gamma\vec{M_0}\times\vec{h}) + \frac{i\alpha\omega}{M_0}\vec{m}\times\vec{M_0} \quad \text{①}'$$

In the linear regime, $|\vec{m}| \ll |\vec{M}|$, $|\vec{h}| \ll |\vec{H}|$.

We write the dynamic components as three-dimensional vectors:

$$\vec{m} = m_x\hat{x} + m_y\hat{y} + m_z\hat{z}$$

$$\vec{h} = h_x\hat{x} + h_y\hat{y} + h_z\hat{z}$$

and $m_z = 0$, $h_z = 0$

so Eq. ① ~~can~~ is equivalent to:

$$\begin{cases} i\omega m_x + (\omega_H + i\frac{M_0}{M}\alpha\omega)m_y = \gamma M_0\mu_0 h_y \ ; \\ i\omega m_y + (\omega_H + i\frac{M_0}{M}\alpha\omega)m_x = \gamma M_0\mu_0 h_x . \end{cases}$$

here $\omega_H = \gamma\mu_0 H_0$

The high-frequency magnetic susceptibility describing the time-dependent relations is written as a non-symmetric second-rank tensor firstly presented as Polder tensor

$$\overleftrightarrow{\chi} = \begin{vmatrix} \chi & i\chi_a & 0 \\ -i\chi_a & \chi & 0 \\ 0 & 0 & \chi_{11} \end{vmatrix}$$

where χ_{11} is either zero or a very small non-resonant value when the ferromagnets are saturated.

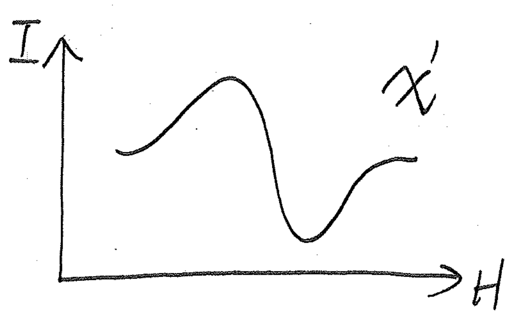
The solution of the linearized equation is the tensor $\overleftrightarrow{\chi}$ projected onto coordinate axes as $\vec{m} = \overleftrightarrow{\chi}\vec{h}$ and

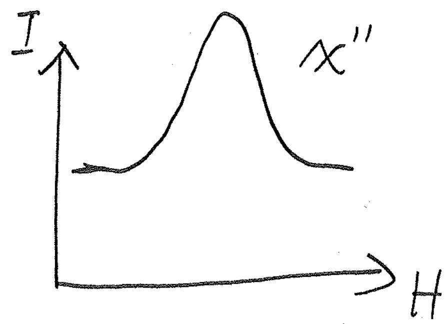
$$\chi' = \frac{\gamma M_0 \omega_H [\omega_H^2 - (1-\alpha^2)\omega^2]}{[\omega_H^2 - (1+\alpha^2)\omega^2] + 4\alpha^2\omega^2\omega_H^2},$$

$$\chi'' = \frac{\alpha\gamma M_0 \omega [\omega_H^2 + (1+\alpha^2)\omega^2]}{[\omega_H^2 - (1+\alpha^2)\omega^2] + 4\alpha^2\omega^2\omega_H^2},$$

$$\chi_a' = \frac{\gamma M_0 \omega [\omega_H^2 - (1+\alpha^2)\omega^2]}{[\omega_H^2 - (1+\alpha^2)\omega^2] + 4\alpha^2\omega^2\omega_H^2}, \text{ and}$$

$$\chi_a'' = 2\alpha\gamma M_0 \omega^2 \omega_H .$$





Solution to DMI calculation

We take the example of small-k regime which covers both dipole and exchange term:

$$\begin{cases} \dfrac{\omega}{\gamma\mu_0} = \sqrt{\;\cdots\cdots\;} + p\cdot\dfrac{2D}{\mu_0 M_s}k \\ \omega = 2\pi f \end{cases}$$

$$\Rightarrow \quad f = \frac{\gamma\mu_0}{2\pi}\sqrt{\;\cdots\cdots\;} + p\cdot\frac{\gamma D}{\pi M_s}k$$

SO:

$$\Delta f = |f(+k) - f(-k)|$$

$$\Delta f = \left|p\cdot\frac{\gamma D}{\pi M_s} - p\cdot\frac{\gamma D}{\pi M_s}(-k)\right|$$

$$\Delta f = \left|p\cdot\frac{2\gamma D k}{\pi M_s}\right|, \quad p = \pm 1 \text{ (sign of DMI)}$$

$$\Delta f = \frac{2\gamma D k}{\pi M_s} \quad \Rightarrow \quad D = \frac{d(\Delta f)}{dk}\cdot\frac{\pi M_s}{2\gamma}$$

$$D\ \cancel{\Delta f} = 0.0105\ \frac{\text{GHz}}{\frac{\text{rad}}{\mu\text{m}}} \times \frac{\pi \times 230\ \frac{\text{kA}}{\text{m}}}{2\times 2\pi \times 28\ \frac{\text{GHz}}{\text{T}}}$$

$$D = 1.044 \times 10^{-4}\ \text{J/m}^2$$