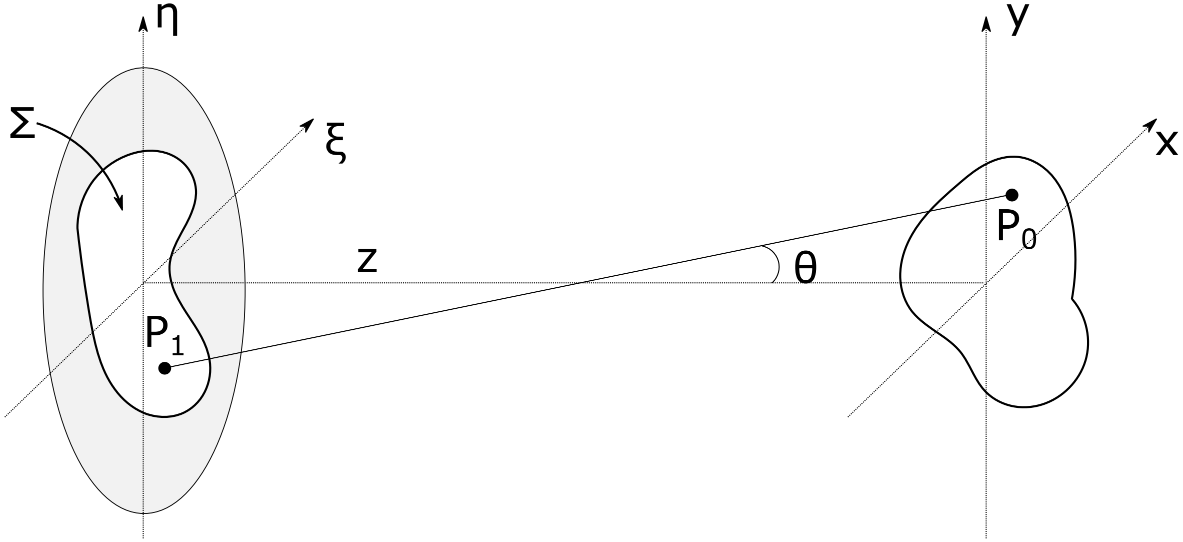
Scalar Diffraction Theory: Basic Approximations

- Maxwell's equations operate on vector quantities
- Scalar wave equation is obeyed if the media is
  - Linear
  - Isotropic and homogeneous
  - Nondispersive and Nonmagnetic
- *Boundaries do not normally satisfy these conditions*
- Good approximation in two conditions:
  - The aperture must be large compared with a wavelength.
  - The fields must not be observed too close to the aperture.
- Our treatment is not good for
  - Subwavelength gratings
  - Photonic crystals
  - Waveguides and dispersive media
  - Small pits on optical recording media

The Huygens-Fresnel Principle

- Knowing the optical field over any given plane in vacuum or an ideal dielectric medium, the field at any other plane can be expressed as a superposition of "secondary" spherical waves, known as Huygens wavelets, originating from each point in the first plane.



$$U(x,y) = \frac{1}{j\lambda} \int\int_{\Sigma} U(\xi,\eta) \frac{\exp(jkr_{01})}{r_{01}} \cos\theta \, d\xi \, d\eta$$

$$r_{01} = \sqrt{z^2 + (x-\xi)^2 + (y-\eta)^2}$$

Fresnel Approximation

- If θ is within the paraxial region

$$r_{01} \approx z\left[1+\frac{1}{2}\left(\frac{x-\xi}{z}\right)^2+\frac{1}{2}\left(\frac{y-\eta}{z}\right)^2\right]$$

- Then, the Huygens-Fresnel principle becomes

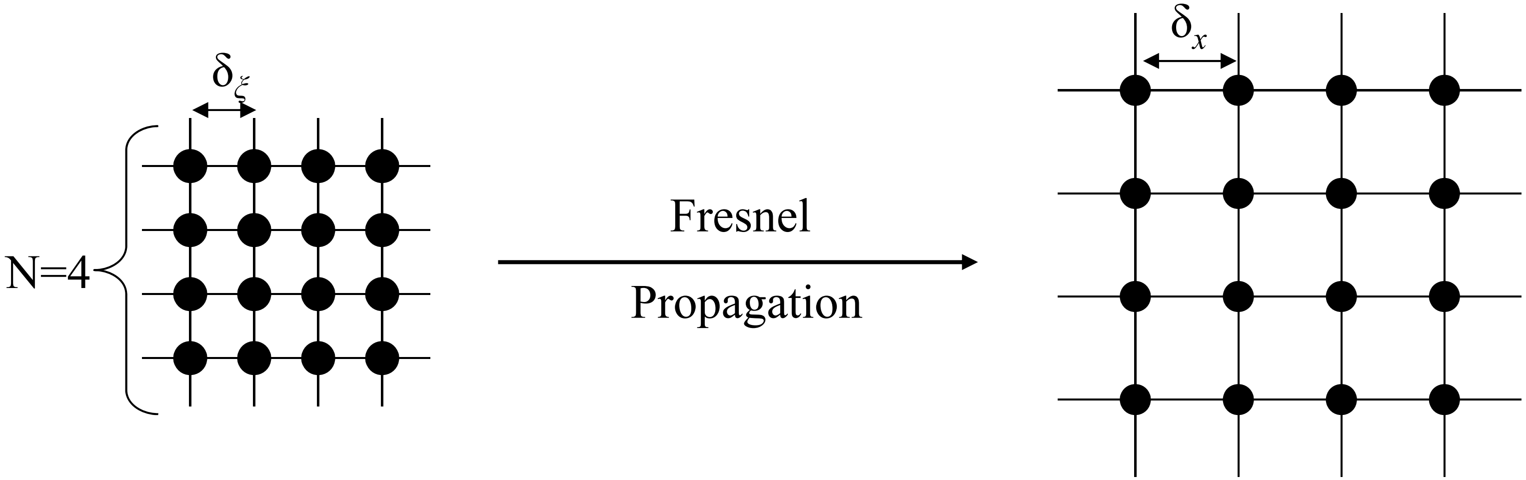
$$U(x,y)=\frac{e^{jkz}}{j\lambda z}\int\int_{-\infty}^{\infty}U(\xi,\eta)\exp\left\{j\frac{k}{2z}\left[(x-\xi)^2+(y-\eta)^2\right]\right\}d\xi d\eta$$

- After factorization of the exponent within the integral

$$U(x,y)=\frac{e^{jkz}}{j\lambda z}e^{j\frac{k}{2z}(x^2+y^2)}\underbrace{\int\int_{-\infty}^{\infty}\left\{U(\xi,\eta)\,e^{j\frac{k}{2z}(\xi^2+\eta^2)}\right\}e^{-j\frac{2\pi}{\lambda z}(x\xi+y\eta)}d\xi d\eta}_{Fourier Transformation}$$

Optical Modeling with Wave Optics

- Numerical solution of Fresnel integrals
- Procedure (for MATLAB®)
  - Form a 1D/2D grid (you can use the *meshgrid* command)
  - Form the complex aperture function on the grid
  - Propagation algorithm (you can use the 2D *fft* command)
  - Scaling of the grid
- Two propagation methods
  - The convolutional approach
  - The Fourier transformation approach



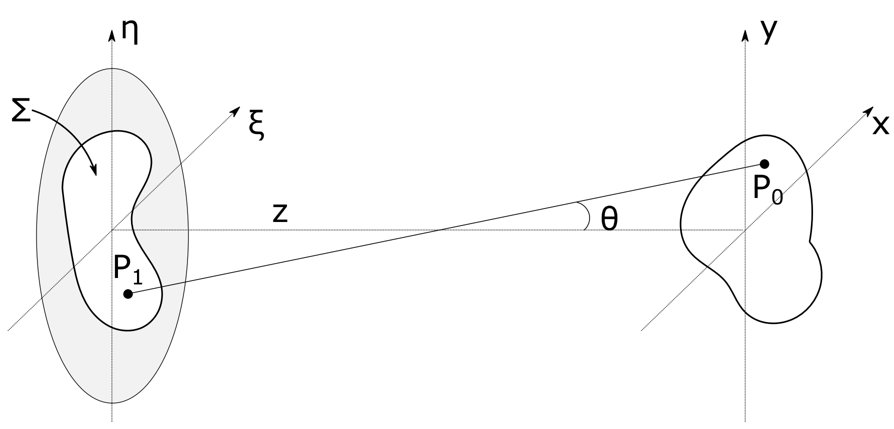
Propagation with Wave Optics: Convolution

$U(\xi,\eta)$ → Fourier Transform → $\times \ \exp\left[-\frac{j2\pi^2}{k}\left(f_\xi^2+f_\eta^2\right)z\right]$ → Inverse Fourier Transform → $U(x,y)$

- The Fresnel integral is a convolution
- $f(x) * g(x) = \mathcal{F}^{-1}\{F(u)G(u)\}$
- Useful for small z: $z < N\frac{\delta_\xi^{\ 2}}{\lambda}$
- Can you derive the Fourier transform of the propagation kernel?

$$\mathscr{F}\left[\frac{1}{j\lambda z}\exp\left(j\frac{k}{2z}\left(\xi^2+\eta^2\right)\right)\right]$$
$$=\exp\left(-\frac{j2\pi^2}{k}\left(f_\xi^2+f_\eta^2\right)z\right)$$

Fresnel approximation of the diffraction profile



$$U(x,y)=\frac{e^{jkz}}{j\lambda z}\int\int_{-\infty}^{\infty}U(\xi,\eta)\exp\left\{j\frac{k}{2z}\left[(x-\xi)^2+(y-\eta)^2\right]\right\}d\xi\, d\eta$$

Propagation with Wave Optics: Fourier Transform

$$U(\xi,\eta) \longrightarrow \boxed{\times\ e^{j\frac{k}{2z}(\xi^2+\eta^2)}} \longrightarrow \boxed{\text{Fourier Transform}} \longrightarrow \boxed{\times\ \frac{1}{j\lambda z}e^{j\frac{k}{2z}(x^2+y^2)}} \longrightarrow U(x,y)$$

- Mathematically identical to the previous method
- Works better for large z: $z > N\frac{\delta_\xi{}^2}{\lambda}$
- What happens when $z >> k(x^2 + y^2) / 2$?

Fresnel approximation of the diffraction profile in Fourier transform representation

$$U(x,y) = \frac{e^{jkz}}{j\lambda z}e^{j\frac{k}{2z}(x^2+y^2)}\underbrace{\int\int_{-\infty}^{\infty}\left\{U(\xi,\eta)\,e^{j\frac{k}{2z}(\xi^2+\eta^2)}\right\}e^{-j\frac{2\pi}{\lambda z}(x\xi+y\eta)}d\xi d\eta}_{Fourier\,Transformation}$$