Homework # 3

Exercise 1

In this exercise, we delve into matrix mechanics, another formulation of quantum physics developed by Werner Heisenberg, Max Born, and Pascual Jordan in 1925. Heisenberg's pivotal contributions [1] to this framework laid the foundation for understanding quantum phenomena through matrices, which is entirely analogous to Schrödinger's wave formulation.

A) Consider an operator $\hat{A}$ in the two-dimensional Hilbert space, which in the matrix representation has the form:

$$A = E \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix},$$

where E is a real and positive constant.

Determine the eigenvalues and eigenvectors of $\hat{A}$.

Can $\hat{A}$ be associated with an observable quantity? If yes, then suppose that a measure of $\hat{A}$ gives as a result one of its eigenvalues: what is the wave function of the system just after the measurement?

B) Consider a "two-states" system which is charecterized by the Hamiltonian $\hat{H}$ which in the matrix representation reads:

$$H = E_0 \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

where E_0 is a real and positive constant.

Do the operators $\hat{A}$ and $\hat{H}$ commute? If yes/no, what can you say about the eigenfunctions of $\hat{A}$ and $\hat{H}$?

[1] Heisenberg, W. Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen.. Z. Physik 33, 879–893

Exercise 2

A Hermitian operator A has three normalized and non-degenerate eigenfunctions ψ_1, ψ_2, ψ_3 with corresponding eigenvalues ϵ_1, ϵ_2 and ϵ_3 respectively. Assume that there is a 50 % chance that a measure A gives ϵ_1 and 25% chance for ϵ_2 and ϵ_3

(a) Write down the explicit expression of a normalized wave function ϕ in terms of the eigenfunctions of A which satisfies the above conditions.

(b) Prove that the $\phi' = e^{i\vartheta}\phi$ (ϑ is a real number) still satisfies the same conditions.

(c) Prove that the expectation values of a generic operator B on ϕ and ϕ' are the same. Notice that this means that ϕ and ϕ' contain the same physical information, thus they represent the same physical state.