From single asperity to multi asperity contact

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Outline

https://c4science.ch/source/Tribology_Course_nb/

- Lecture 1
  - Introduction of lecturer and laboratory LSMS (lsms.epfl.ch)
  - Introduction to tribology (my vision)
  - From da Vinci to rate and state friction laws

- Exercise 1 (optional HW): Some fundamental solutions in mechanics of solids

- Lecture 2
  - Surface roughness, self-affine roughness
  - Single asperity contact: Hertz contact theory
  - Multiple asperities contact, rough contact mechanics

- Exercise 2 (optional HW)
  - Generation with open-source software Tamaas of rough surfaces
  - Resolution of Hertz contact with Tamaas

- Lecture 3: From friction to wear

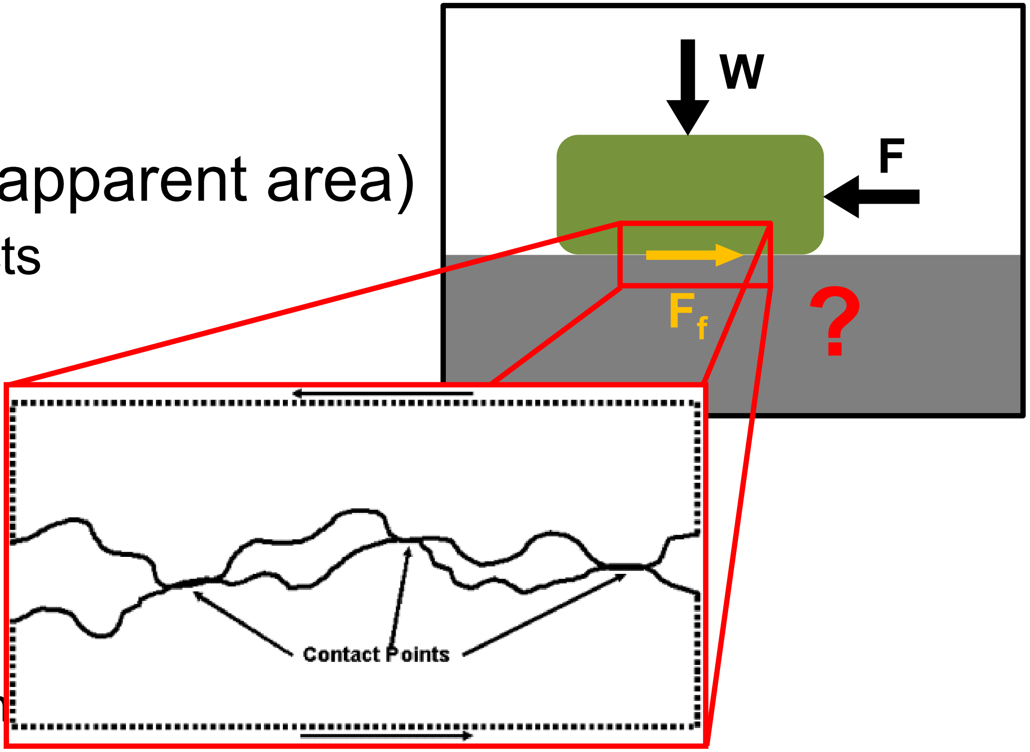
- Exercise 3 (optional HW): Resolution of rough contact mechanics with Tamaas

Short recap

- Bowden and Tabor (1950)
  - Real contact area
    (much smaller than apparent area)
    - Sum of microcontacts
  - Friction force

$$F_f = \sigma_s \cdot \Sigma_r$$

    - Σ_r real contact area
    - σ_s shear strength of th

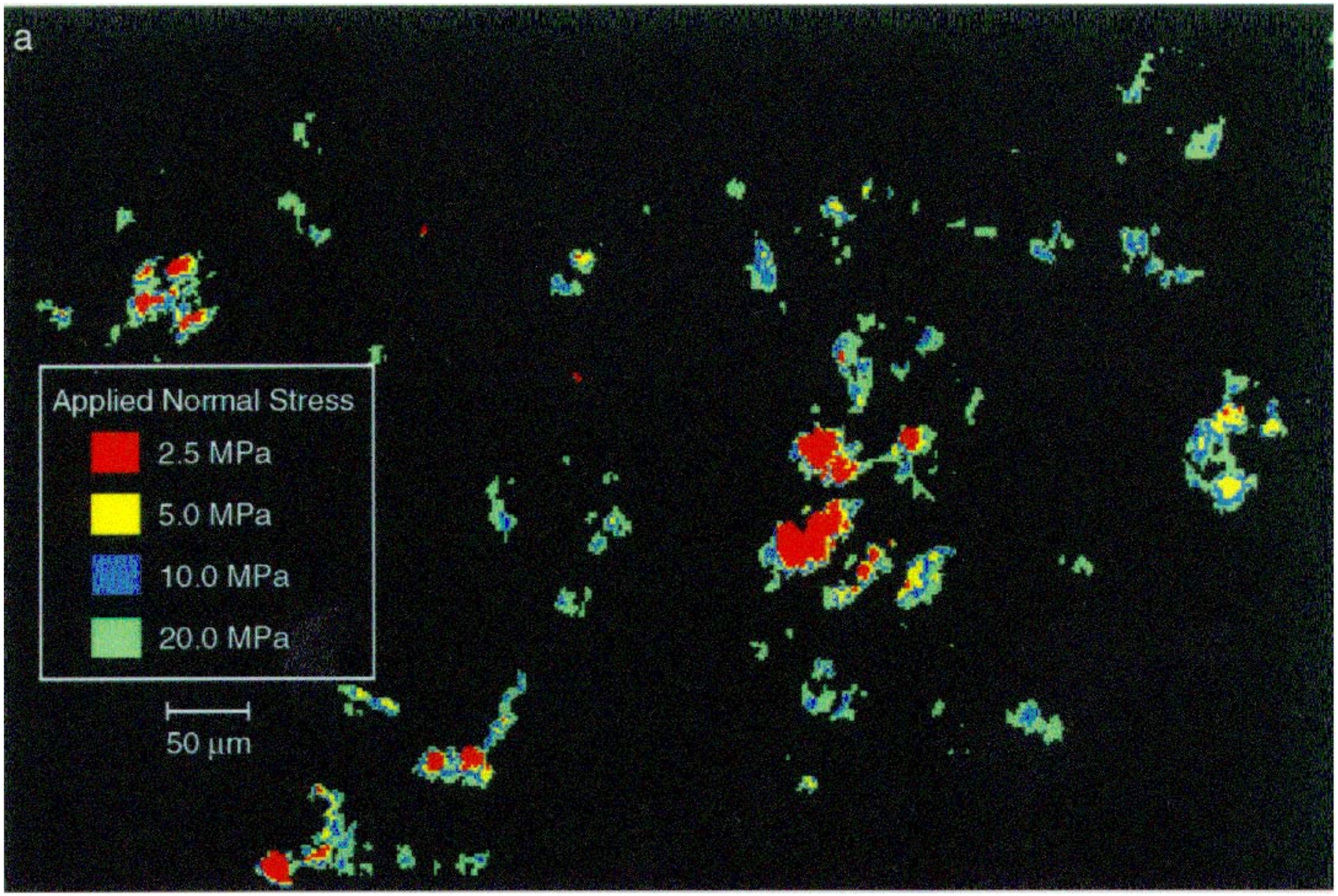


W increases $\Rightarrow$ Σ_r increases (linearly for small W)

$\Rightarrow$ friction force increases

Real Contact Area of Random Surfaces

Confirmed with optical images by Dieterich-Kilgore (1994)

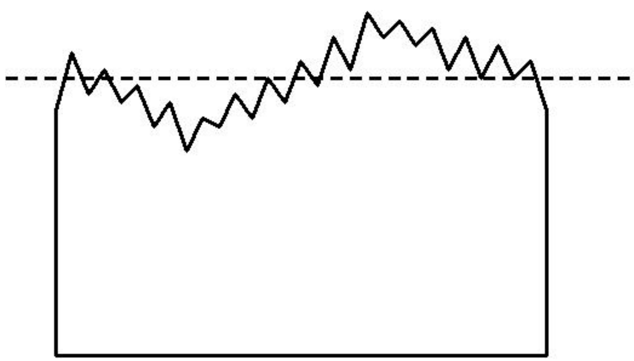


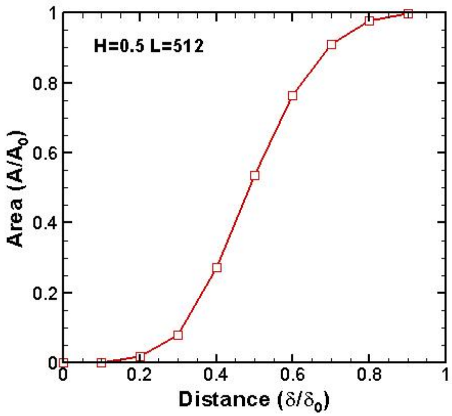
Theoretical Models

For $\Sigma_r \propto W$

Hertz: elastic contact of elastic sphere (no friction, no adhesion) $\Sigma_r \propto W^{\frac{2}{3}}$

Simplest model for roughness: the overlap (or cutoff) model (no mechanics)





Overlap model fails to capture linearity

Linearity captured (many asperities of various heights + mech):

- Greenwood and Williamson, 1966 (distribution of heights of spheres);
- Bush et al., 1975 (distribution of radii and aspherical asperities);
- Persson, 2001 (self-affine surfaces);
- Borri-Brunetto et al., 2001, many others…

Self-Affine Fractal Surfaces

- Surface roughness power spectra

$$C(q) = \frac{1}{(2\pi)^2} \int d^2 r \langle h(r+r')h(r') \rangle e^{-iq.r}$$

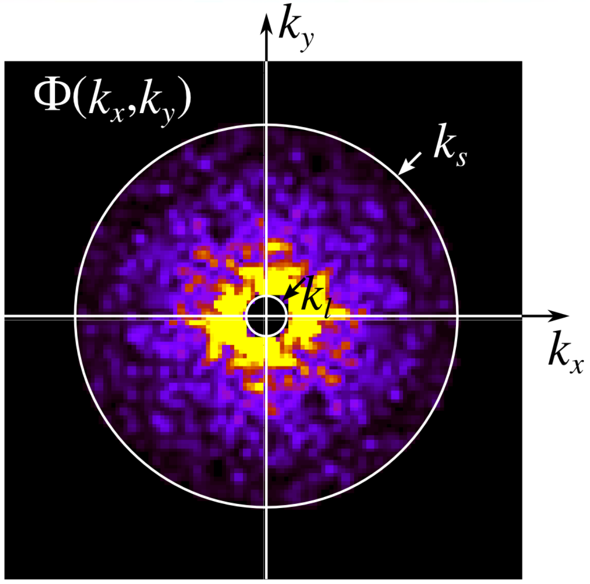
$$z = h(x)$$

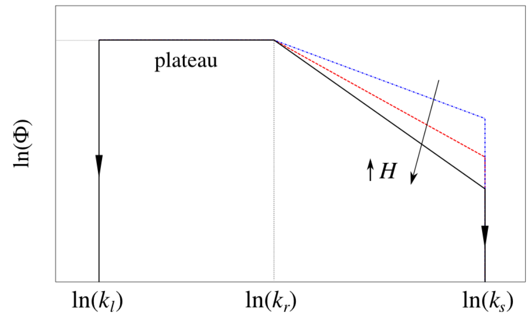
$$\lambda_0 = 2\pi / q_0$$

$$q < q_0, C(q) = C_0$$

$$q > q_0, C(q) = C_0 \left(\frac{q}{q_0} \right)^{-2(H+1)}$$

- Variations in height over a lateral length scale of L rise as L^H
- (H<1, means surfaces look smooth macroscopically)





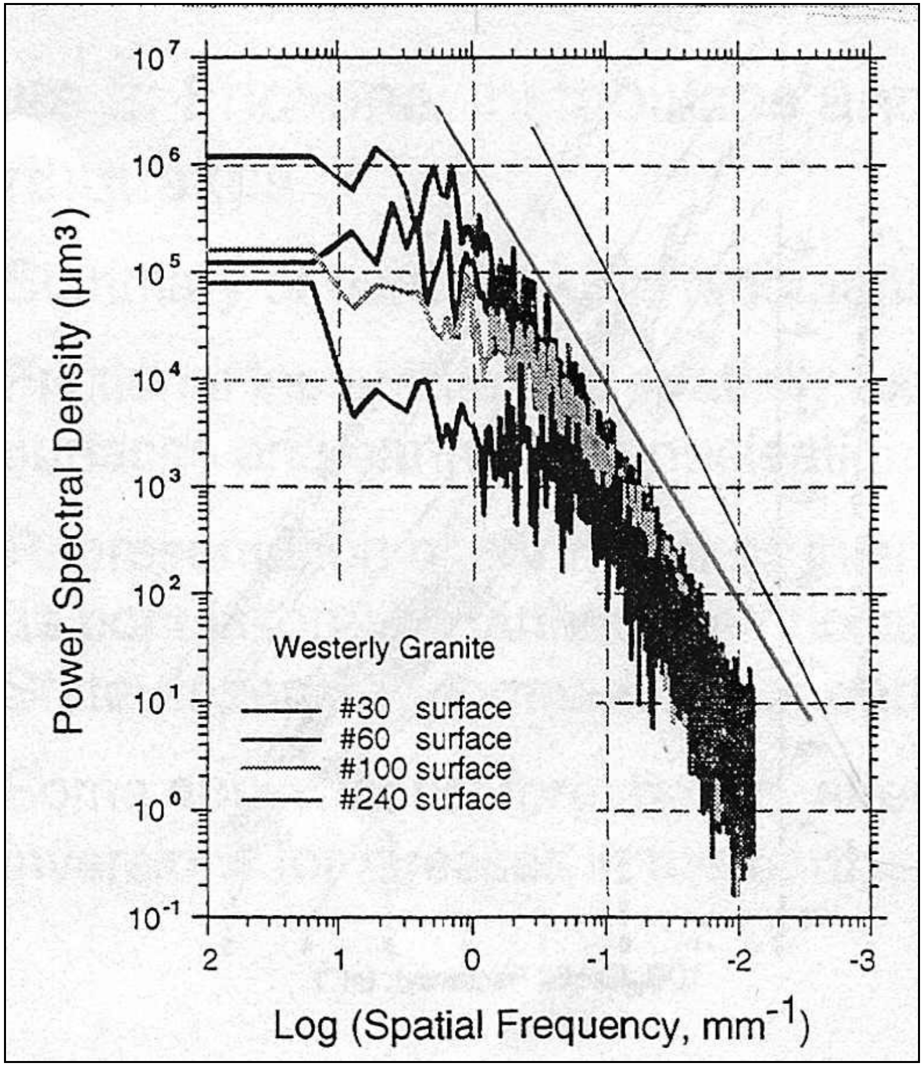
Cutoffs in spectrum

Isotropic (versus anistropic surface)

Figure from Yastrebov

Comparison with Experiments

Natural surfaces in comparison to self-affine fractal surfaces (Dieterich)

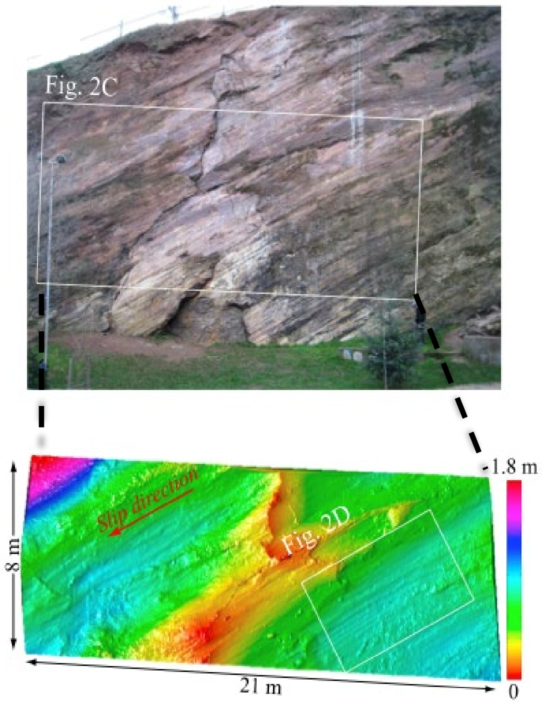


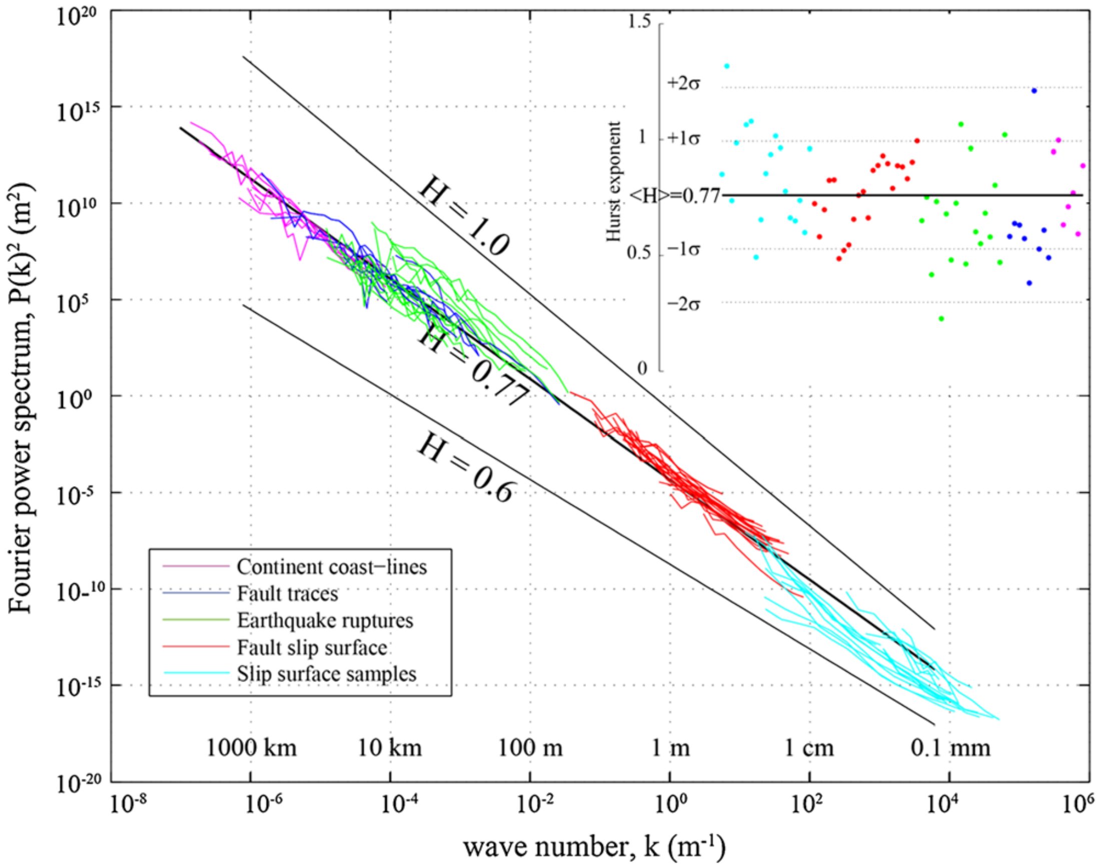
Roughness of natural surfaces

Self-affine fractals

$$C(\mathbf{q}) = \frac{1}{2\pi}\int h(\mathbf{x})h(\mathbf{0})e^{-i\mathbf{q}\cdot\mathbf{x}}d\mathbf{x} \qquad C(\mathbf{q}) \sim \mathbf{q}^{-2(H+1)}$$

H<1 Hurst exponent

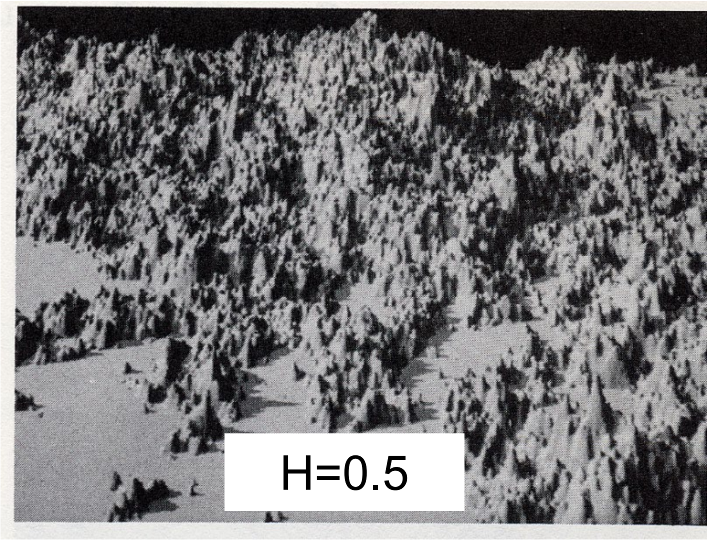


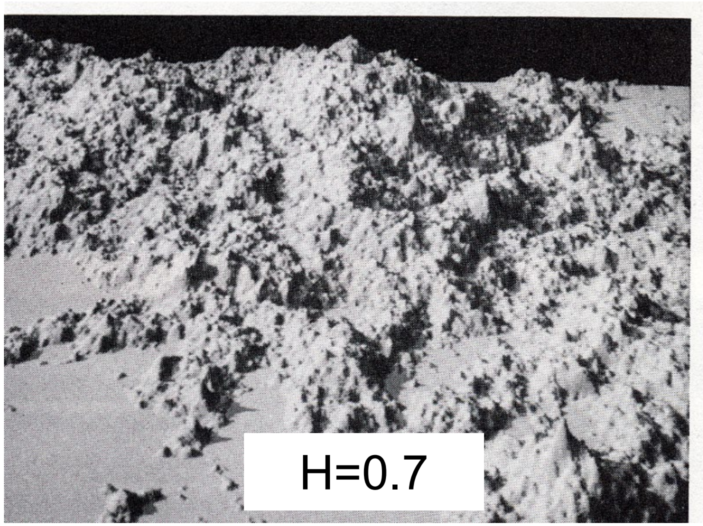


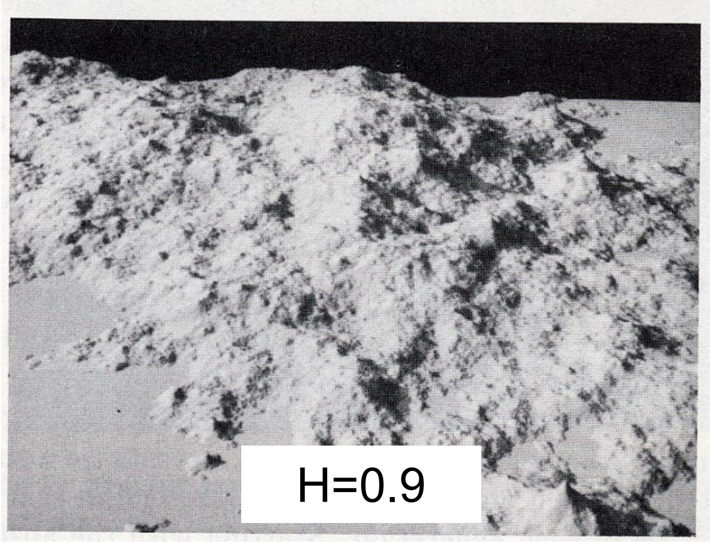
Renard et al., 2013, Geophysical Research Letters

Scaling over 10 decades! (from 0.1 mm to 1000 km)

Self-Affine Fractal Surfaces







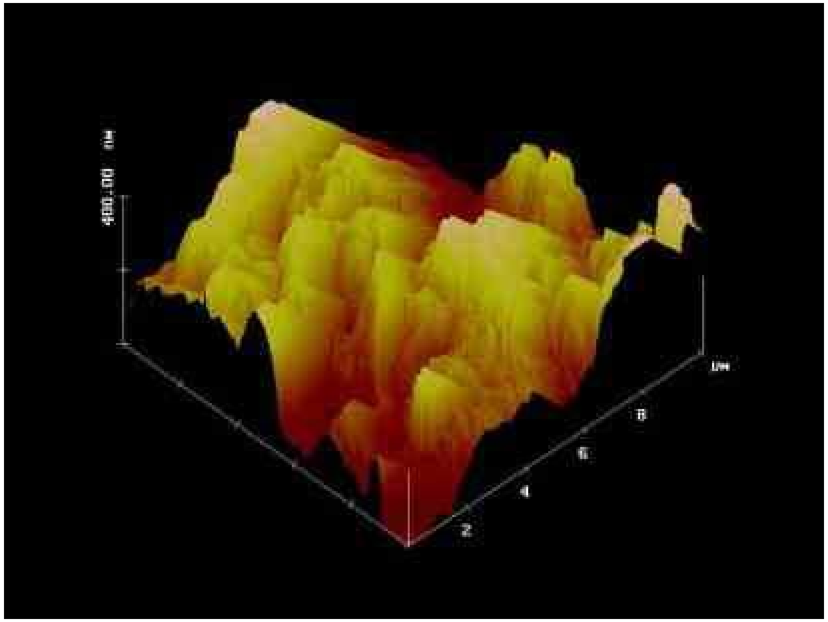
H: Hurst
(or roughness)
exponent

From Boger et al., 1987

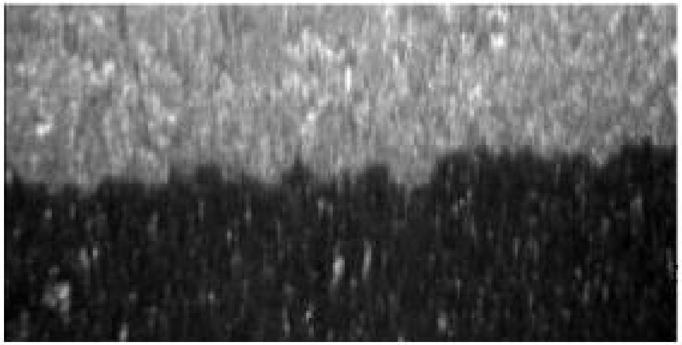
More Examples

H=0.8 (www.phys.ntnu.no)

Mount Everest



10x10 micrometer^2 AFM image of clay surface



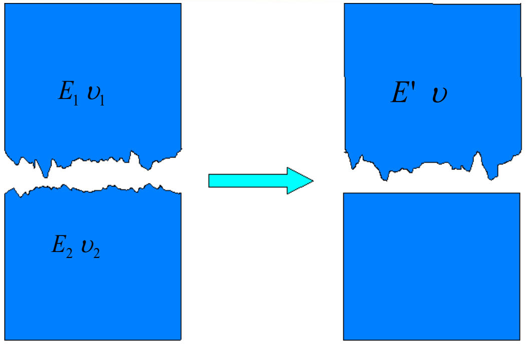
Propagation of a crack in plexiglas block with artificially controlled thickness

Continuum Mechanics

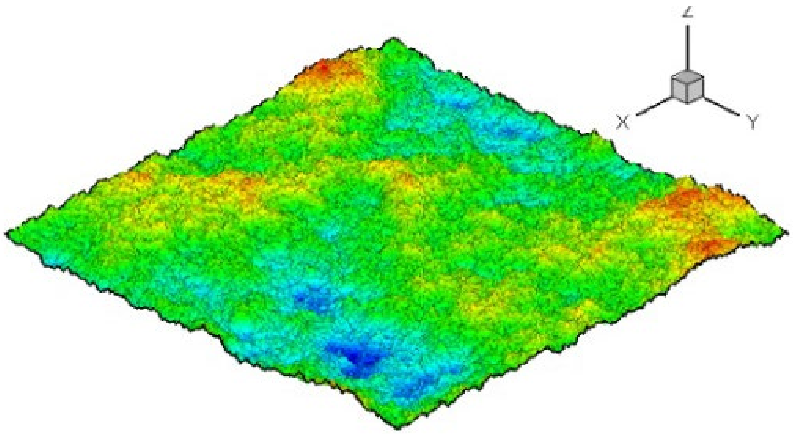
Early finite element simulations

Elastic or J2 elasto/perfectly-plastic solid with a rough surface contacting a rigid surface
L = 512 nodes per side; **periodic BCs**
Full range of H and roughness amplitude

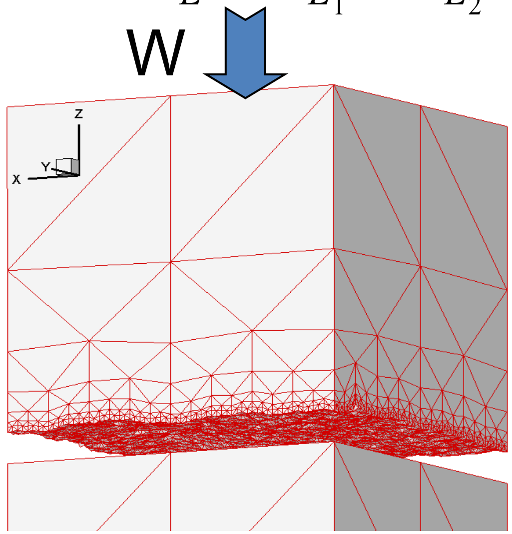
No Friction
No plasticity
(Johnson, 1985)



$$\frac{1-\upsilon^2}{E'} = \frac{1-\upsilon_1^{\,2}}{E_1} + \frac{1-\upsilon_2^{\,2}}{E_2}$$



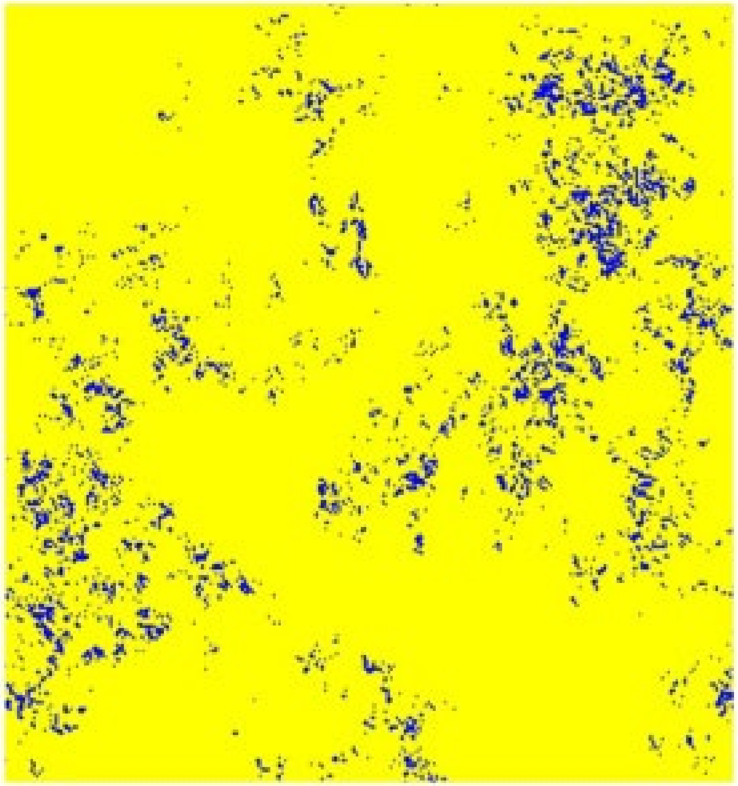
H=0.7 (D = 3 - H = 2.3), 256x256 grid; generated by successive Random Addition Rule (RF Voss)



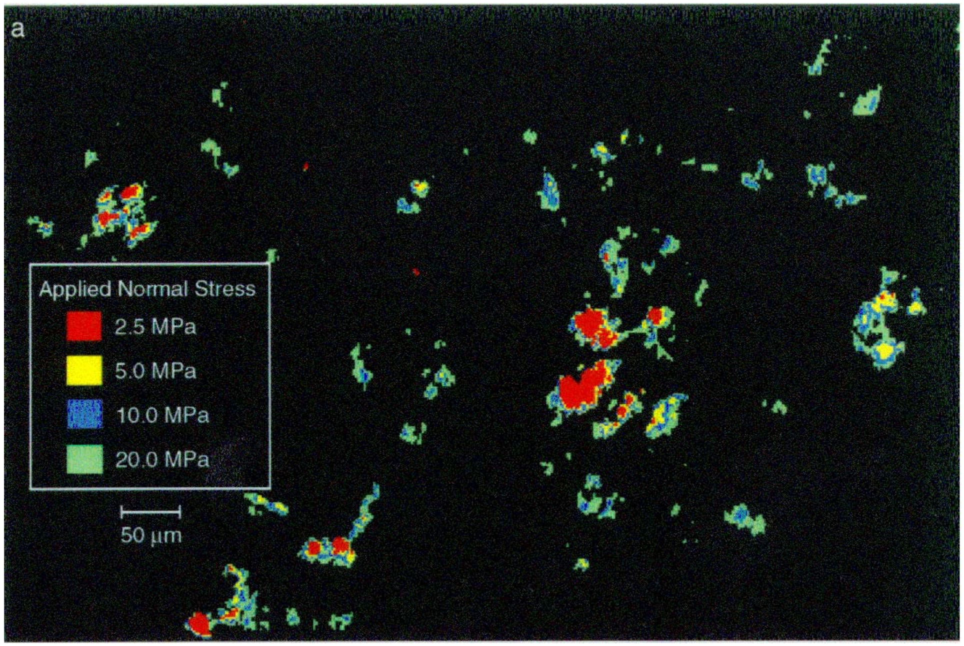
- S. Hyun, L. Pei, J.F. Molinari, and M.O. Robbins, “Finite-element analysis of contact between **elastic** self-affine surfaces”, *Phys. Rev. E*, 2004
- L. Pei, S. Hyun, J.F. Molinari, and M.O. Robbins, “Finite-element analysis of contact between **elasto-plastic** self-affine surfaces”, *Journal of the Mechanics and Physics of Solids, 2006*

Quick overview of results

Complex contact morphology



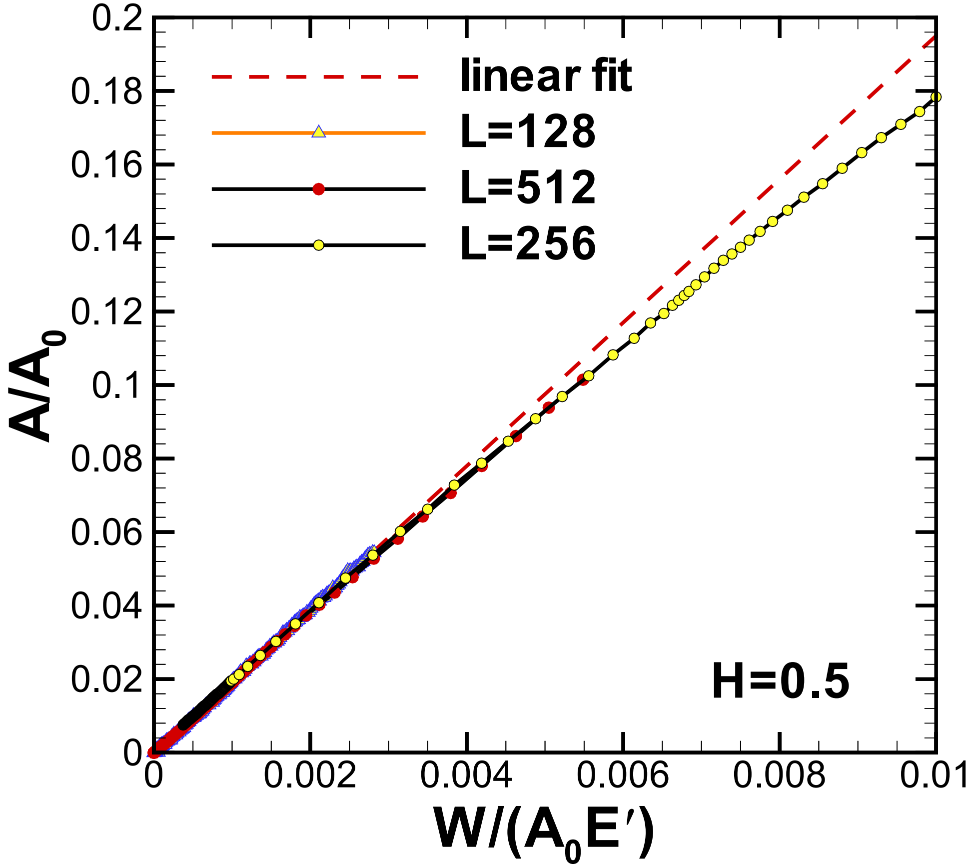
Contact contour, L=256, H=0.5, Δ=0.08, ν=0, A/A_0=0.1

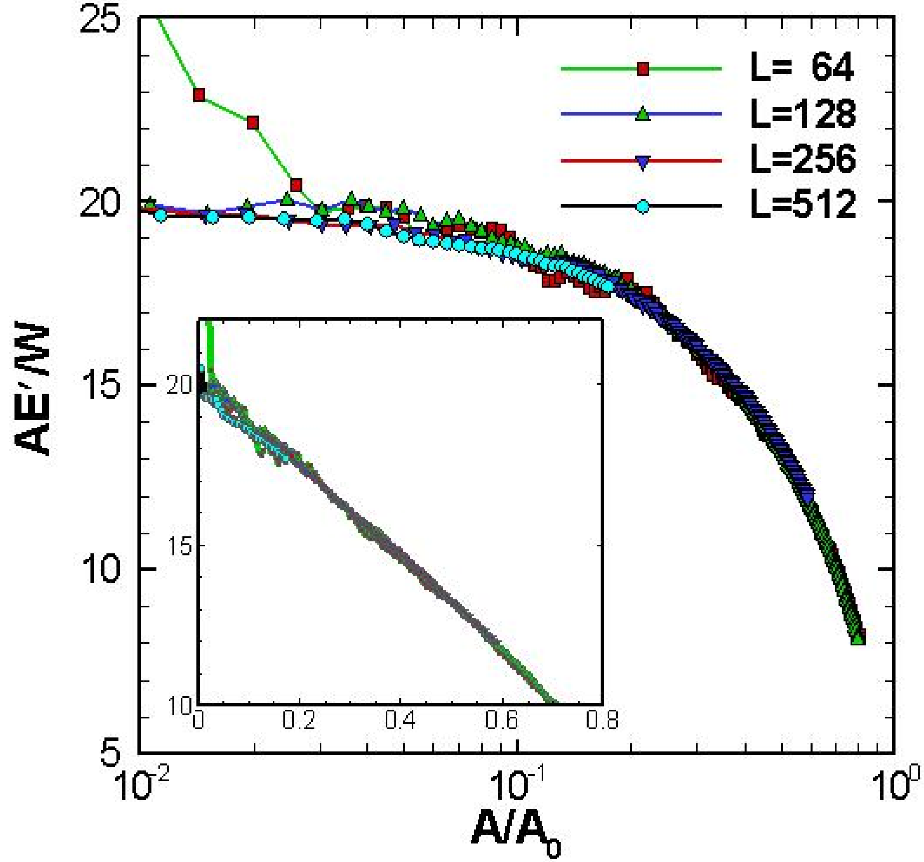


Complex Contact Geometry:
J.H. Dieterich and B.D. Kilgore, *Tectonophysics* 256, 219 (1996). Image contacts between self-affine surfaces of transparent materials (quartz, calcite, soda-lime glass, acrylic plastic).

Quick overview of results

Real contact area versus load

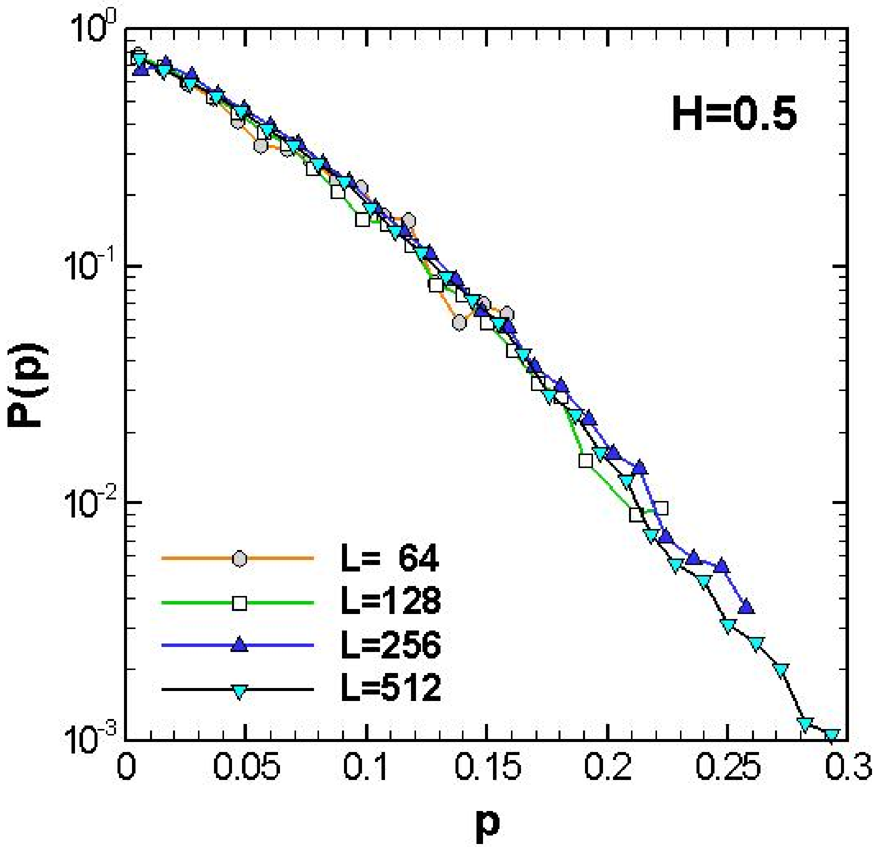


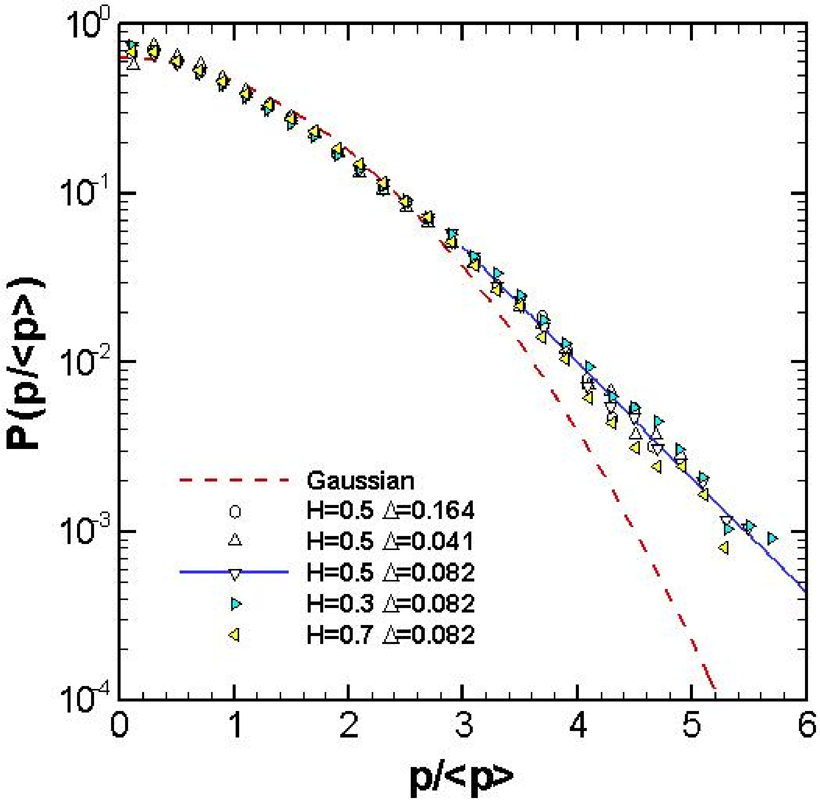


- A proportional to load (consistent with theory)
- AE'/W constant up to 8% $\Rightarrow$ In this domain, mean pressure constant
- Slope depends on roughness parameters (unlike analytical theories)

Quick overview of results

Statistics of micro contacts



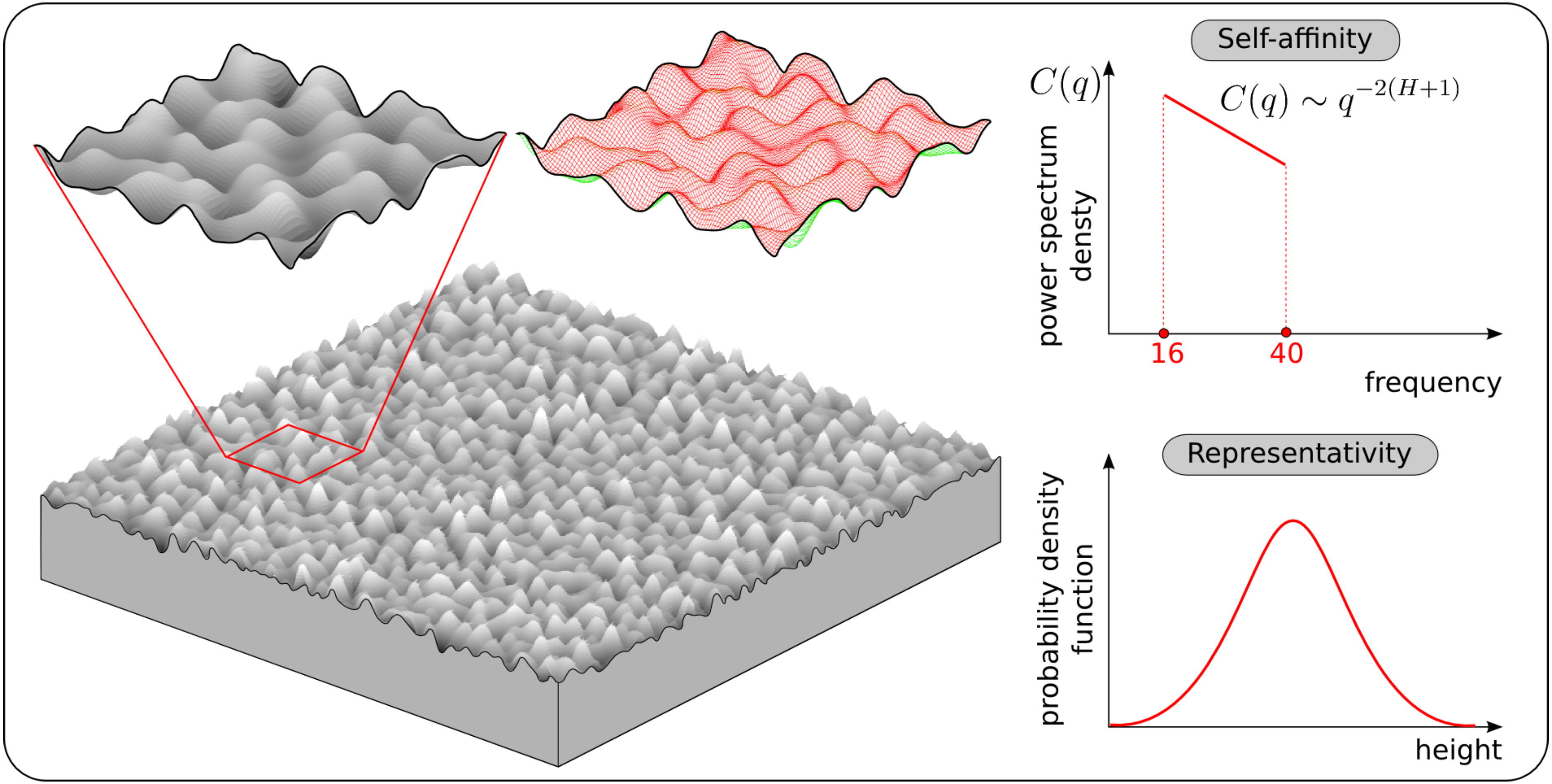


Universal curve (for all H and Δ)!

Exponential decay, not Gaussian ⇒ we predict larger pressures than previous models (may reflect correlations in the loads carried by different asperities, which is not captured in analytical theories)

Smooth representative surfaces

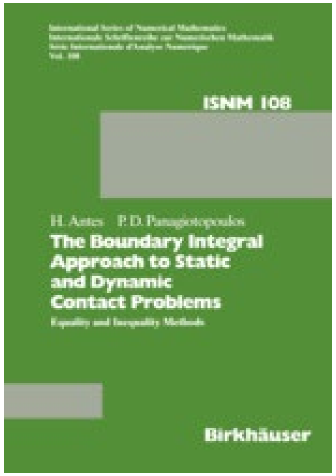
Yastrebov Anciaux Molinari IJSS 2015



Requires many asperities at many scales, and a fine discretization…

Change gun: the BEM

Boundary Element Method or Boundary Integral Equation Method



Building blocks:

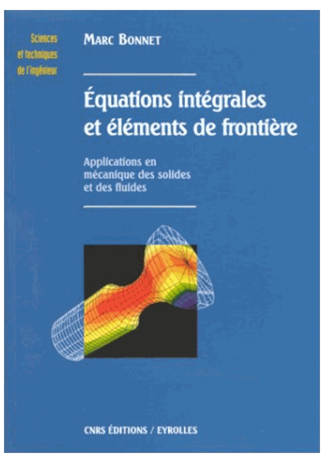
1) Integral representation (to obtain solution at any point in the interior from solution on the boundary)
2) Fundamental solutions (Boussinesq, Westergaard, …)
3) If complex domain, discretization of boundary for numerical solution

Advantages:

Concentrate discretization at the surface (reduction of dimension)

High Precision

Efficiency



Disadvantages:

Hard to include heterogeneities in bulk (and loss of reduction of dimension)

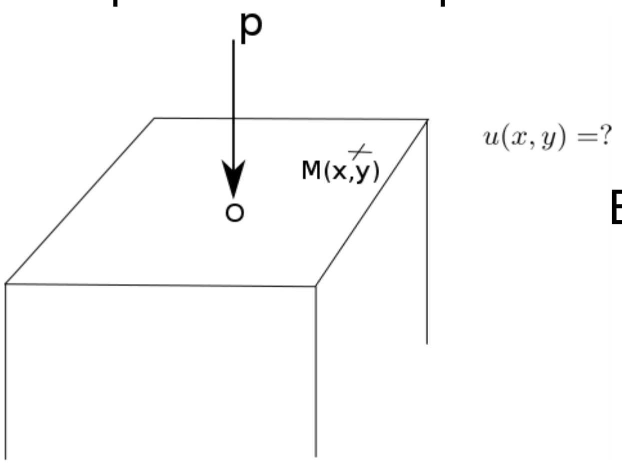
Linear elasticity: hard to include non-linear behavior in bulk

(FEM better for this)

Fundamental solutions

For contact problems

Displacement at a point M due to a normal load applied at O



Boussinesq's solution : $u(x,y) = \frac{2p(1-2\nu)}{4\pi G\sqrt{x^2+y^2}}$

Westergaard's (1939, 1D contact line) and Johnson's (1985, 2D contact plane) solution:

$$p(x,y) = \cos\left(\frac{2\pi n x}{L}\right)\cos\left(\frac{2\pi m y}{L}\right)$$

$$u(x,y) = \frac{2}{E^*\sqrt{n^2+m^2}}p(x,y)$$

Minimization problem

Under constraints

Find p and u admissible such that :

$$\frac{1}{2}\int_S pudS$$ is minimum

and $p \geq 0$ (contact constraint)

and $\int_S pdS = p_{target}$ (loading)

Discretization with a uniform grid

Periodic solution for FFT (requires periodic asperities);
Convolution in Fourier space is a product

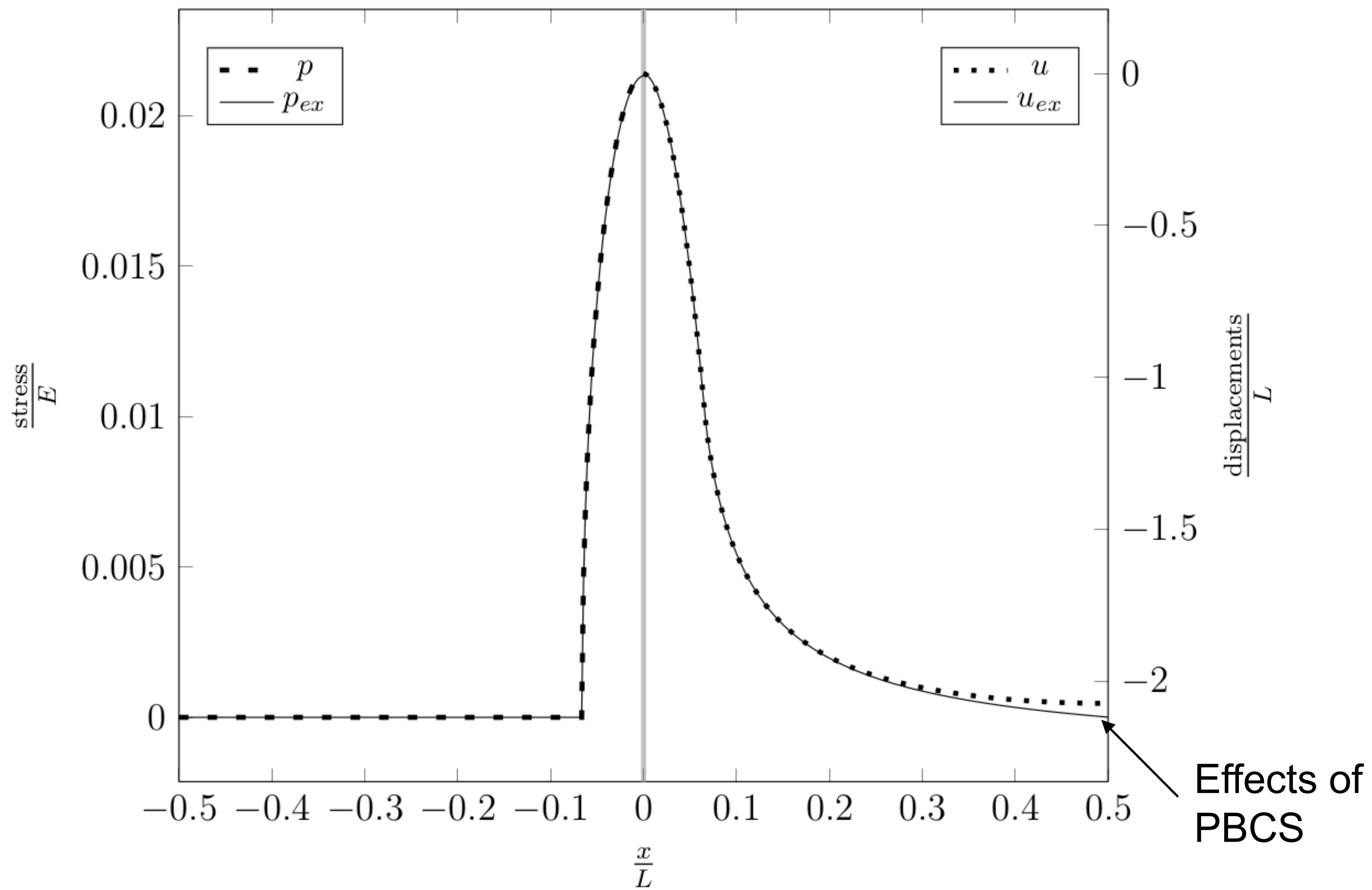
Relation between displacement and pressure is computed in Fourrier's space

$$\mathbf{u} = \mathrm{FFT}^{-1}(\mathbf{K}(\mathrm{FFT}(\mathbf{p})))$$

Minimize energy under constraints: Stanley and Kato, Polonsky and Keer

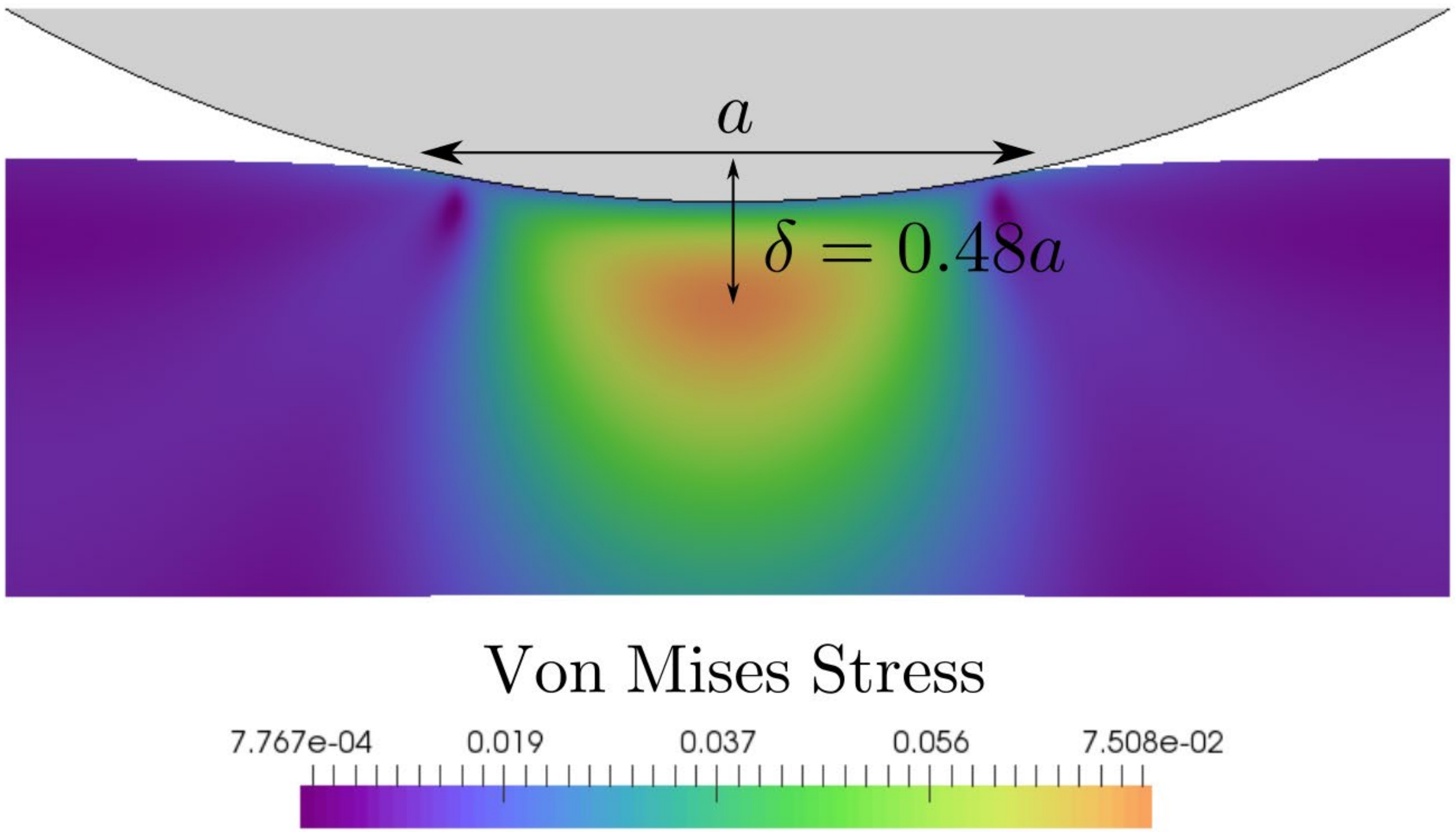
Hertz contact

Comparison between computed and exact solutions



Pressure of Hertz contact

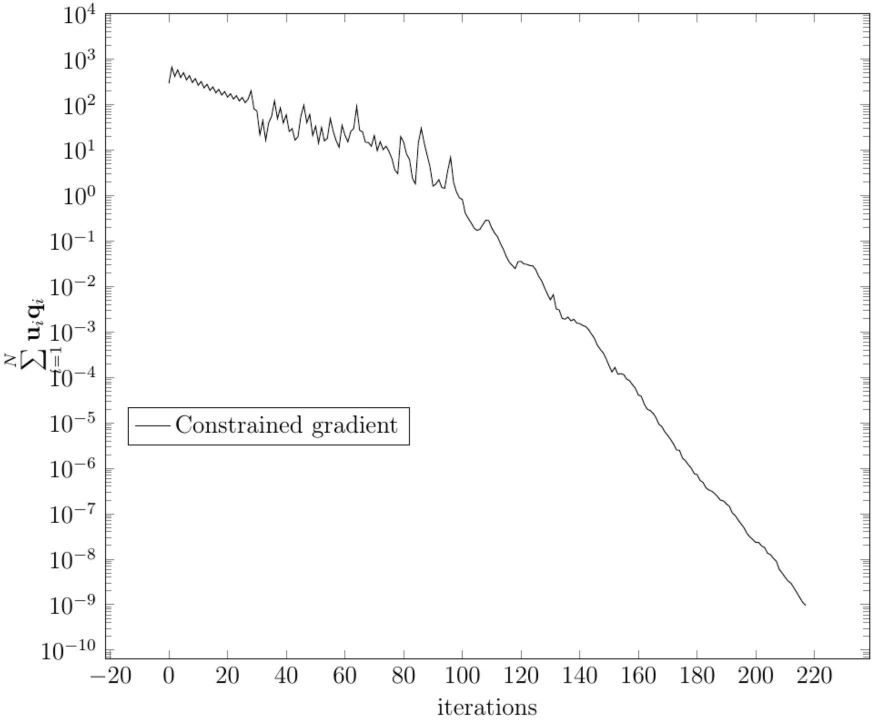
Solution can be computed at any point



Convergence of solution

grid size	64x64	128x128	256x256	512x512	1024x1024	2048x2048
η	$1.40\ 10^{-2}$	$4.08\ 10^{-3}$	$7.78\ 10^{-4}$	$3.66\ 10^{-4}$	$1.20\ 10^{-4}$	$1.66\ 10^{-5}$

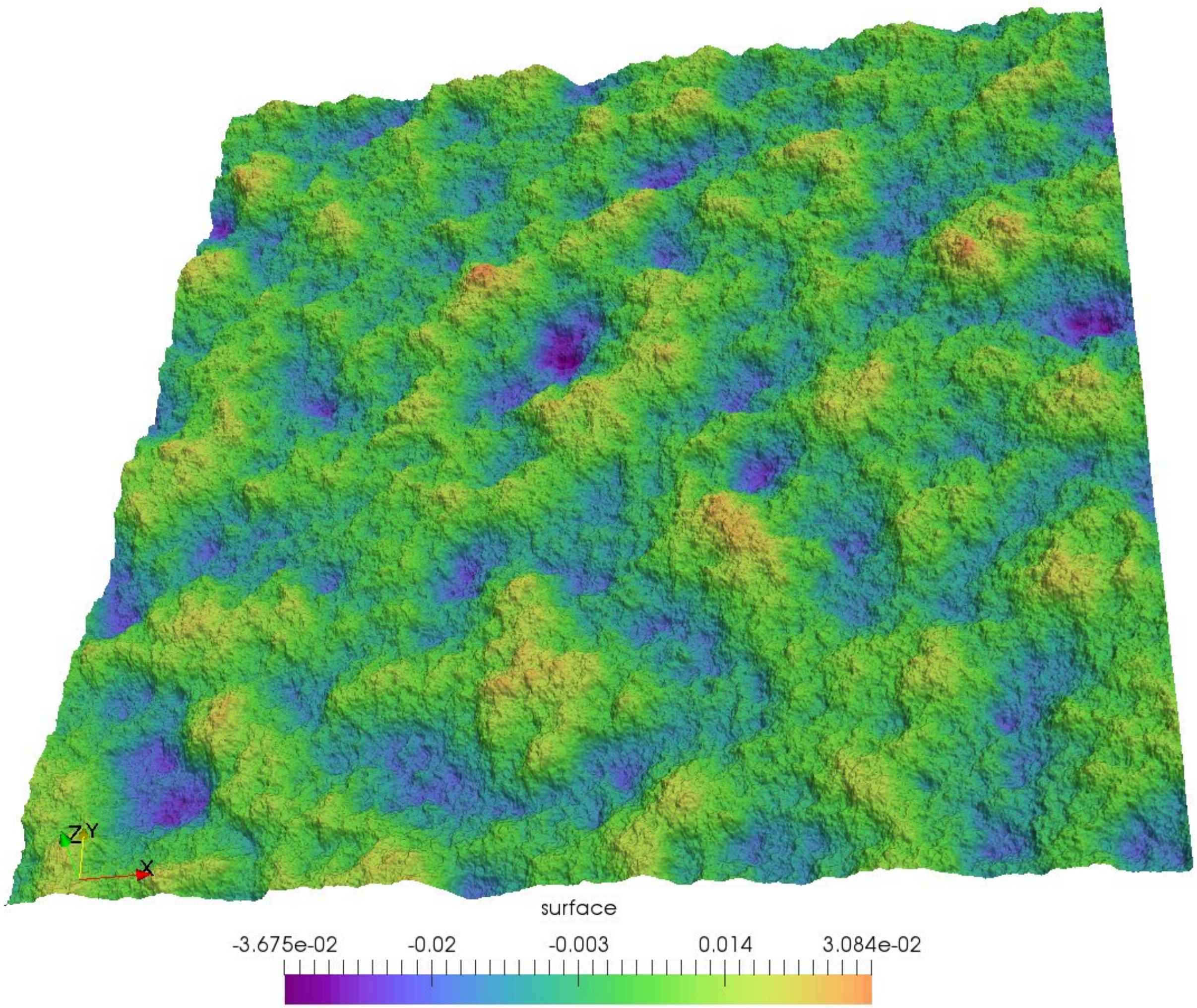
Table 1: Error in energetic norm between computed and exact Hertz solution for different grid sizes: 64x64 grid already gives satisfying results. Numerical solution tends to the analytical solution when l tends to 0.



$$\eta^2 = \frac{\int\limits_S (u - u_{ex})(p - p_{ex})dS}{\int\limits_S u_{ex} p_{ex} dS}$$

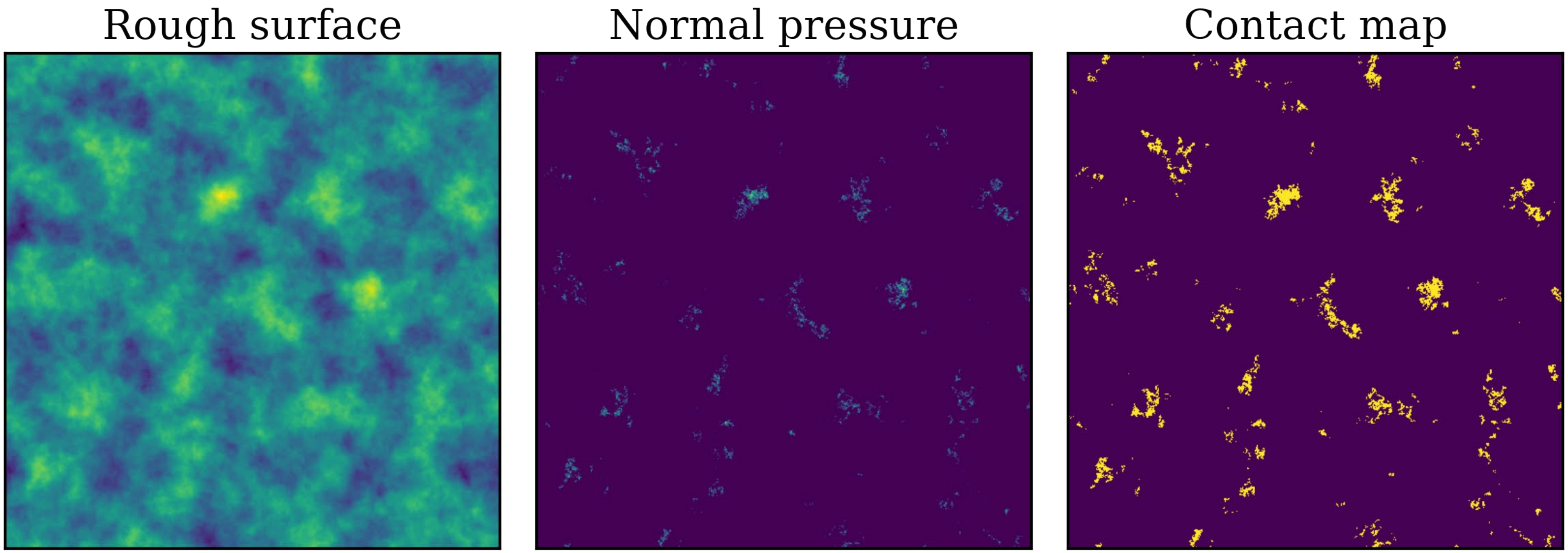
Rough surface

3D view



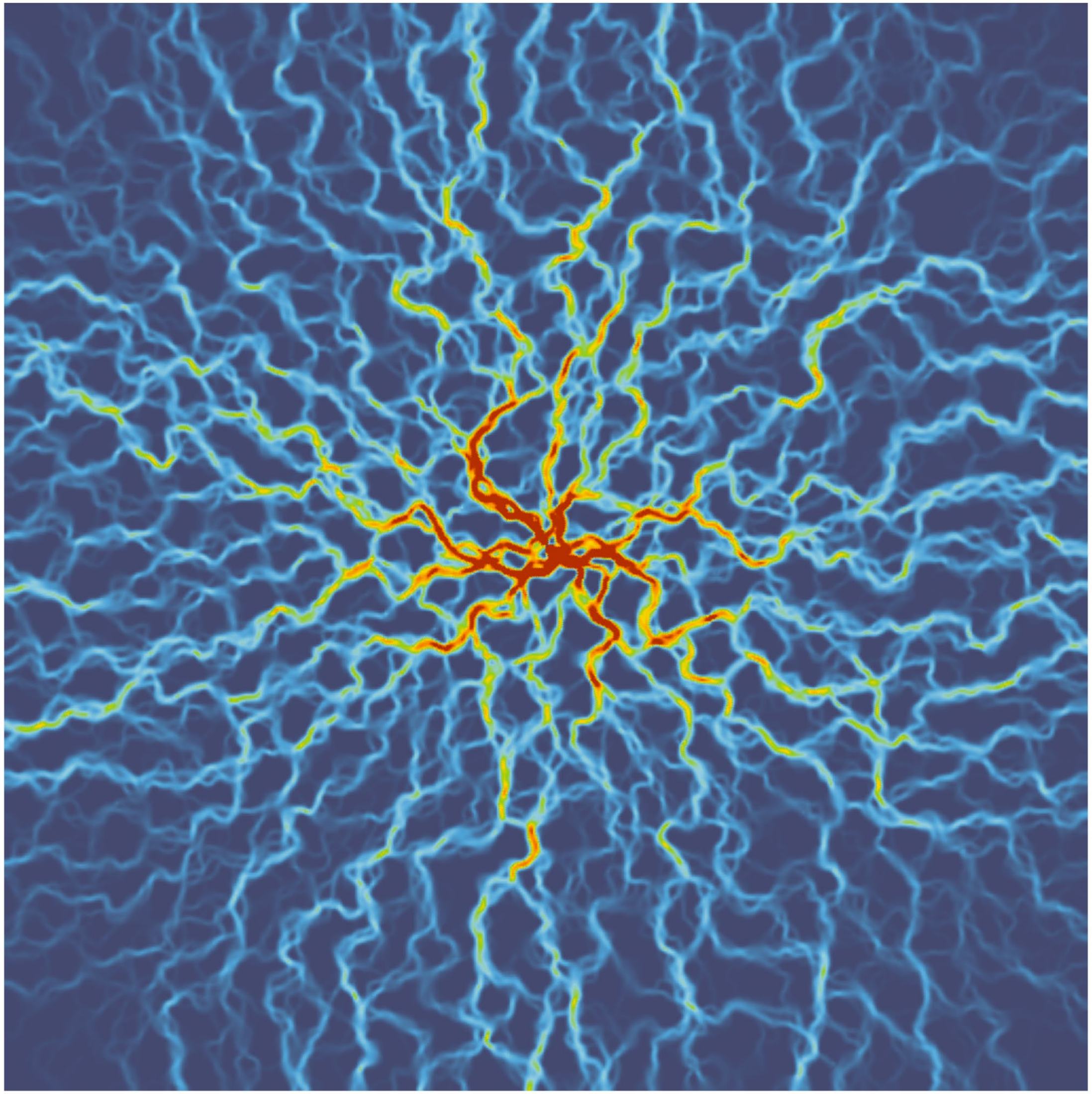
Rough contact

Solution of simulations



Fluid injection at a rough surface

Yastrebov et al. in preparation



Summary comments

- Roughness; self-affine roughness
- Hertz contact theory
- Multi-asperity contact
- Power of BEM for mesoscale modeling of rough surfaces: access to statistical data of micro contact sizes and local pressures
- Limitation of BEM: difficulty of including non-linearity and heterogeneities