Homework 1

Exercise 1

In thermodynamics a function in the form of $y = x \ln x$ is used extensively. Calculate the first and second derivatives of the function.

Exercise 2

You are given the function $y = \frac{1}{6}x^3 - x^2 - 6x + 2$

a) Calculate the local minimum and maximum of the function.

b) Calculate the inflection point of the function.

Exercise 3

In this exercise, we introduce some mathematical notation and concepts that are used in thermodynamics:

If we have a function like $f(x, y) = 3x^2 + 4xy - 7y^2$, we can compute the partial derivatives

$$\left(\frac{\partial f}{\partial x}\right)_y = 6x + 4y, \qquad \left(\frac{\partial f}{\partial y}\right)_x = 4x - 14y$$

Using these, the **(total) differential of** f, also called the **total derivative** of f, is defined as

$$\mathrm{d}f = (6x + 4y)\mathrm{d}x + (4x - 14y)\mathrm{d}y$$

In general

$$\mathrm{d}f = \left(\frac{\partial f}{\partial x}\right)_y \mathrm{d}x + \left(\frac{\partial f}{\partial y}\right)_x \mathrm{d}y$$

Please note that in Analysis III, you will learn about gradients of vector fields which are very closely related but have a different notation.

a) Compute the total derivative of $f(x, y) = x^3 y$.

b) Compute the total derivative of $f(x, y, z) = x^2 y + y^2 z + z^4$.

c) At the intuitive level, df represents how much f (for example, the energy of a material) changes if we change the inputs x and y (for example, x and y could be the temperature and pressure) by very tiny amounts dx and dy. How can we interpret the two terms in $df = \left(\frac{\partial f}{\partial x}\right)_y dx + \left(\frac{\partial f}{\partial y}\right)_x dy$?

Exercise 4

The ideal gas law states: $pV = nRT$

a) Calculate the following partial derivatives:

$$\left(\frac{\partial p}{\partial T}\right)_V ; \left(\frac{\partial T}{\partial V}\right)_p ; \left(\frac{\partial V}{\partial p}\right)_T$$

b) What does the following equation equal to?

$$\left(\frac{\partial p}{\partial T}\right)_V \cdot \left(\frac{\partial T}{\partial V}\right)_p \cdot \left(\frac{\partial V}{\partial p}\right)_T =?$$

c) Calculate the total derivative of pressure as a function of T and V.(dp =?)