
Exercise Set 7

Goals

- 1) Perform different tests for the mean.
- 2) Learn the roles of different error types in statistical testing.

For a T-table, see for example www.tdistributiontable.com. For a Z-Table, www.z-table.net. In the exam, these tables will be provided if needed.

1 Photodetector efficiency [normal]

You are working on producing a new type of photodetector. Your aim is to beat your company's current best detector for 2500nm light, made of InGaAs, which has a responsivity of 1.2A/W.

Your company policy is that improved prototypes are only considered for further development if they are shown to provide an improvement with 99% probability.

You have measured your device and found 1.3A/W, using a testing method that has a standard deviation of 10% of the measured value (from that, you can compute the standard deviation in A/W). You repeat the measurement 4 times. Now you want to convince your company to consider further development.

- a) Choose the correct test.
- b) Perform the test. Does your photodiode fulfil the requirements?
- c) The conclusion of such a test depends on the choice of the confidence level. Here we took the choice of 0.99. Compute the confidence level at which the reference value of 1.2A/W would be exactly at the edge. In that case, what would be the probability, that your photodiode does actually perform worse than the reference value, and your measurement was just a statistical fluctuation?
- d) How many measurements would you have to perform to get just over the 99% level?

2 Novel diet for a healthy lifestyle [normal]

Two groups of twelve people that share the exact same weight of 105kg were selected to test 2 new diets. They followed their diets strictly for 4 months and their weights after the diet phase are shown in the table below. For each diet separately, using the appropriate test, determine if the diets were effective on the $\alpha = 0.05$ level of significance.

Diet 1	86	96	82	103	91	88	94	90	97	87	97	105
Diet 2	106	118	106	91	96	102	100	100	108	105	118	93

3 Error of type 1 (false discovery/positive) and 2 (false negative), power of a test [advanced]

Let $Y_1, Y_2, \dots, Y_n$ be independent and identically distributed (i.i.d) random variables that follow the Gaussian/normal $\mathcal{N}(\mu = \theta, \sigma^2 = 1)$ distribution. We are interested to test if $\theta = 0$. There are many tests possible, and they differ in the amount and relative ratio of the type 1 and type 2 errors.

We propose a certain test for $\theta = 0$ is proposed: The test accepts $\theta = 0$ if the measured average times the square-root of the number of measurements, $|\sqrt{n}\bar{Y}| \leq 2$, and rejects is otherwise (hence implying $\theta \neq 0$).

- a) Lets assume that the true value of θ is zero, i.e. the real distribution is the standard (centered and normalized) normal law $\mathcal{N}(\mu = 0, \sigma^2 = 1)$. What is the probability of the test to incorrectly state $\theta \neq 0$ when only one single measurement is performed?
- b) How does that probability change if 9 measurements are made?
- c) We now assume that $\theta = 1$ and still take 9 measurements. Compute the type 1 (false discovery/positive) and type 2 errors.
- d) Compute the, depending on which is appropriate, the type 1 or the 2 error (as well as the power of the test, corresponding to 1 minus the probability for a type 2 error), when $\theta = 0, 0.5, 1, 1.5$ and 2.