
Exercise Set 2

Goals

- 1) Exercise basic probability operations
- 2) Understand the standard deviation and variance, and their difference
- 3) Familiarize yourself with random variables and their plots in Python

1 Elementary events operation

- a) Let A and B be independent events. Prove that the complementary events $\bar{A}$ and $\bar{B}$ are also independent.
- b) Let E , F and G be three events and assume their individual probabilities $P(E)$, $P(F)$, $P(G)$ are known. Express the probabilities of the following composite events in terms of the probabilities of E, F , and G as well as their intersections:

- 1) "none of them is realized"?
- 2) "at maximum one is realized"
- 3) "exactly two are realized"
- 4) "E and F are realized but not G"

Drawing Venn diagrams can help you with this task.

2 Your odds at Roulette

Roulette lives from composite events. In the course, a roulette board was shown. Assume that all numbers $[0,36]$ can be realized with equal probability (elementary events).

- a) Compute the following probabilities:

- 1) Single number.
- 2) Red $[32,19,21,25,34,27,36,30,23,5,16,1,14,9,18,7,12,3]$.
- 3) First dozen.
- 4) Even.

- b) Are "Red" and "Even" independent events?

- c) Write a Python script that randomly generates roulette numbers with equal probability. Once a number is chosen, make it announce (print) all the events that apply from the following: pair/impair, red/black, manque/passe, 1st/2nd/3rd dozen. Challenge yourself to make the code as compact as

possible!

d) Starting with 10 coins, write a while loop which keeps betting 1 coin on impair, and only stops once you have reached zero coins. At the end of the loop, plot your money vs. time.

e) Run this loop 1000 times and plot a histogram showing how many rounds you were able to play before you lose all your coins. Compute the mean and standard deviation of that distribution. Comment on whether mean and standard deviation are enough to describe this result?

3 Modeling a random coin game experiment

In this game, we throw a coin four times and record the results: heads or tails?

- a) Describe the space Ω of all possible events.
- b) How many events does Ω contain?
- c) A useful way to record this game is a vector (coin1,coin2,coin3,coin4), assigning the number 1 for heads and 0 for tails. For example, (1,0,0,1) denotes the elementary event "first coin heads, second and third coin tails, fourth coin heads". Write down the following composite events in terms of the elementary ones:
 - "We get heads at the second and fourth throw"
 - "We get heads only at the second and fourth throw"
 - "We get at least one heads out of the four throws"
 - "We get at least two heads out of the four throws"
- d) Let assume the coin is fair and both sides have equal probability. Compute the probabilities of all the events the you just expressed.
- e) Lets think about a completely different situation. We now consider a family with four children. Using the oversimplified assumption of a 50% probability for giving birth to a boy and a 50% probability for giving birth to a girl, what are the probabilities to have:
 - only boys?
 - only girls?
 - two boys and two girls?
- f) Comment on whether we could use the same model for the hair colour of four children.

4 Revisiting the steel cable data (Exercise Set 1)

And keeping the outlier as a real data point for this exercise.

Data (in MPa):

10.1	12.2	9.3	12.4	13.7	10.8	11.6	10.1	11.2	11.3
12.2	12.6	11.5	9.2	14.2	11.1	13.3	11.8	7.1	10.5

Compute the variance and the standard deviation sd from this dataset, by hand or in Python. Last exercise, you calculated the mean. How many datapoints fall into the window $[mean - sd, mean + sd]$?