Homework 1

Exercise 1

In thermodynamics a function of the form of $y = f(x) = x \ln x$ is used extensively. Calculate the first and second derivatives of the function.

Exercise 2

You are given the function $y = f(x) = 1/6x^3 - x^2 - 6x + 2$.

- (a) Calculate the local minimum and maximum.
- (b) Calculate the inflection point.

Exercise 3

In this exercise, we introduce some standard notation used in thermodynamics for concepts that you already know: In Analysis II, we wrote vector fields like

$$f(x,y) = \begin{pmatrix} x^2 + 3y \\ x + 5y \end{pmatrix}. \tag{1}$$

In thermodynamics, we write this vector field as

$$\delta f = (x^2 + 3y)\mathrm{d}x + (x + 5y)\mathrm{d}y \tag{2}$$

and call this object a **differential form**. The symbol δ in front of δf indicates that this is a differential form.

If we have a function $f: \mathbb{R}^2 \to \mathbb{R}$ like $f(x,y) = 3x^2 + 4xy - 3y^2$, we can generate a vector field by taking the gradient

$$\nabla f(x,y) = \begin{pmatrix} 6x + 4y \\ 4x - 6y \end{pmatrix}. \tag{3}$$

In thermodynamics, we say that we compute the **differential of** f, also called the **total derivative** of f, written as

$$df = (6x + 4y)dx + (4x - 6y)dy. (4)$$

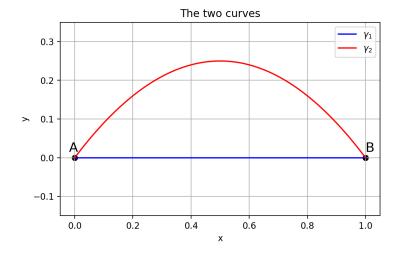
We can compute the integral of a differential form along a curve $\gamma: I \to \mathbb{R}^n$ in the same way as for vector fields along curves since we are only changing the notation. Answer the following questions:

(a) Let $f(x,y) = x^3 - y$ and consider the integral of its total derivative along two different curves

$$I_1 = \int_{\gamma_1} \mathrm{d}f, \qquad I_2 = \int_{\gamma_2} \mathrm{d}f, \tag{5}$$

where $\gamma_1:[0,1]\to\mathbb{R}^2, \gamma_1(s)=(s,0)$ is the straight line segment from A=(0,0) to B=(1,0) and $\gamma_2:[0,1]\to\mathbb{R}^2, \gamma_2(s)=(s,s-s^2)$ is a parabolic curve with the **same endpoints** (see figure).

Try to answer this question without doing any calculations: Is $I_1 = I_2$?



- (b) Can you explain why? (you will need a theorem about line integrals)
- (c) Let $\delta g = xy dx + y^2 dy$ be a differential form that can not be written as a total derivative of a function and consider the two integrals

$$I_3 = \int_{\gamma_1} \delta g, \qquad I_4 = \int_{\gamma_2} \delta g. \tag{6}$$

Again, without doing any calculations: What can you say about the relation between I_3 and I_4 ?

Exercise 4

The ideal gas law states pV = nRT.

(a) Calculate the following partial derivatives:

$$\left(\frac{\partial p}{\partial V}\right)_V, \left(\frac{\partial T}{\partial V}\right)_p, \left(\frac{\partial V}{\partial p}\right)_T$$

(b) What does the following expression equal to?

$$\left(\frac{\partial p}{\partial V}\right)_V \cdot \left(\frac{\partial T}{\partial V}\right)_p \cdot \left(\frac{\partial V}{\partial p}\right)_T = ?$$

(c) Calculate the total derivative of the pressure as a function of T and V.

$$dp = ?$$