
Exercise Set 3

Goals

- 1) Basic probability operations.
- 2) Familiarize yourself with Binomial and Normal law and how they model daily life processes.
- 3) Perform elementary probability distribution computations.

1 Bonus round

In a quiz show, you win the chance to play a bonus game. One dice is rolled, and you receive the number of eyes *squared* in 100 CHF bills. That is, if the dice shows the number i , you win $100 i^2$ CHF.

- a) What are the minimal and maximal possible amounts you can win?
- b) How much can you expect to win on average?
- c) Why is the result in (b) not just $100 (\bar{i})^2$ CHF

2 Binomial distribution and coin game

A fair coin has two possible outcomes, "heads" or "tails", that occur with equal probability. The coin toss is the paradigmatic realization of a Binomial experiment. We denote the number of coin tosses by N , and we are interested only in the total number of "heads" and "tails" in the N tosses, not their order. This is described by the random variable $X = \sum_{i=1}^N X_i$, where X_i denotes the outcome of the i -th coin throw. You define $X_i = 1$ if the i -th throw shows "heads" and $X_i = 0$ if it shows "tails".

- a) You throw a coin ten times. Compute the probabilities of the following events:
 - 1) only "heads" in every throw
 - 2) either only "heads" or only "tails"
 - 3) exactly 5 "heads" and 5 "tails"
- b)
 - 1) Express the probability $P(X = n)$ and $P(X \leq n)$ for any integer n with $0 \leq n \leq N$.
 - 2) Draw the probability distribution and cumulative distribution for $N=10$ in Python. To generate your x-axis (i.e. your values n) you can use `np.arange()` (make sure you use the correct limits!). For the binomial coefficients, you can use `math.comb(N,n)` after importing the "math" package.
 - 3) Compute the expectation value $\mathbb{E}[X]$ and its standard deviation $\sigma = \sqrt{\text{Var}[X]}$ of the Binomial distribution for general N and state the numbers for the example above, $N=10$.
 - 4) The variance apparently is zero for two special cases, $p = 0$ and $p = 1$. Why is that?
 - 5) Consider now an unfair coin. It has been made to show "heads" with a very low probability, $p = \frac{1}{6}$. On average, how many times do you expect to throw the coin before you find the first "heads"?

3 The probability distribution of the dice

- a) Find the probability distribution of rolling a fair, single dice with 6 faces (normal dice). Compute the expectation value, variance and standard deviation.
- b) Now you throw 2 fair dice in parallel, and you play a game that depends on the sum of the faces. Say, X_1 denotes the result of dice 1, and X_2 that of dice 2. You are now interested in their sum, $X = X_1 + X_2$. Find the probability distribution of X , and compute its expectation value μ , variance and standard deviation σ .
- c) Draw the probability distribution and cumulative distribution. Does the distribution looks Gaussian ? Draw a Gaussian variable $Y \sim \mathcal{N}(\mu, \sigma)$ with the same mean and standard deviation on top of your previous plot.
- d) Someone offers you to gamble in the following game: You throw 2 dice. If the sum X is below 10, you have to pay 1 CHF, but if it is 10 or higher, you win 2 CHF. Is it a good idea for you to play this game (is it fair)?

4 The Binomial distribution and the grocer

A grocer buy apples from a farmer. 25% of these apples are not suitable for clients. The grocer asks a worker to make packages with 5 apples, but the worker does not care whether the apple is suitable or not. Whenever a client buys a package, if two or more apples are not suitable the client comes back to the grocer to complain.

- a) Compute the probability distribution of $X =$ number of good apples in the package.
- b) What is the probability for a client to come back and complain?
- c) If 100 clients buy a package this day, how many complaints will there be on average?
- d) The grocers sells the packages for 2CHF and buys them for 1CHF. If the package was bad though, the customer returns to the store, complains and receives a new package for free. Does the grocer make profit on average?