

# Homework 1

## Exercise 1

In thermodynamics a function of the form of  $y = f(x) = x \ln x$  is used extensively. Calculate the first and second derivatives of the function.

## Exercise 2

You are given the function  $y = f(x) = 1/6x^3 - x^2 - 6x + 2$ .

- (a) Calculate the local minimum and maximum.
- (b) Calculate the inflection point.

## Exercise 3

In this exercise, we introduce some standard notation used in thermodynamics for concepts that you already know: In Analysis II, we wrote vector fields like

$$f(x, y) = \begin{pmatrix} x^2 + 3y \\ x + 5y \end{pmatrix}. \quad (1)$$

In thermodynamics, we write this vector field as

$$\delta f = (x^2 + 3y)dx + (x + 5y)dy \quad (2)$$

and call this object a **differential form**. The symbol  $\delta$  in front of  $\delta f$  indicates that this is a differential form.

If we have a function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  like  $f(x, y) = 3x^2 + 4xy - 3y^2$ , we can generate a vector field by taking the gradient

$$\nabla f(x, y) = \begin{pmatrix} 6x + 4y \\ 4x - 6y \end{pmatrix}. \quad (3)$$

In thermodynamics, we say that we compute the **differential of  $f$** , also called the **total derivative** of  $f$ , written as

$$df = (6x + 4y)dx + (4x - 6y)dy. \quad (4)$$

We can compute the integral of a differential form along a curve  $\gamma : I \rightarrow \mathbb{R}^n$  in the same way as for vector fields along curves since we are only changing the notation. Answer the following questions:

- (a) Let  $f(x, y) = x^3 - y$  and consider the integral of its total derivative along two different curves

$$I_1 = \int_{\gamma_1} df, \quad I_2 = \int_{\gamma_2} df, \quad (5)$$

where  $\gamma_1 : [0, 1] \rightarrow \mathbb{R}^2, \gamma_1(s) = (s, 0)$  is the straight line segment from  $A = (0, 0)$  to  $B = (1, 0)$  and  $\gamma_2 : [0, 1] \rightarrow \mathbb{R}^2, \gamma_2(s) = (s, s - s^2)$  is a parabolic curve with the **same endpoints** (see figure).

Try to answer this question without doing any calculations: Is  $I_1 = I_2$ ?



- (b) Can you explain why? (you will need a theorem about line integrals)
- (c) Let  $\delta g = xydx + y^2dy$  be a differential form that can not be written as a total derivative of a function and consider the two integrals

$$I_3 = \int_{\gamma_1} \delta g, \quad I_4 = \int_{\gamma_2} \delta g. \quad (6)$$

Again, without doing any calculations: What can you say about the relation between  $I_3$  and  $I_4$ ?

#### Exercise 4

The ideal gas law states  $pV = nRT$ .

- (a) Calculate the following partial derivatives:

$$\left(\frac{\partial p}{\partial V}\right)_V, \left(\frac{\partial T}{\partial V}\right)_p, \left(\frac{\partial V}{\partial p}\right)_T$$

- (b) What does the following expression equal to?

$$\left(\frac{\partial p}{\partial V}\right)_V \cdot \left(\frac{\partial T}{\partial V}\right)_p \cdot \left(\frac{\partial V}{\partial p}\right)_T = ?$$

- (c) Calculate the total derivative of the pressure as a function of T and V.

$$dp = ?$$