

CLOSE TO COLLAPSE



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Last week: Postulates

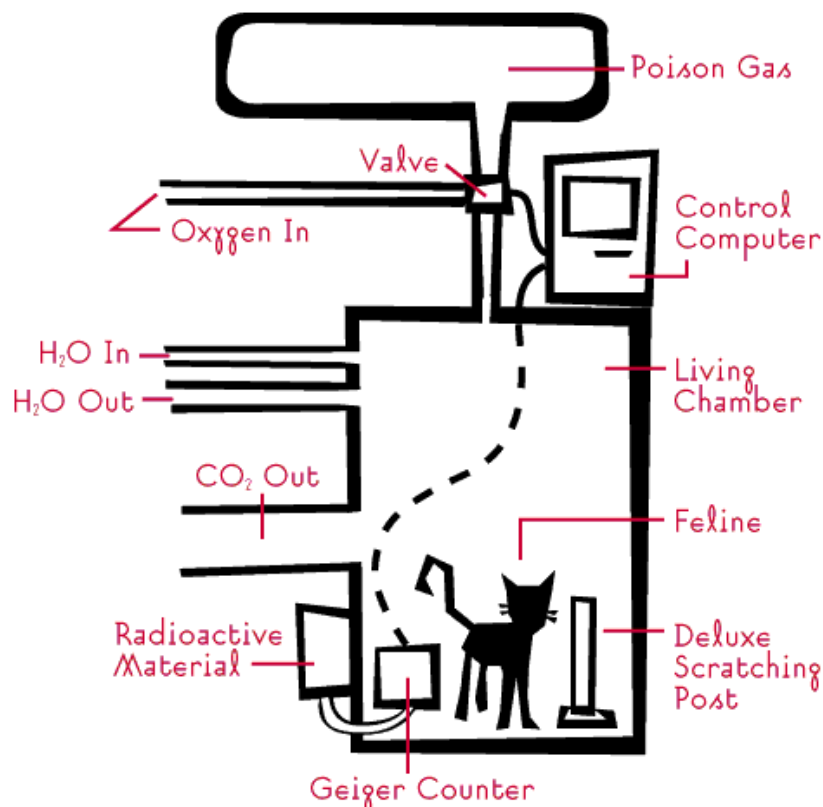
- Operators, eigenvalues, eigenfunctions, expectation values
- Linear operators, Hermitian operators
- Products of operators, and commutators
- Commuting operators and common eigenfunctions
- The five postulates of quantum mechanics
 - I. **Wavefunctions**
 - II. **Operators**
 - III. **Measurements**
 - IV. **Expectation values, probabilities**
 - V. **...**

Fifth postulate: collapse

- If the measurement of the physical quantity A gives the result a_n , the wavefunction of the system immediately after the measurement is the eigenvector $|\varphi_n\rangle$

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When scientists turn bad...



Cat wavefunction

$$|\Psi_{cat}(t)\rangle = |\Psi_{alive}\rangle \left(\exp\left(-\frac{t}{\tau}\right) \right)^{\frac{1}{2}} + |\Psi_{dead}\rangle \left(1 - \exp\left(-\frac{t}{\tau}\right) \right)^{\frac{1}{2}}$$

- There is not a value of the observable until it's measured (a conceptually different “statistics” from thermodynamics)

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Reimagining of Schrödinger's cat breaks quantum mechanics — and stumps physicists

In a multi-‘cat’ experiment, the textbook interpretation of quantum theory seems to lead to contradictory pictures of reality, physicists claim.

Daide Castelvecchi

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Top Three List

- **Albert Einstein:** *“Gott wurfelt nicht!” [God does not play dice!]*
- **Werner Heisenberg** *“I myself . . . only came to believe in the uncertainty relations after many pangs of conscience. . .”*
- **Erwin Schrödinger:** *“Had I known that we were not going to get rid of this damned quantum jumping, I never would have involved myself in this business!”*

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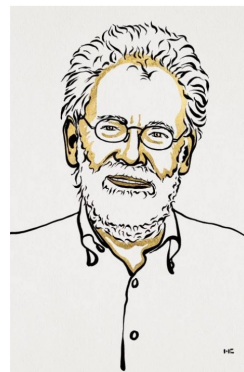
The Nobel Prize in Physics 2022



III. Niklas Elmehed © Nobel Prize Outreach
Alain Aspect
Prize share: 1/3



III. Niklas Elmehed © Nobel Prize Outreach
John F. Clauser
Prize share: 1/3



III. Niklas Elmehed © Nobel Prize Outreach
Anton Zeilinger
Prize share: 1/3

The Nobel Prize in Physics 2022 was awarded jointly to Alain Aspect, John F. Clauser and Anton Zeilinger "for experiments with entangled photons, establishing the violation of Bell inequalities and pioneering quantum information science"

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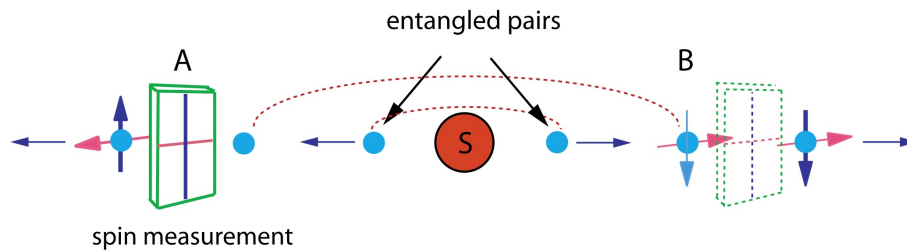
Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in

quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.



<https://www.nobelprize.org/uploads/2022/10/advanced-physicsprize2022.pdf>

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Uncertainties, and Heisenberg's Indetermination Principle

$$(\Delta A)^2 = \langle (A - \langle A \rangle)^2 \rangle = \langle A^2 \rangle - \langle A \rangle^2$$

$$\Delta A \Delta B \geq \frac{1}{2} |\langle [A, B] \rangle|$$

$$\left[x, -i\hbar \frac{d}{dx} \right] = i\hbar$$

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Linewidth Broadening

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$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

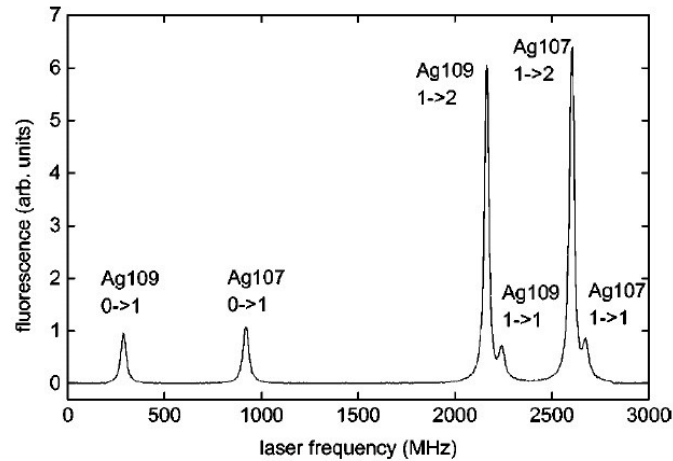


FIG. 2. The $^2S_{1/2}-^2P_{3/2}$ spectrum, obtained with a frequency-doubled diode laser aligned perpendicular to the thermal atomic beam.

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Good Quantum Numbers

$$\frac{d\langle A \rangle}{dt} = \frac{d\langle \Psi | \hat{A} | \Psi \rangle}{dt} = \frac{1}{i\hbar} \langle [\hat{A}, \hat{H}] \rangle$$

If A commutes with the Hamiltonian, and it does not depend on time, its expectation value does not change with time (it's a constant of motion – if we are in an eigenstate, that quantum number will remain constant)

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Variational Principle

$$E[\Psi] = \frac{\langle \Psi | \hat{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

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Variational Principle

$$E[\Psi] = \frac{\langle \Psi | \hat{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

$$E[\Psi] \geq E_0$$

If $E[\Psi] = E_0$, then Ψ is the ground state wavefunction, and viceversa...

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Matrix Formulation (I)

$$\hat{H}|\psi\rangle = E|\psi\rangle$$

$$|\psi\rangle = \sum_{n=1,k} c_n |\varphi_n\rangle \quad \{|\varphi_n\rangle\} \text{ orthogonal}$$

$$\langle\varphi_m|\hat{H}|\psi\rangle = E\langle\varphi_m|\psi\rangle$$

$$\sum_{n=1,k} c_n \langle\varphi_m|\hat{H}|\varphi_n\rangle = E c_m$$

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Matrix Formulation (II)

$$\sum_{n=1,k} H_{mn} c_n = E c_m$$

$$\begin{pmatrix} H_{11} & \dots & H_{1k} \\ \vdots & & \vdots \\ H_{k1} & \dots & H_{kk} \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ \vdots \\ c_k \end{pmatrix} = E \begin{pmatrix} c_1 \\ \vdots \\ c_k \end{pmatrix}$$

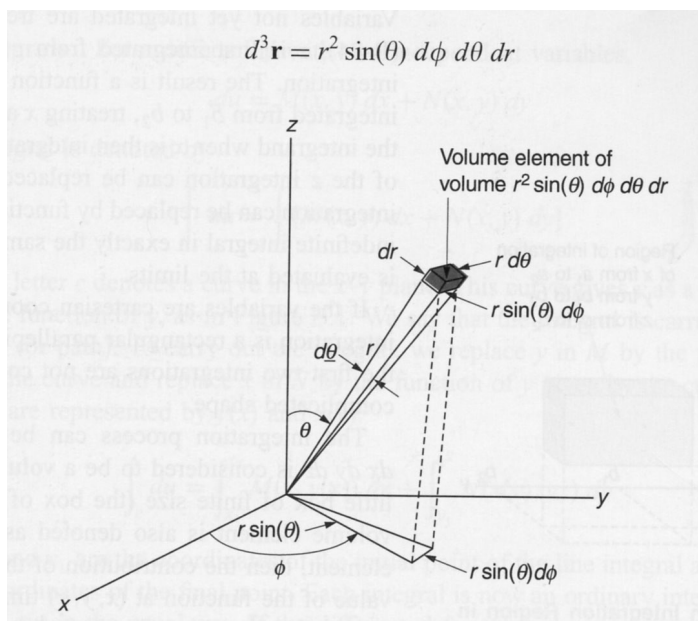
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Matrix Formulation (III)

$$\det \begin{pmatrix} H_{11} - E & \dots & H_{1k} \\ \cdot & H_{22} - E & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ H_{k1} & \dots & H_{kk} - E \end{pmatrix} = 0$$

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Spherical Coordinates



$$x = r \sin \theta \cos \varphi$$

$$y = r \sin \theta \sin \varphi$$

$$z = r \cos \theta$$

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Angular Momentum

Classical

Quantum

$$\vec{L} = \vec{r} \times \vec{p}$$

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Commutation Relation

$$\hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$$

$$\left[\hat{L}^2, \hat{L}_x \right] = \left[\hat{L}^2, \hat{L}_y \right] = \left[\hat{L}^2, \hat{L}_z \right] = 0$$

$$\left[\hat{L}_x, \hat{L}_y \right] = i\hbar \hat{L}_z \neq 0$$

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Angular Momentum in Spherical Coordinates

$$\hat{L}_z = -i\hbar \frac{\partial}{\partial \varphi}$$

$$\hat{L}^2 = -\hbar^2 \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right)$$

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Simultaneous eigenfunctions of L^2 , L_z

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Simultaneous eigenfunctions of L^2 , L_z

$$Y_l^m(\theta, \varphi) = \begin{cases} (-1)^m \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos \theta) e^{im\varphi} & m \geq 0 \\ \sqrt{\frac{2l+1}{4\pi} \frac{(l+m)!}{(l-m)!}} P_l^{-m}(\cos \theta) e^{im\varphi} & m < 0 \end{cases}$$

$$l = 0, 1, 2, \dots \quad m = -l, -l+1, \dots, -1, 0, 1, \dots, l-1, l$$

$$Y_0^0(\theta, \phi) = \frac{1}{(4\pi)^{1/2}}$$

$$Y_1^0(\theta, \phi) = \left(\frac{3}{4\pi}\right)^{1/2} \cos \theta$$

$$Y_1^{\pm 1}(\theta, \phi) = \left(\frac{3}{8\pi}\right)^{1/2} \sin \theta e^{\pm i\phi}$$

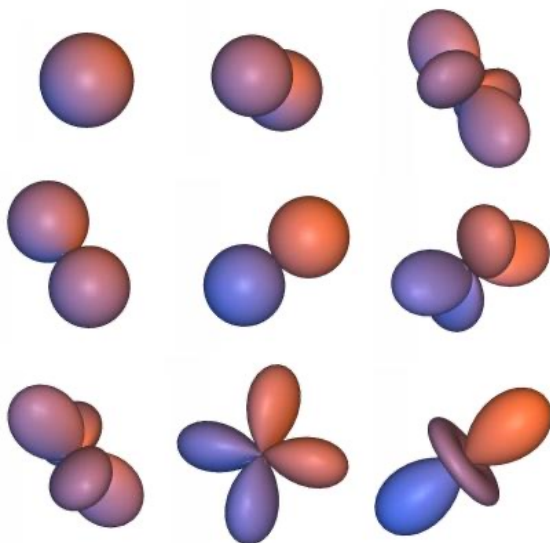
$$Y_2^0(\theta, \phi) = \left(\frac{5}{16\pi}\right)^{1/2} (3 \cos^2 \theta - 1)$$

$$Y_2^{\pm 1}(\theta, \phi) = \left(\frac{15}{8\pi}\right)^{1/2} \sin \theta \cos \theta e^{\pm i\phi}$$

$$Y_2^{\pm 2}(\theta, \phi) = \left(\frac{15}{32\pi}\right)^{1/2} \sin^2 \theta e^{\pm 2i\phi}$$

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Spherical Harmonics in Real Form



$$p_x = \frac{1}{\sqrt{2}} (Y_1^1 + Y_1^{-1}) = \sqrt{\frac{3}{4\pi}} \sin \theta \cos \phi$$

$$p_y = \frac{1}{\sqrt{2} i} (Y_1^1 - Y_1^{-1}) = \sqrt{\frac{3}{4\pi}} \sin \theta \sin \phi$$

$$p_z = Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta$$

$$d_{z^2} = Y_2^0 = \sqrt{\frac{5}{16\pi}} (3 \cos^2 \theta - 1)$$

$$d_{xz} = \frac{1}{\sqrt{2}} (Y_2^1 + Y_2^{-1}) = \sqrt{\frac{15}{4\pi}} \sin \theta \cos \theta \cos \phi$$

$$d_{yz} = \frac{1}{\sqrt{2} i} (Y_2^1 - Y_2^{-1}) = \sqrt{\frac{15}{4\pi}} \sin \theta \cos \theta \sin \phi$$

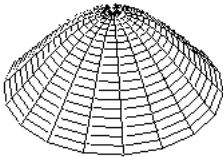
$$d_{x^2-y^2} = \frac{1}{\sqrt{2}} (Y_2^2 + Y_2^{-2}) = \sqrt{\frac{15}{16\pi}} \sin^2 \theta \cos 2\phi$$

$$d_{xy} = \frac{1}{\sqrt{2} i} (Y_2^2 - Y_2^{-2}) = \sqrt{\frac{15}{16\pi}} \sin^2 \theta \sin 2\phi$$

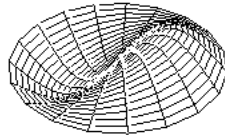
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Same as a beating drum...

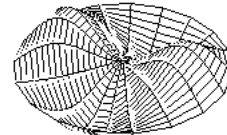
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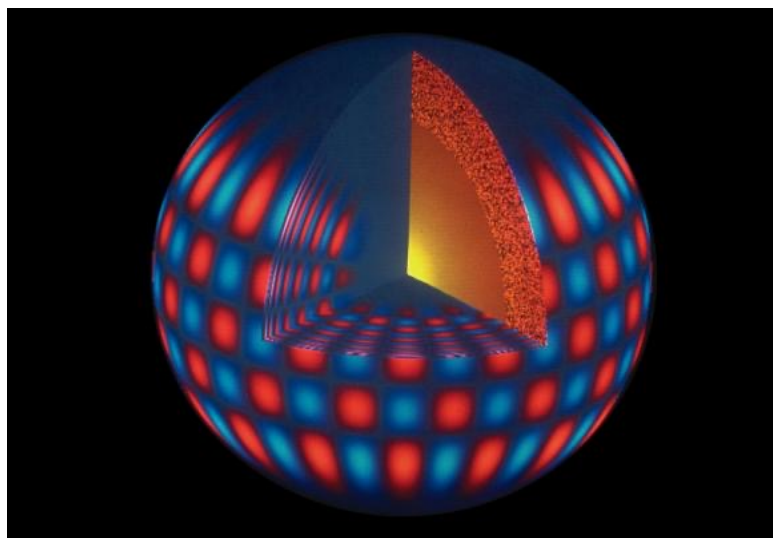


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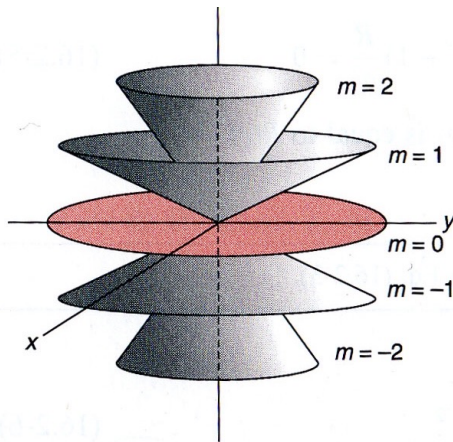
...for the career helioseismologist



Normal modes (i.e. sound, or seismic waves) for the Sun (basically jello in a 3d spherical box)

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Angular Momentum, then...



$$L^2 = l(l+1)\hbar^2 = 0, \quad 2\hbar^2, \quad 6\hbar^2 \dots$$

$$L_z = 0, \quad \pm\hbar, \quad \pm 2\hbar, \quad \pm 3\hbar \dots$$

Figure 16.4. Cones of Possible Angular Momentum Directions for $l=2$. These cones are similar to the cones of precession of a gyroscope, and represent possible directions for the angular momentum vector. The z component is arbitrarily chosen as the one component that can have a definite value.

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An electron in a central potential (I)

$$\hat{H} = -\frac{\hbar^2}{2\mu} \nabla^2 + V(\vec{r}) \quad \nabla^2 \text{ needs to be in spherical coordinates}$$

$$\hat{H} = -\frac{\hbar^2}{2\mu} \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \vartheta} \frac{\partial}{\partial \vartheta} \left(\sin \vartheta \frac{\partial}{\partial \vartheta} \right) + \frac{1}{r^2 \sin^2 \vartheta} \frac{\partial^2}{\partial \varphi^2} \right] + V(r)$$

An electron in a central potential (II)

$$\hat{H} = -\frac{\hbar^2}{2\mu} \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{d}{dr} \right) + \frac{\hat{L}^2}{2\mu r^2} + \hat{V}(r)$$

$$\psi_{nlm}(\vec{r}) = R_{nlm}(r) Y_{lm}(\vartheta, \varphi)$$

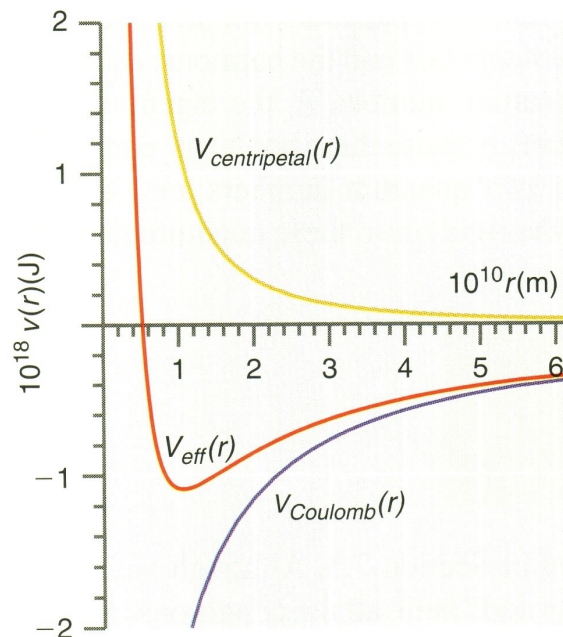
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An electron in a central potential (III)

$$u_{nl}(r) = r R_{nl}(r) \quad V_{eff}(r) = \frac{\hbar^2}{2\mu} \frac{l(l+1)}{r^2} - \frac{Ze^2}{4\pi\epsilon_0 r}$$

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What is the $V_{\text{eff}}(r)$ potential ?



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The Radial Wavefunctions for Coulomb $V(r)$

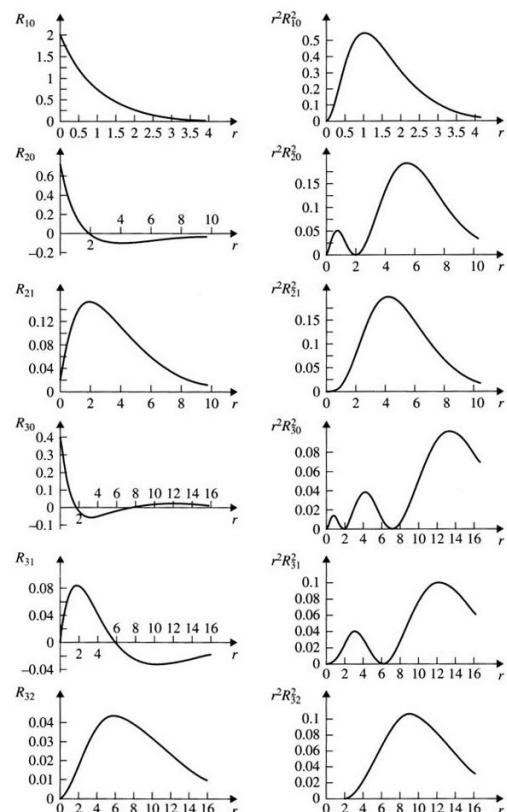
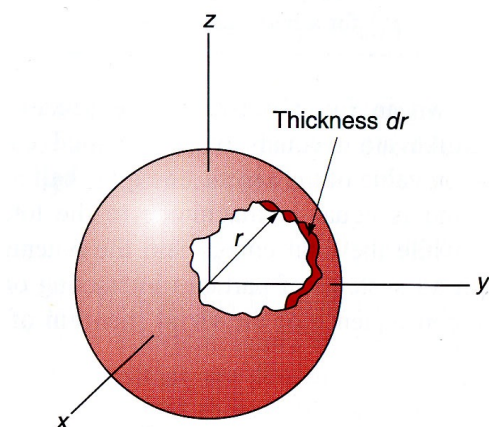


Figure 7.9 Radial functions $R_{nl}(r)$ and radial distribution functions $r^2 R_{nl}^2(r)$ for atomic hydrogen. The unit of length is $a_\mu = (m/\mu)\alpha_0$, where α_0 is the first Bohr radius (1.66).

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