MICRO-561

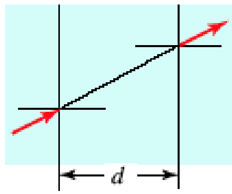
Biomicroscopy I

Syllabus (tentative)

Lecture 1	Introduction & Ray Optics-1
Lecture 2	Ray Optics-2 & Matrix Optics-1
Lecture 3	Matrix Optics-2
Lecture 4	Matrix Optics-3 & Microscopy Design-1
Lecture 5	Microscopy Design-2
Lecture 6	Microscopy Design-3
Lecture 7	Resolution-1
Lecture 8	Resolution-2
Lecture 9	Resolution-3 & Contrast
Lecture 10	Fluorescence-1
Lecture 11	Fluorescence-2
Lecture 12	Fluorescence-3, Sources, Filters
Lecture 13	Detectors
Lecture 14	Bio-application Examples

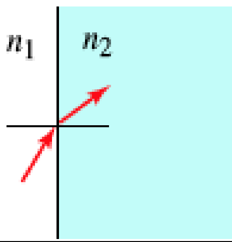
Reminder: Summary of matrix optics for basic functions & components

Propagation



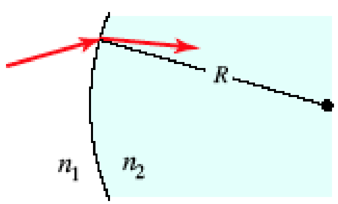
$$\mathbf{M} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Planar boundary



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

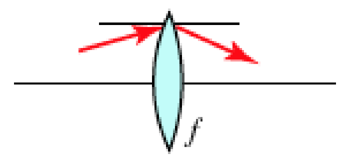
Spherical boundary



Convex, $R > 0$; concave, $R < 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{(n_2 - n_1)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$$

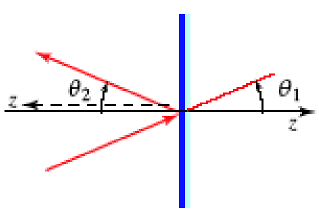
Lens



Convex, $f > 0$; concave, $f < 0$

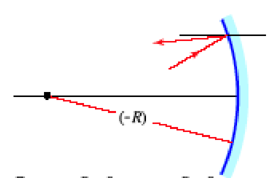
$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

Planar mirror



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Spherical mirror

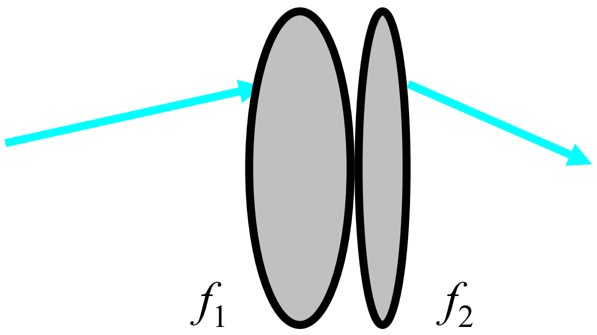


Concave, $R < 0$; convex, $R > 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$

Example: Consecutive Lenses

- Suppose we have two thin lenses right next to each other with no space in between.
- How does it behave?



When light enters this system, it experiences:

1) Initially, 1st lens with focal length of f_1
2) Then, 2nd lens with focal length of f_2

$$M_1 = M_{thin\ lens\ with\ f_1}$$

$$M_1 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix}$$

$$M_2 = M_{thin\ lens\ with\ f_2}$$

$$M_2 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix}$$

$$M_{total} = M_2 \times M_1$$

$$M_{total} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix}$$

$$M_{total} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} - \frac{1}{f_2} & 1 \end{bmatrix}$$

$$M_{total} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_{total}} & 1 \end{bmatrix}$$

Two consecutive lenses act as one lens whose focal length is computed by the “resistive sum” ➔

$$\frac{1}{f_{total}} = \frac{1}{f_1} + \frac{1}{f_2}$$

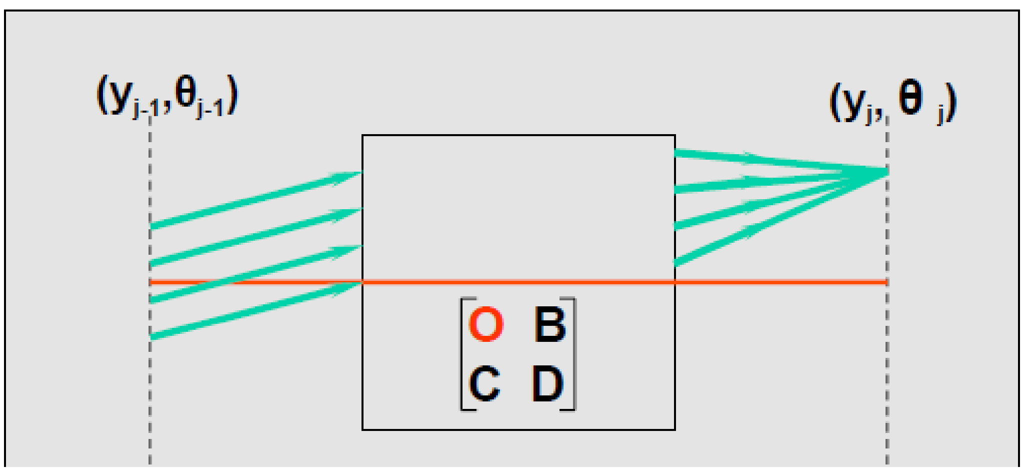
System Properties for "**A=0**"

$$\begin{bmatrix} y_j \\ \theta_j \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} * \begin{bmatrix} y_{j-1} \\ \theta_{j-1} \end{bmatrix}$$

If A=0 ➔ $y_j = 0.y_{j-1} + B.\theta_{j-1}$

Under this condition the equation is **independent of positions** (y_{j-1})

➔It means all possible rays entering the system from **different positions** but with same angle (θ_{j-1}), will cross each other at the same point at the exit the system.



A=0 corresponds to FOCUSING

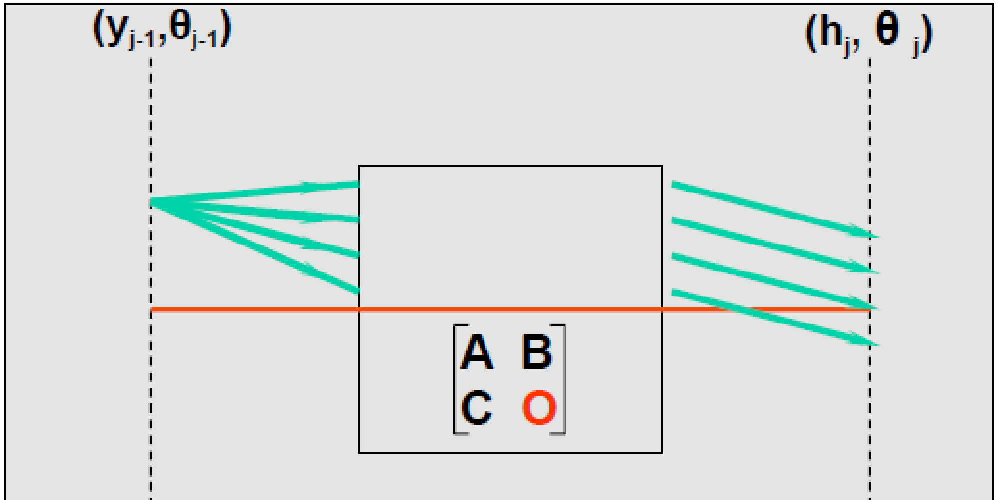
System Properties for "**D=0**"

$$\begin{bmatrix} y_j \\ \theta_j \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} * \begin{bmatrix} y_{j-1} \\ \theta_{j-1} \end{bmatrix}$$

If D=0 ➔ $\theta_j = C.\, y_{j-1} + 0.\, \theta_{j-1}$

Under this condition the equation is **independent of angle** (θ_{j-1})

➔It means all possible rays entering the system from different angles but originated from the same position (y_{j-1}), will exit the system with the same angle.



D=0 represents COLLIMATING

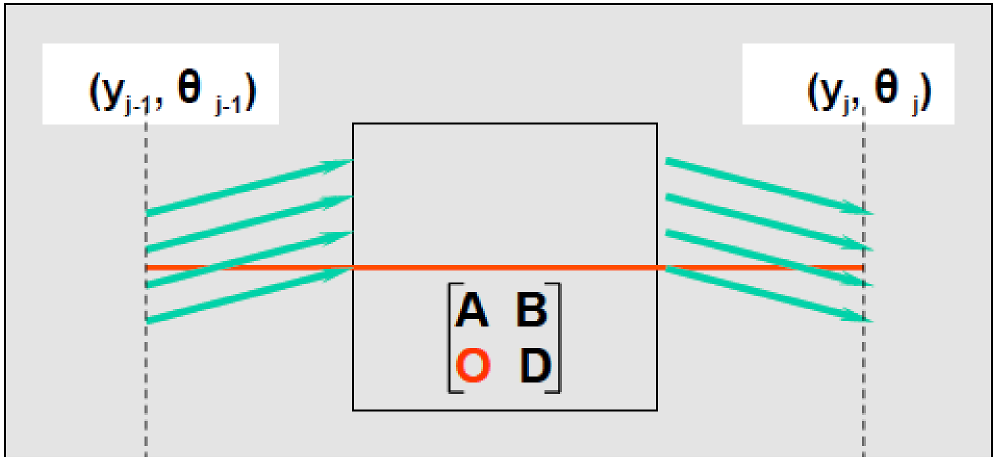
System Properties for "**C=0**"

$$\begin{bmatrix} y_j \\ \theta_j \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} * \begin{bmatrix} y_{j-1} \\ \theta_{j-1} \end{bmatrix}$$

If C=0 ➔ $\theta_j = 0.y_{j-1} + D.\theta_{j-1}$

Under this condition the equation is **independent of position** (y_{j-1}).

➔It means all possible rays entering the system from different entrance points but with same entrance angle (θ_{j-1}), will exit the system with the same exit angle.



C=0 corresponds to deviation

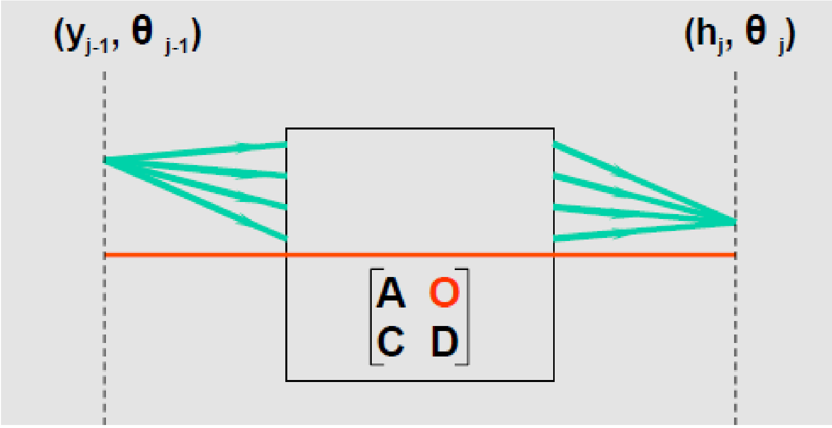
System Properties for “**B=0**”

$$\begin{bmatrix} y_j \\ \theta_j \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} * \begin{bmatrix} y_{j-1} \\ \theta_{j-1} \end{bmatrix}$$

If B=0 ➔ $y_j = A.y_{j-1} + 0.\theta_{j-1}$

Under this condition the equation is **independent of angle** (θ_{j-1})

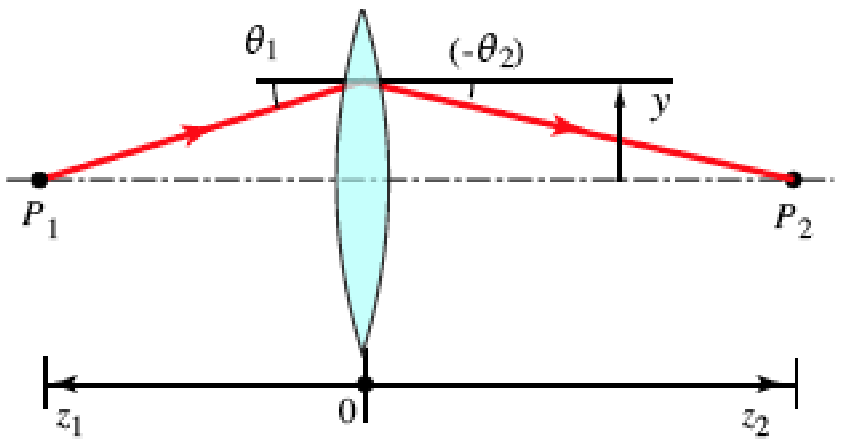
➔ It means all possible rays entering the system with different angles but from the same point (y_{j-1}), will cross each other at the same point at the exit of the system.



B=0 corresponds to IMAGING condition
It means these two planes (indicated by dashed vertical line) are conjugate

How can we connect ray tracing & matrix representation?

RECALL THIN LENS EXAMPLE:



From Ray Tracing

Ray Deflection :

$$\theta_2 = \theta_1 - \frac{y}{f},$$

Focal length:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Imaging Condition:

$$\frac{1}{f} = \frac{1}{z_{obj}} + \frac{1}{z_{im}}$$

Magnification:

$$\text{mag} = \frac{z_{im}}{z_{obj}}$$

From Matrix Representation

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix} \rightarrow \begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix} \rightarrow$$

Ray Deflection :

$$\theta_{out} = \theta_{in} - \frac{y_{in}}{f}$$

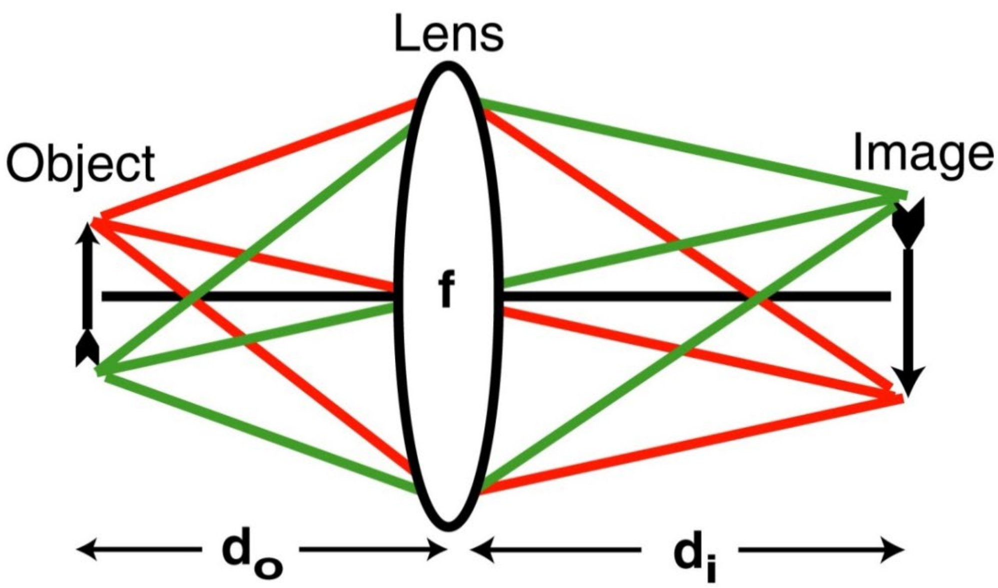
According to Matrix Representation:

An optical system forms an **image** of an object when "$B = 0$"

When $B = 0$ ➔ $\boxed{y_{out} = A\, y_{in}}$

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} A & 0 \\ C & D \end{bmatrix} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix} = \begin{bmatrix} A y_{in} \\ C y_{in} + D\theta_{in} \end{bmatrix}$$

Independent of their angle, all rays from a point y_{in} arrive at the same point y_{out}

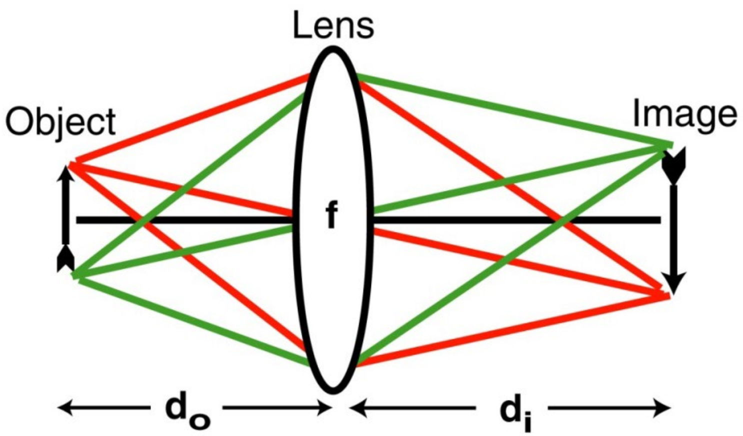


When $B = 0$, $\boxed{y_{out} = A\, y_{in}}$

A is the **magnification.**

$\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$ ➔ this is called "**conjugate**" matrix ➔ it describes an "**imaging**" optical set-up

Example:
Find the conjugate matrix of an imaging system based on single thin-lens



Q: How can we write the "conjugate matrix" of this one thin-lens imaging system?

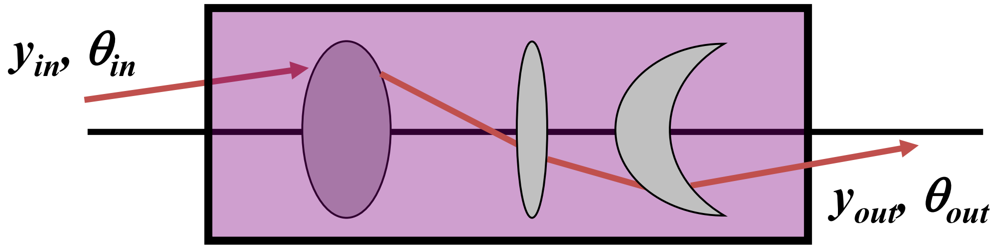
From the object plane to the image plane, light rays :

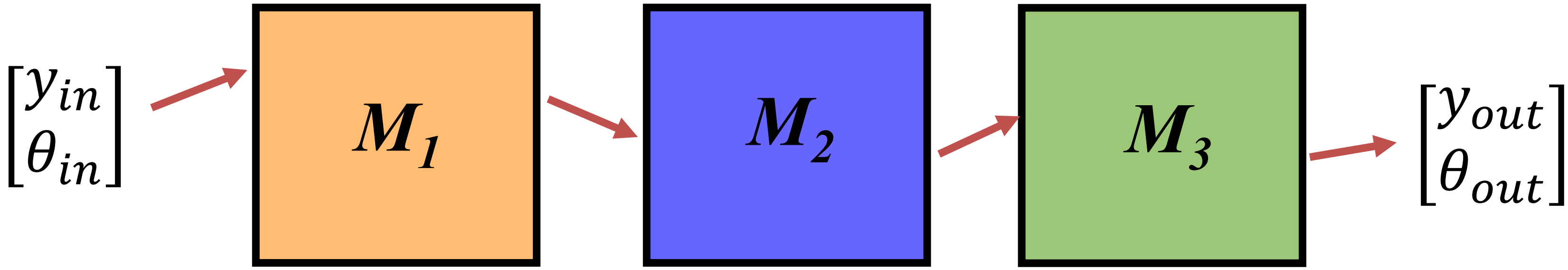
1)**Propagate** over a distance of d_o

2) Then, **encounter a thin lens** of focal length f

3) Finally, **propagate** over a distance of d_i

Reminder: Cascaded Elements





we simply multiply ray matrices.

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = M_3 \left\{ M_2 \left(M_1 \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix} \right) \right\} = M_3 M_2 M_1 \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix}$$

Notice the order !!

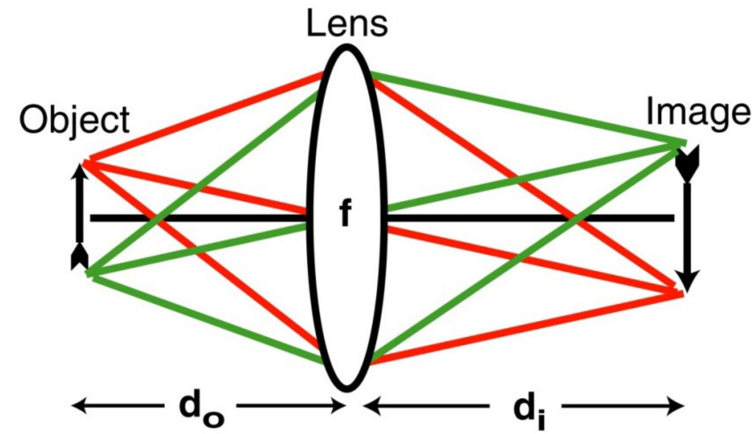
Example: Conjugate matrix for 1 thin lens imaging system

From the object plane to the image plane, light rays :

1)**Propagate** over a distance of d_o

2) Then, encounter a thin lens of focal length f

3) Finally, propagate over a distance of d_i



$$M_1 = \begin{bmatrix} 1 & d_o \\ 0 & 1 \end{bmatrix}$$

$$M_2 = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix}$$

$$M_3 = \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix}$$

$$M_{system} = M_3 \times M_2 \times M_1$$

$$M_{system} = \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \times \begin{bmatrix} 1 & d_o \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & d_o \\ -1/f & 1-d_o/f \end{bmatrix}$$

$$= \begin{bmatrix} 1-d_i/f & d_o+d_i-d_od_i/f \\ -1/f & 1-d_o/f \end{bmatrix}$$

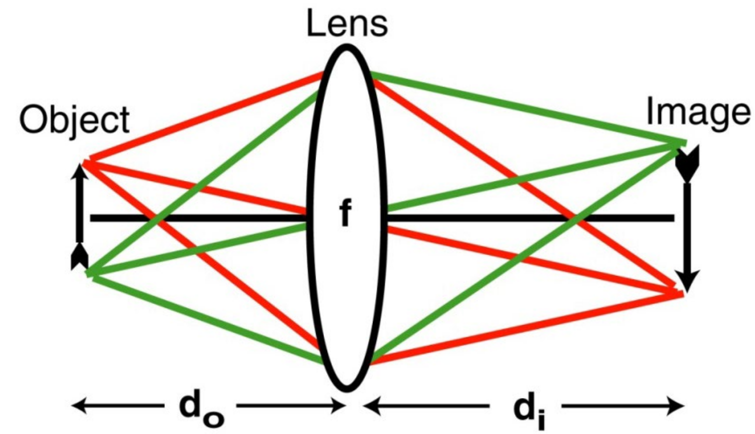
Example: Conjugate matrix for 1 thin lens imaging system

From the object plane to the image plane, light rays :

1)**Propagate** over a distance of d_o

2) Then, encounter a thin lens of focal length f

3) Finally, propagate over a distance of d_i



$$\left.\begin{aligned} M_1 &= \begin{bmatrix} 1 & d_o \\ 0 & 1 \end{bmatrix} \\ M_2 &= \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \\ M_3 &= \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \end{aligned}\right\}$$

$$M_{system} = M_3 \times M_2 \times M_1$$

$$= \begin{bmatrix} 1 - d_i / f & \boxed{d_o + d_i - d_o d_i / f} \\ -1/f & 1 - d_o / f \end{bmatrix}$$

For imaging, set B as 0 ➔ $B = d_o + d_i - d_o d_i / f = 0$

It means, $d_o d_i \left[1/d_o + 1/d_i - 1/f \right] = 0$, and this happens only if :

$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$$

This is the imaging condition (a.k.a. Lens law)

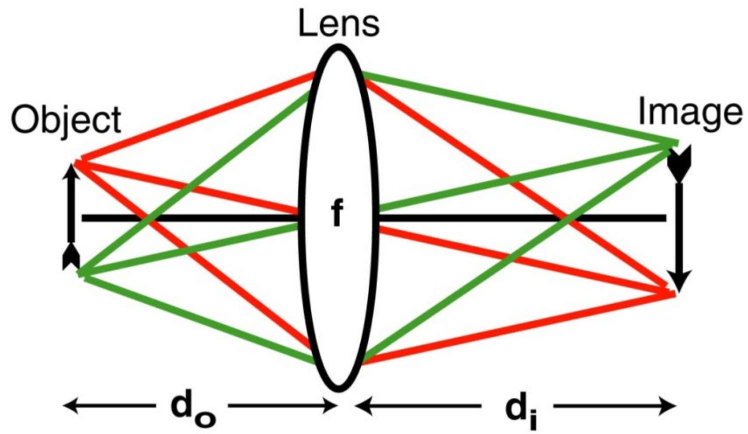
Example: Conjugate matrix for 1 thin lens imaging system

From the object plane to the image plane, light rays :

1)**Propagate** over a distance of d_o

2) Then, **encounter a thin lens** of focal length f

3) Finally, **propagate** over a distance of d_i



At the imaging condition:

$$M_{system} = M_3 \times M_2 \times M_1$$

$$= \begin{bmatrix} 1 - d_i / f & 0 \\ -1/f & 1 - d_o / f \end{bmatrix}$$

$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$$

If the imaging condition is satisfied then:

$$A = 1 - d_i / f$$

$$= 1 - d_i \left[\frac{1}{d_o} + \frac{1}{d_i} \right]$$

$$= -\frac{d_i}{d_o}$$

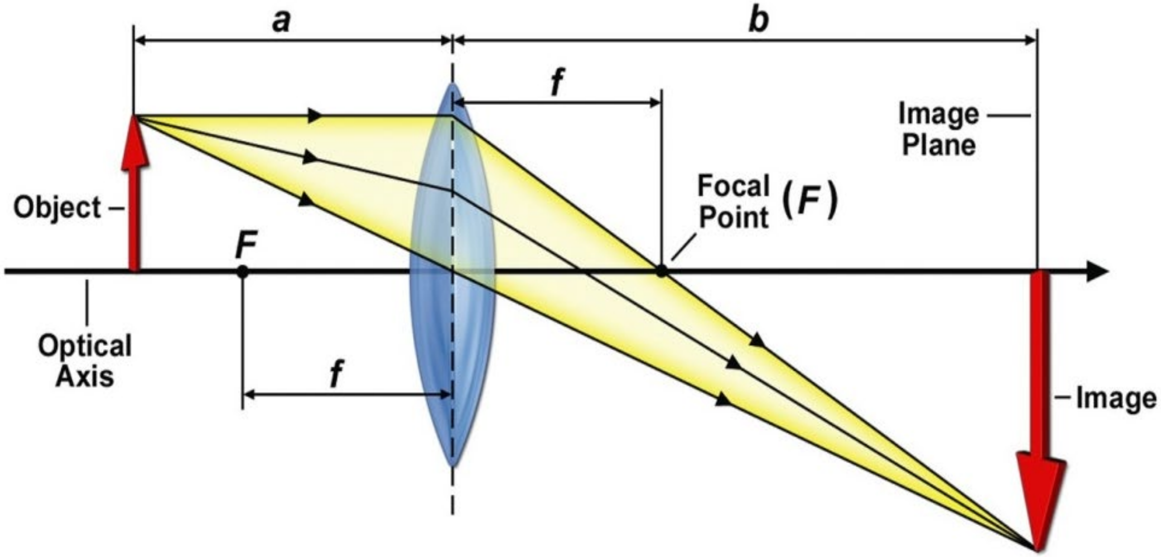
Magnification:

$A = M$

$$\boxed{M = -\frac{d_i}{d_o}}$$

How can we connect ray tracing & matrix representation?

Recall the relevant relations/equations for the example of thin lens based on Ray Optics:

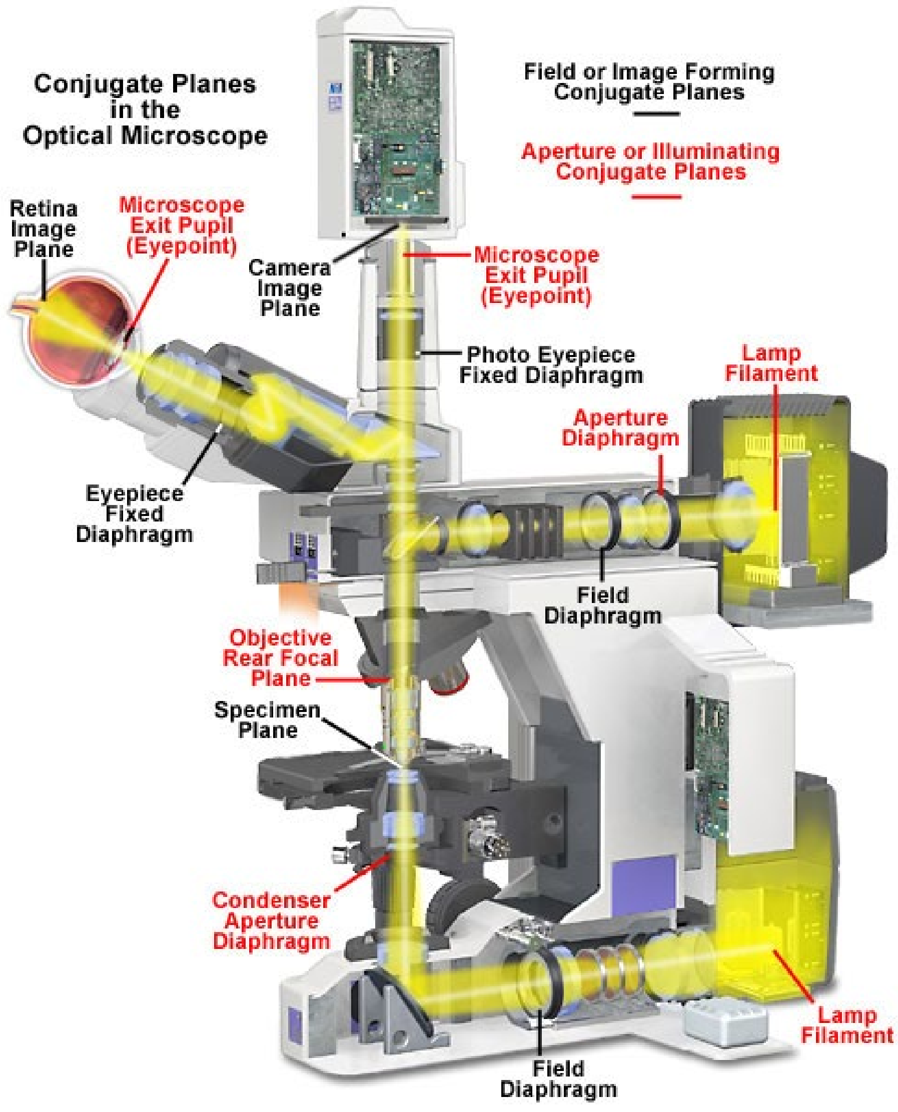


$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$$ Imaging condition

$$M = \frac{b}{a}$$ Magnification

$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$ Focal length (lens maker's formula)

Generalization: How to trace an image in an optical system?

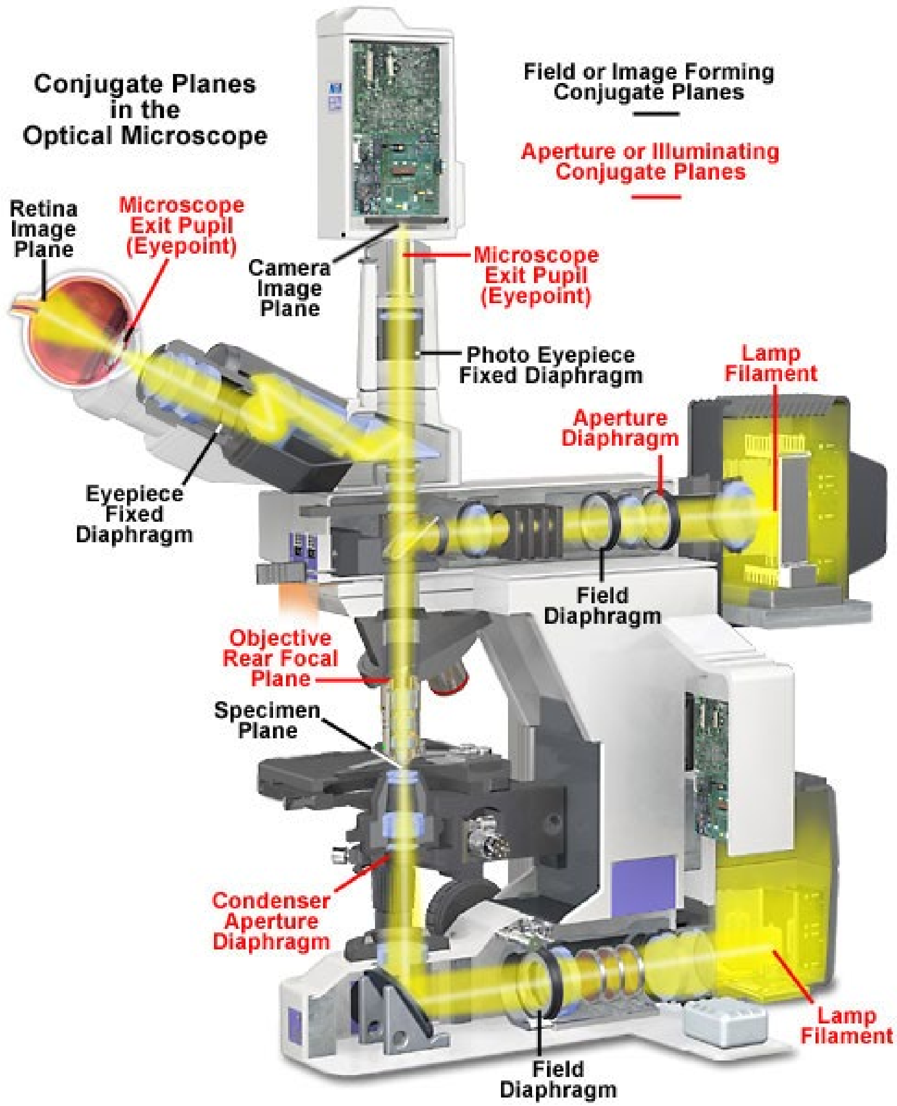


Optical systems (i.e. microscope) contain multiple lenses.

Q: What is the connection between matrix optics & ray tracing?

A: Use <u>cardinal planes</u>

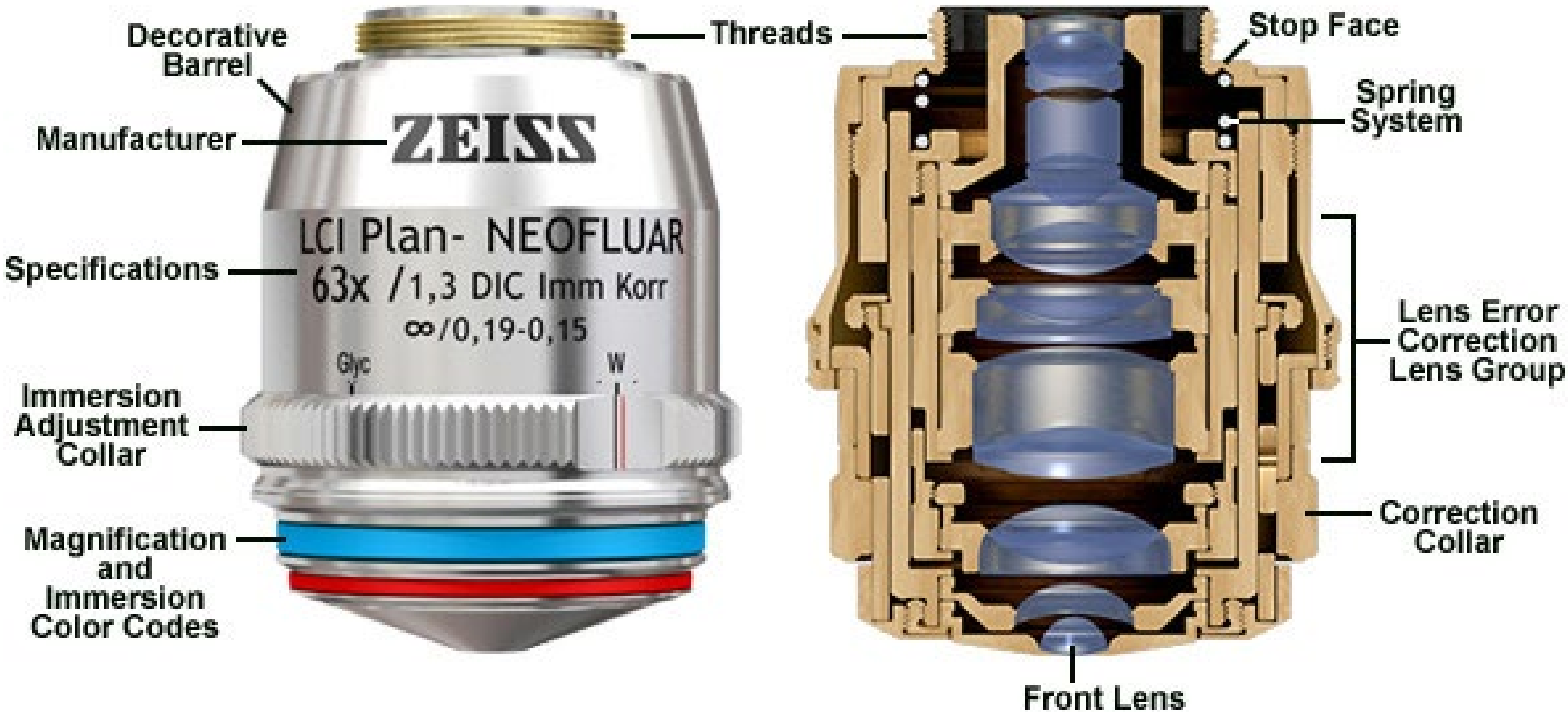
Microscope – An Optical System



Let's start with an important optical component:

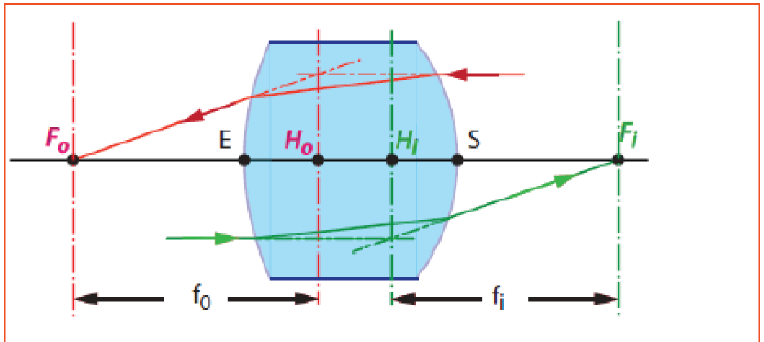
- The Objective

Anatomy of the Microscope Objective Lens



- It is not a single (and thin) lens
- It contains multiple THICK lenses
- Effectively this is an optical system itself

Thick lens



ray propagation across a thick lens and the principal planes

<u>A thick lens is an "optical system":</u>

- Refraction at entrance & exit surfaces
- Propagation inside the lens

The thick lens can be simplified by representing that **<u>as if</u>** the refraction is happening at **the principal planes**:

1) Two principal planes: $\mathbf{H_o}$ and $\mathbf{H_i}$ ➔ **virtual planes where the lens appears to bend the rays**

2) Two focal planes: $\mathbf{F_o}$ and $\mathbf{F_i}$

These are the FOUR CARDINAL <u>PLANES</u>.

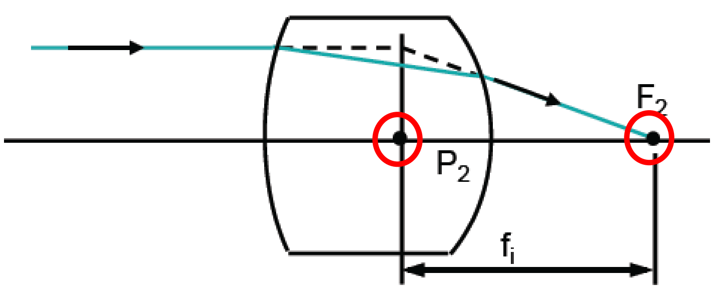
There are also FOUR CARDINAL <u>POINTS</u>:

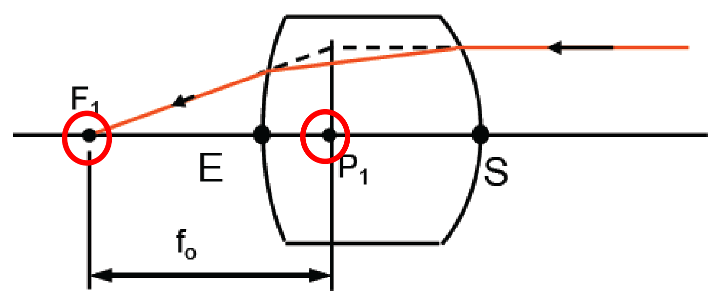
1) Two principal points: P_1 and P_2

which are the intersection points of the principal planes with the optical axis.

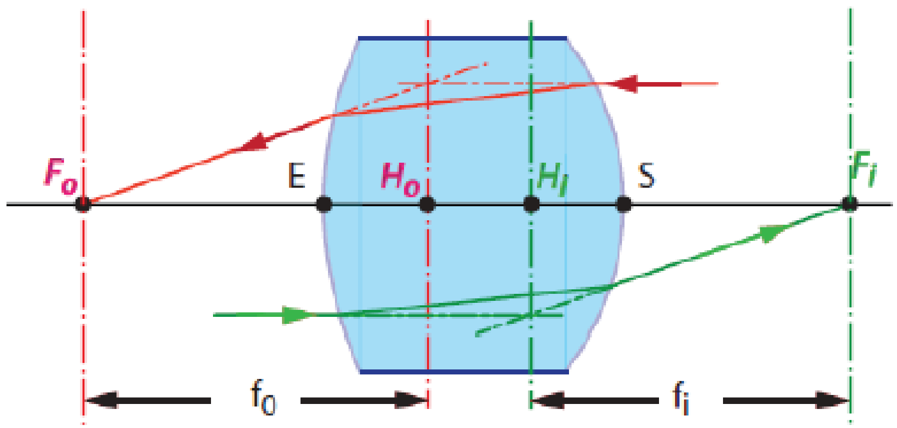
2) Two focal points: F_1 and F_2

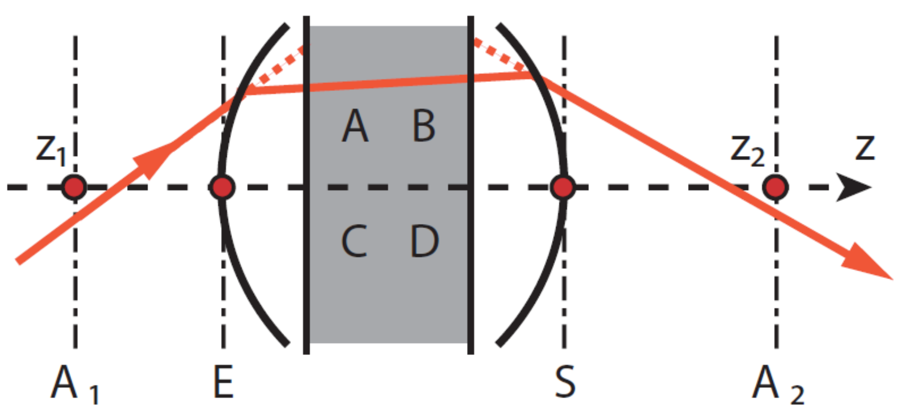
which are the intersection points of the focal planes with the optical axis.





The thick lens between planes E & S is represented with [ABCD] matrix.



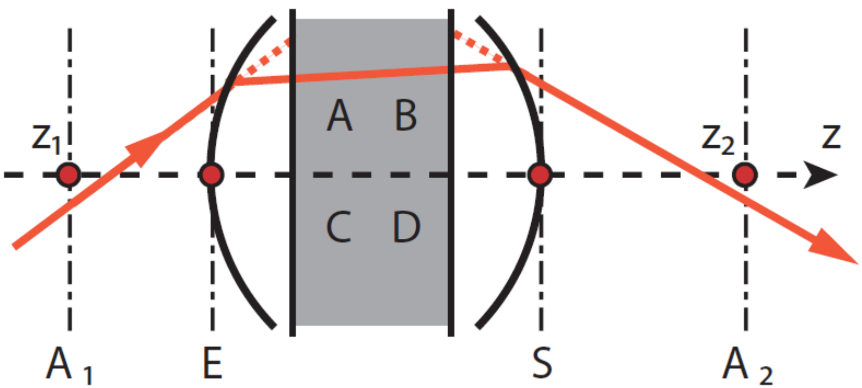


Generalization… beyond the thick lens case

Cardinal planes (two focal planes & two principal planes) are helpful concepts for ABCD analysis
➔ **They are not limited to thick lens and can be used <u>for any optical system in general</u>.**

To generalize, one needs to first realize that a system between the planes E & S is described with [ABCD].

<u>Next</u>, for the imaging condition, we need to find transfer from plane A_1 to plane A_2 by considering propagations and [ABCD] of the system between planes E & S, as it will be analyzed in the next slides.



$$z_1 = \overline{EA_1}$$

$$z_2 = \overline{SA_2}$$

<u>Some useful rules</u>

1. Light is traveling from left to right
2. Directed distance
   + if measured from reference plane to destination plane along propagation direction

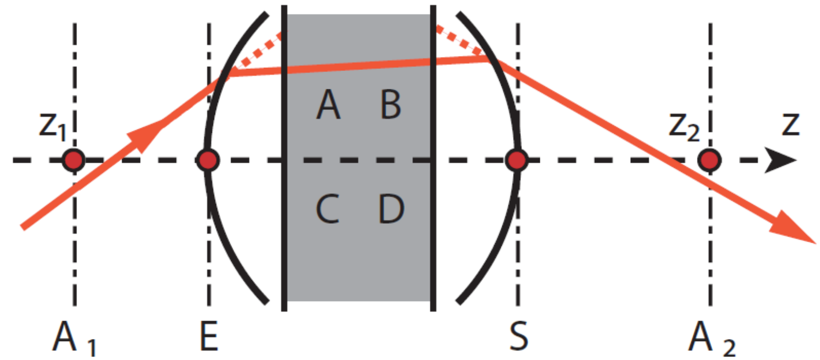
   - if measured from reference plane to destination plane against propagation direction
3. Radius of curvature
   + if center of curvature after surface
   - if center of curvature before surface

Here, plane E is the origin (reference)

Generalized analysis with [ABCD] matrix & principal planes

We use the following three steps to transfer from plane A_1 to plane A_2 :

1) Transfer from $A_1 \rightarrow E$ by *propagation* over $z_1 = T(A_1E)$
2) Transfer from $E \rightarrow S$ by [ABCD] optical system = T(ES)
3) Transfer from $S \rightarrow A_2$ by *propagation* over $z_2 = T(SA_2)$



$$z_1 = \overline{EA_1} \quad \& \quad z_2 = \overline{SA_2}$$

$$T_{A_1E} = \begin{bmatrix} 1 & -z_1 \\ 0 & 1 \end{bmatrix}$$

$$T_{ES} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$T_{SA_2} = \begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix}$$

$$T(A_{1\rightarrow}A_2) = T(SA_2) \times T(ES) \times T(A_1E)$$

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} 1 & -z_1 \\ 0 & 1 \end{bmatrix}$$

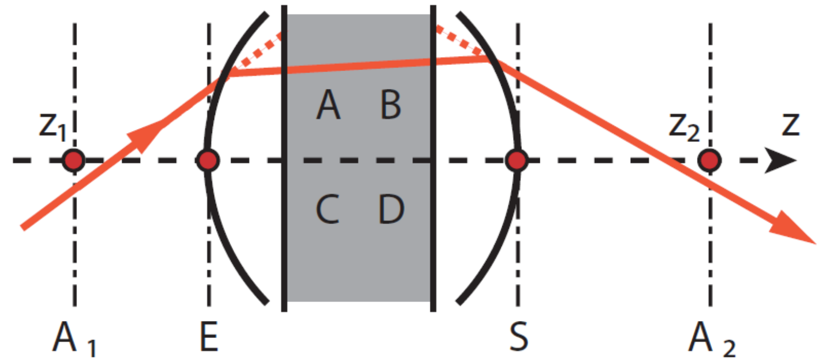
$$= \begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} A & B - z_1 A \\ C & D - z_1 C \end{bmatrix}$$

$$= \begin{bmatrix} A + Cz_2 & -Az_1 + B + z_2(-Cz_1 + D) \\ C & D - Cz_1 \end{bmatrix}$$

Generalized analysis with [ABCD] matrix & principal planes

We use the following three steps to transfer from plane A_1 to plane A_2 :

1) Transfer from $A_1 \rightarrow E$ by *propagation* over z_1 = $T(A_1E)$
2) Transfer from $E \rightarrow S$ by [ABCD] optical system = $T(ES)$
3) Transfer from $S \rightarrow A_2$ by *propagation* over z_2 = $T(SA_2)$



$$T(A_{1\rightarrow}A_2) = T(SA_2) \times T(ES) \times T(A_1E)$$

$$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} 1 & z_2 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} 1 & -z_1 \\ 0 & 1 \end{bmatrix}$$

$$T_{11} = A + Cz_2$$

$$T_{12} = -Az_1 + B + z_2(-Cz_1 + D)$$

$$T_{21} = C$$

$$T_{22} = D - Cz_1$$

1st observation:

T_{21} is independent of A_1 and A_2

Therefore, it is only a system property

Vergence $\mathbf{V} = -\boldsymbol{C}$

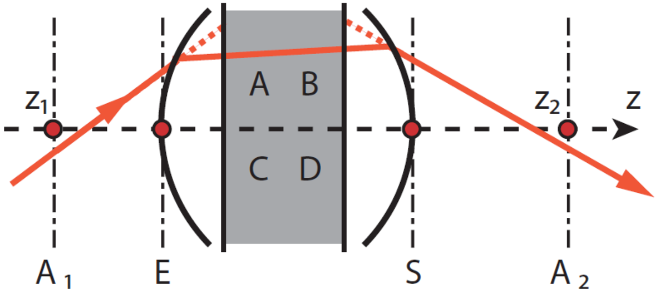
if $V > 0$ convergence or (+) system

if $V < 0$ divergence or (-) system

if $V = 0$ afocal system

Consider Imaging (Conjugation) Case:

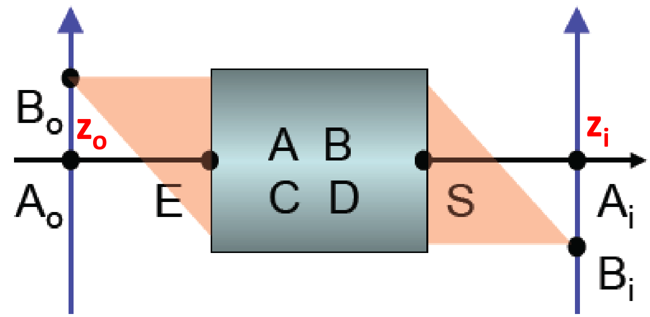
Transfer from plane A_1 to plane A_2



$$T(A_oA_i) = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}$$

$$= \begin{bmatrix} A - Vz_i & (-Az_o + B) + z_i(Vz_o + D) \\ -V & D + Vz_o \end{bmatrix}$$

Now, lets consider its application to an imaging case:



$z_o = \overline{EA_o}$

$z_i = \overline{SA_i}$

- The corresponding notation in the above figure is as follows:
  $z_1 = z_o$
  $z_2 = z_i$
  $A_1 = (A_o, B_o, \ldots)$
  $A_2 = (A_i, B_i, \ldots.)$

- It maps **o**bject points (A_o,B_o...) to **i**mage points (A_i,B_i...)

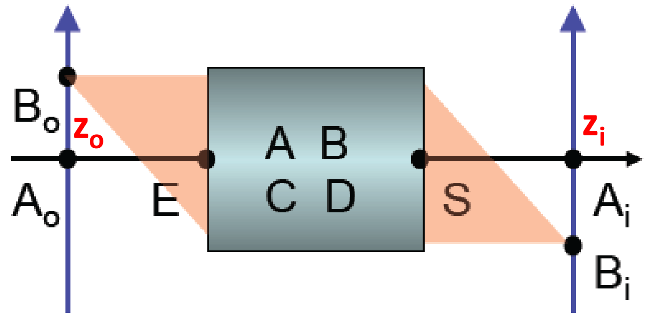
- If we define ray vectors as $\vec{X}_i$ & $\vec{X}_o$, we can write that:

$$\vec{X}_i = T(A_oA_i).\vec{X}_o$$

$$\vec{X}_i = \begin{bmatrix} y_i \\ \theta_i \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ -V & T_{22} \end{bmatrix}\begin{bmatrix} y_o \\ \theta_o \end{bmatrix}$$

$$= \begin{bmatrix} T_{11}y_o + T_{12}\theta_o \\ -Vy_o + T_{22}\theta_o \end{bmatrix}$$

Consider Imaging (Conjugation) Case:



- If we define ray vectors as $\vec{X}_i$ & $\vec{X}_o$, we can write that:

$$\vec{X}_i = \mathrm{T}(A_o A_i).\vec{X}_o$$

$$\vec{X}_i = \begin{bmatrix} y_i \\ \theta_i \end{bmatrix} = \begin{bmatrix} T_{11} y_o + T_{12}\theta_o \\ -V y_o + T_{22}\theta_o \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ -V & T_{22} \end{bmatrix} \begin{bmatrix} y_o \\ \theta_o \end{bmatrix}$$

Imaging requires that:

$T_{12} = 0$

$$\vec{X}_i = \begin{bmatrix} y_i \\ \theta_i \end{bmatrix} = \begin{bmatrix} T_{11} y_o \\ -V y_o + T_{22}\theta_o \end{bmatrix}$$

$T_{11} = M_t = y_i / y_o$

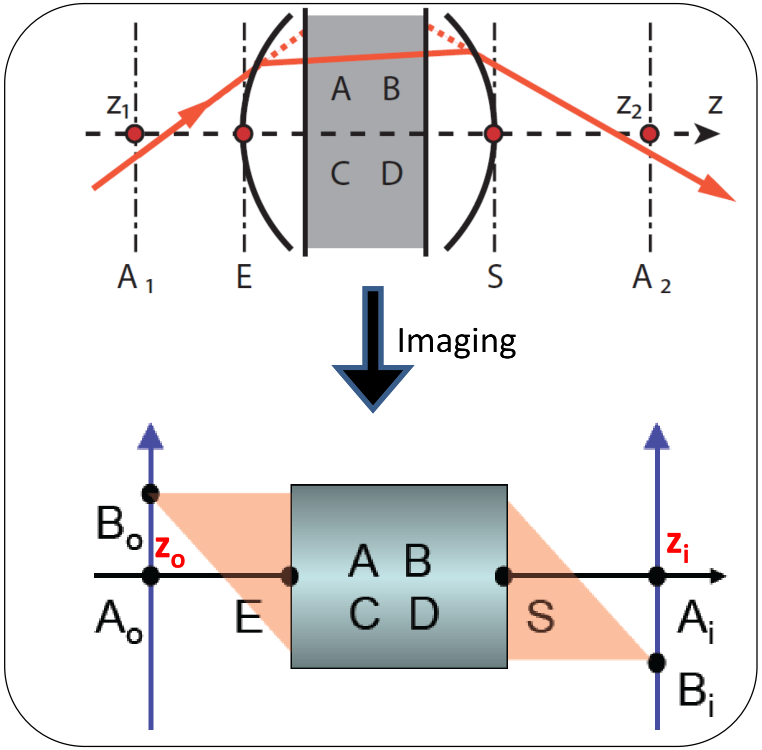
It corresponds to lateral magnification

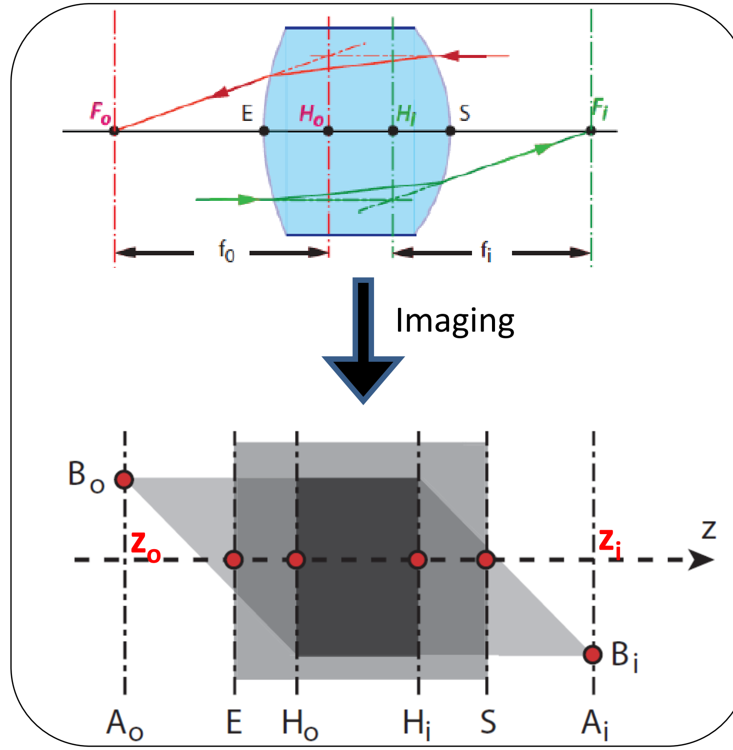
$T_{22} = M_\alpha = \theta_i / \theta_o$

It corresponds to angular magnification

$$\mathrm{T}(A_o A_i) = \begin{bmatrix} M_t & 0 \\ -V & M_\alpha \end{bmatrix}$$

Conjugated System: Connection to the **Principal Planes** (H_o & H_i)





The two principal planes (H_o and H_i) are conjugated:
➔Object-image relation holds between the planes H_o and H_i with the condition that $M_t=M_\alpha=1$

➔ When we apply $\mathrm{T}(A_oA_i)$ for $\mathrm{H}(H_oH_i)$, then we get:

Recall:

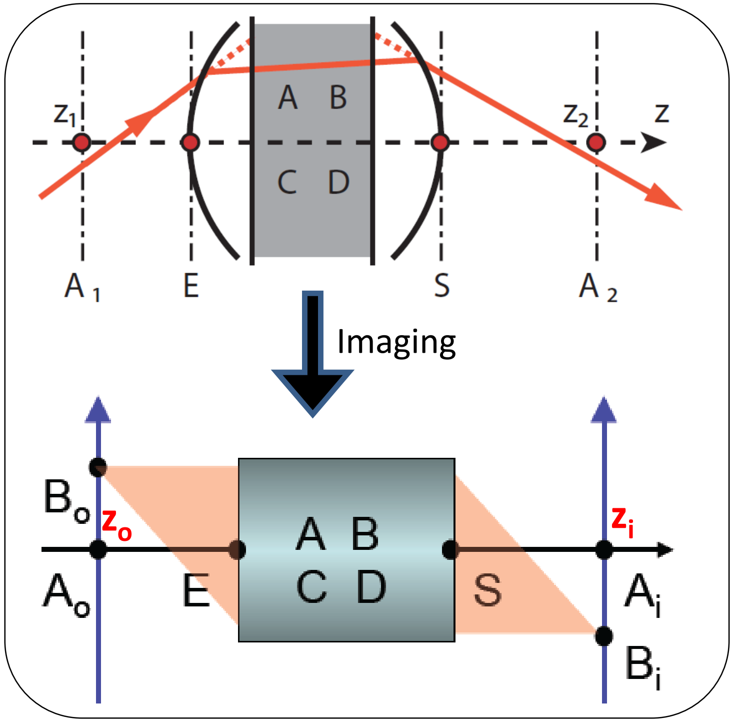
$$\mathrm{T}(A_oA_i) = \begin{bmatrix} M_t & 0 \\ -V & M_\alpha \end{bmatrix}$$

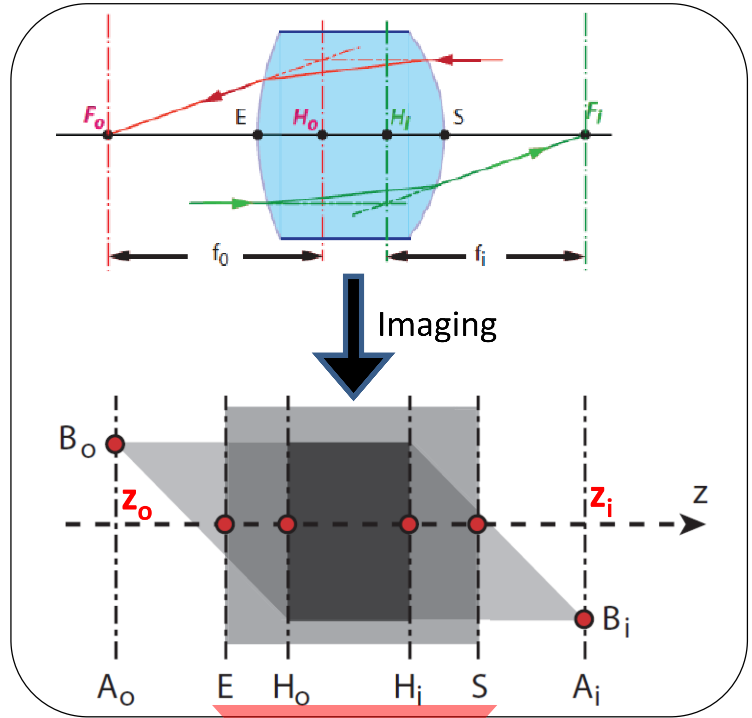
In this case, M = 1 ➔

Transfer matrix H between the principal planes H_o and H_i as:

$$H\,(H_oH_i) = \begin{bmatrix} 1 & 0 \\ -V & 1 \end{bmatrix}$$

Conjugated System: Connection to the **Principal Planes** (H_o & H_i)





$C = -V$

For the region between planes E & S, the operation is represented by [ABCD]:

$$M_1 = M_{EH_o} = \begin{bmatrix} 1 & \overline{EH_o} \\ 0 & 1 \end{bmatrix}$$

$$M_2 = M_{H_oH_i} = \begin{bmatrix} 1 & 0 \\ -V & 1 \end{bmatrix}$$

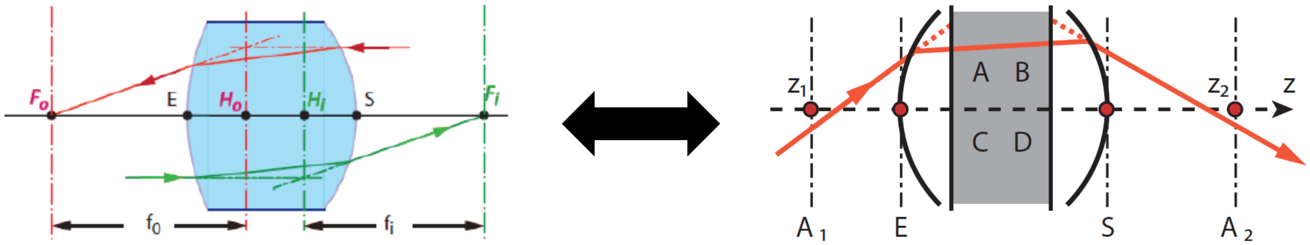
$$M_3 = M_{H_iS} = \begin{bmatrix} 1 & \overline{H_iS} \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ -V & D \end{bmatrix} = M_3 \times M_2 \times M_1$$

$$= \begin{bmatrix} 1 & \overline{H_iS} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -V & 1 \end{bmatrix} \times \begin{bmatrix} 1 & \overline{EH_o} \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 - V\overline{H_iS} & \overline{H_iS} + \overline{EH_o} - V\overline{EH_o}\,\overline{H_iS} \\ -V & 1 - V\overline{EH_o} \end{bmatrix}$$

Conjugated System: Object Principal Plane & Object Focal Plane



For the object side (indicated by the red line), the relevant components are H_o, F_o & f_o

For finding the location of object principle plane (H_o):

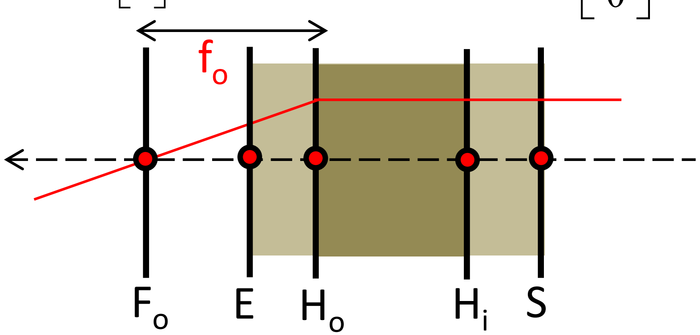
$$D = 1 - V\overline{EH_o} \Rightarrow \overline{EH_o} = \frac{1}{V}(1 - D)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - V\overline{H_iS} & \overline{H_iS} + \overline{EH_o} - V\overline{EH_o}\,\overline{H_iS} \\ -V & 1 - V\overline{EH_o} \end{bmatrix}$$

Object principle plane: $\boldsymbol{H_o} = \overline{EH_o} = \frac{1}{C}(D - 1)$

For finding the location of object focal plane (F_o) & object focal length (f_o):

$$X_{in} = \begin{bmatrix} 0 \\ \theta \end{bmatrix} \qquad X_{out} = \begin{bmatrix} y_0 \\ 0 \end{bmatrix}$$



$$\begin{bmatrix} y_o \\ 0 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} 1 & \overline{F_oE} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ \theta \end{bmatrix}$$

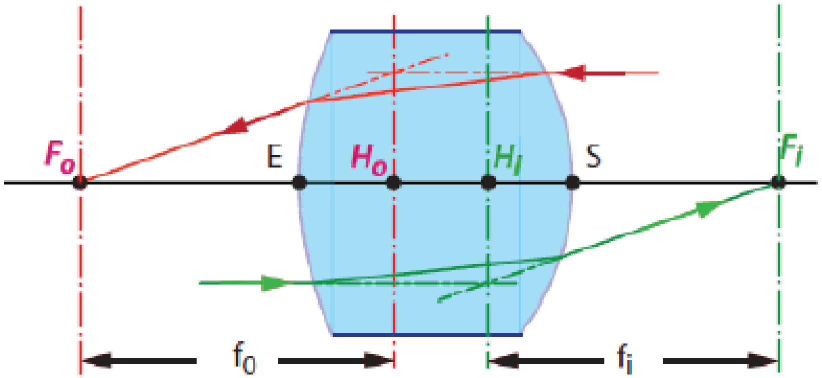
$$\begin{bmatrix} y_o \\ 0 \end{bmatrix} = \begin{bmatrix} \theta(B + \overline{F_oE}A) \\ \theta(D + \overline{F_oE}C) \end{bmatrix}$$

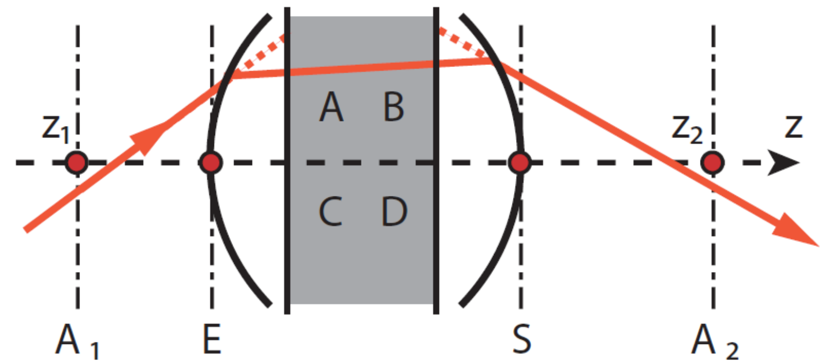
This can hold only if: $D + C\overline{F_oE} = 0$

Object focal point: $\boldsymbol{F_o} = \overline{EF_o} = D/C$

Object focal length: $\boldsymbol{f_o} = \overline{\boldsymbol{H_oF_o}} = \overline{\boldsymbol{H_oE}} + \overline{\boldsymbol{EF_o}} = -\frac{1}{C}(D - 1) + \frac{D}{C} = +\frac{1}{C}$

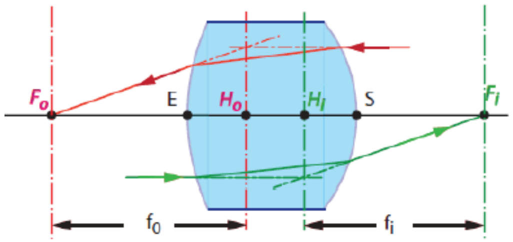
Summary: Cardinal Points & Planes for Object

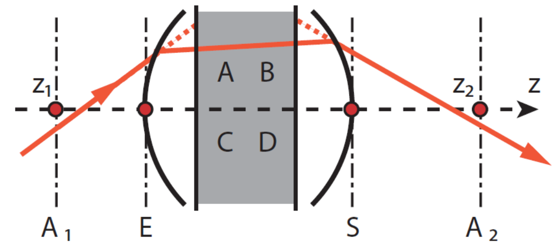




Distances	Notation	Directed Distances	ABCD elements
Object Focal Point	F_o	$\overline{EF_o}$	D/C
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$
Object Principle Plane	H_o	$\overline{EH_o}$	$(D-1)/C$

Conjugated System: Image Principal Plane & Image Focal Plane





For the image side (indicated by the green line), the relevant terms are H_i, F_i & f_i

For finding the location of image principle plane (H_i):

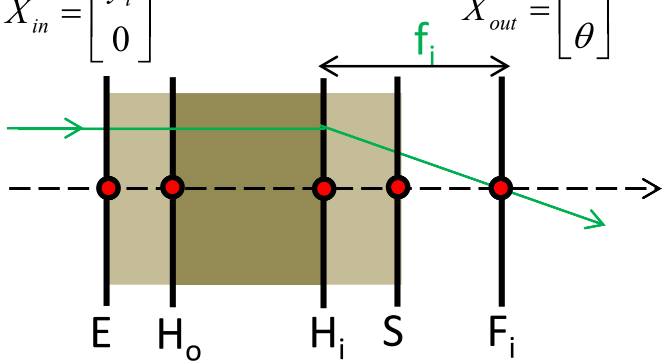
$$A = 1 - V\overline{H_iS} \Rightarrow \overline{H_iS} = \frac{1}{V}(1 - A)$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - V\overline{H_iS} & \overline{H_iS} + \overline{EH_o} - V\overline{EH_o}\,\overline{H_iS} \\ -V & 1 - V\overline{EH_o} \end{bmatrix}$$

Image principle plane: $\boldsymbol{H_i} = \overline{SH_i} = \frac{1}{C}(1 - A)$

For finding the location of image focal plane (F_i) & image focal length (f_i):

$X_{in} = \begin{bmatrix} y_i \\ 0 \end{bmatrix}$ $X_{out} = \begin{bmatrix} 0 \\ \theta \end{bmatrix}$



$$\begin{bmatrix} 0 \\ \theta \end{bmatrix} = \begin{bmatrix} 1 & \overline{SF_i} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} A & B \\ C & D \end{bmatrix} \times \begin{bmatrix} y_i \\ 0 \end{bmatrix}$$

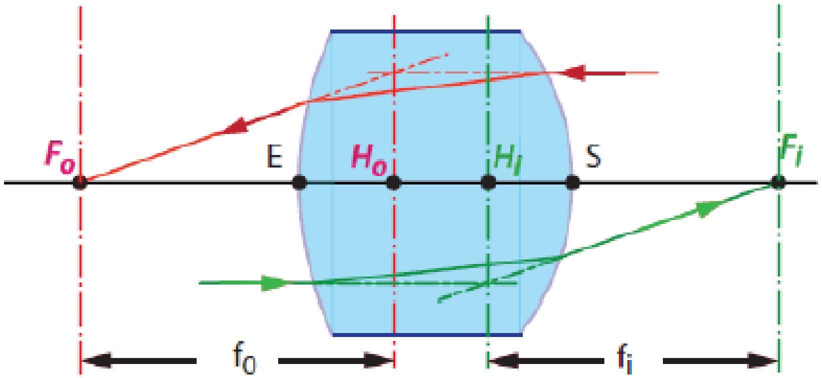
$$\begin{bmatrix} 0 \\ \theta \end{bmatrix} = \begin{bmatrix} (A + C\overline{SF_i})y_i \\ Cy_i \end{bmatrix}$$

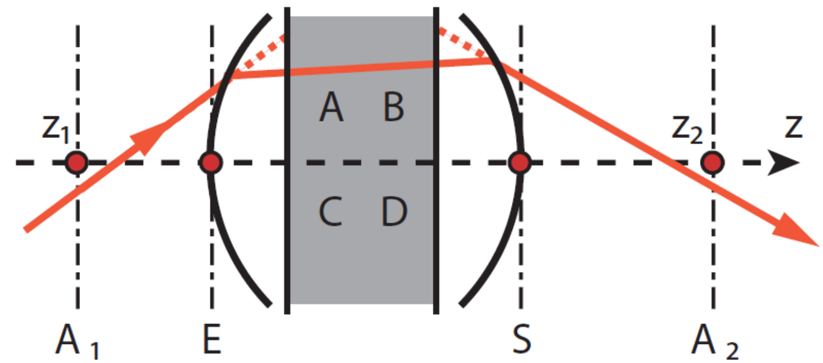
This can hold only: $A + C\overline{SF_i} = 0$

Image focal point: $\boldsymbol{F_i} = \overline{SF_i} = -A/C$

Image focal length: $\boldsymbol{f_i} = \overline{\boldsymbol{H_iF_i}} = \overline{\boldsymbol{H_iS}} + \overline{\boldsymbol{SF_i}} = -\frac{1}{C}(1 - A) - \frac{A}{C} = -\frac{1}{C}$

Summary: Cardinal Points & Planes for Image

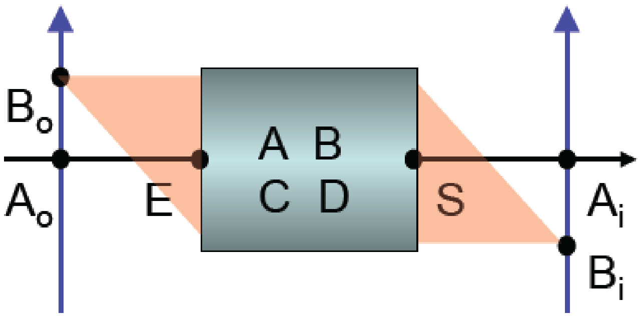


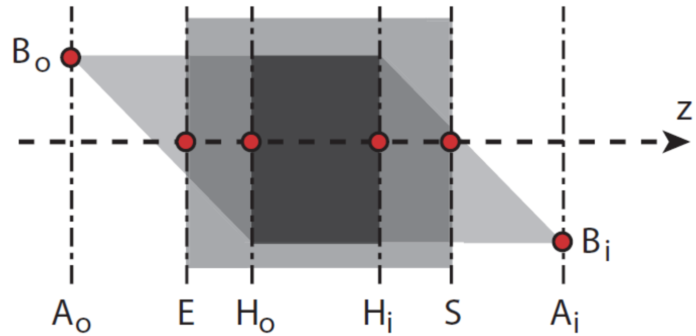


Distances	Notation	Directed Distances	ABCD elements
Image Focal Point	F_i	$\overline{SF_i}$	$-A/C$
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$
Image Principle Plane	H_i	$\overline{SH_i}$	$(1-A)/C$

The analysis (and the table) is also valid for **generalized** optical systems

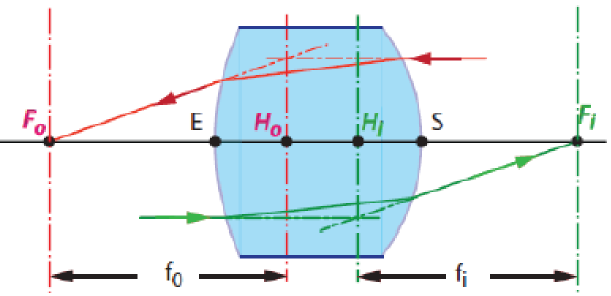
Cardinal points/planes can be used to find the image of the complex (i.e. cascaded) optical systems and the corresponding rays ➔ This is an alternative to ray tracing method.

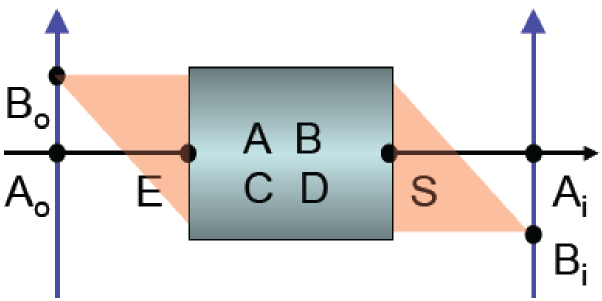


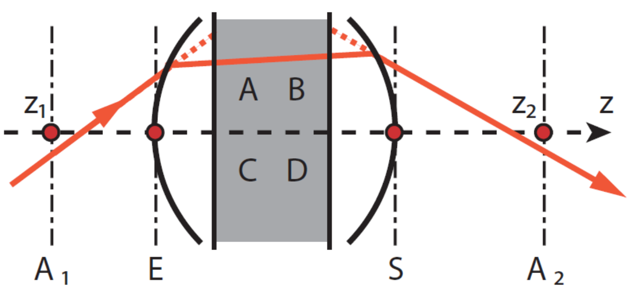


Distances	Notation	Directed Distances	ABCD elements
Object Focal Point	F_o	$\overline{EF_o}$	D/C
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$
Object Principle Plane	H_o	$\overline{EH_o}$	$(D-1)/C$
Image Focal Point	F_i	$\overline{SF_i}$	$-A/C$
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$
Image Principle Plane	H_i	$\overline{SH_i}$	$(1-A)/C$

Example: Cardinal Points & Planes of a Thick lens







Lens thickness = $\overline{ES} = e$

Lens index = n Outside index =1

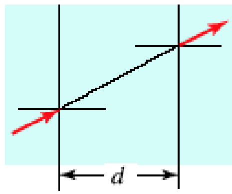
Opt. Power of 1st Surface $\Phi_1 = (n-1)/nR_1$

Opt. Power of 2nd Surface $\Phi_2 = (1-n)/R_2$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = ?$$

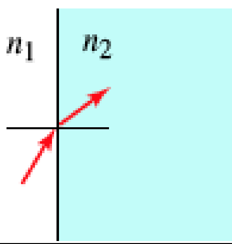
Remember: Matrix optics description of basic functions & components

Propagation



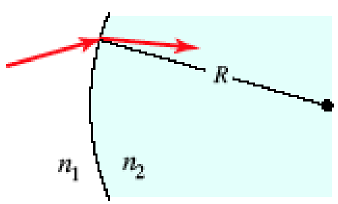
$$\mathbf{M} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Planar boundary



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

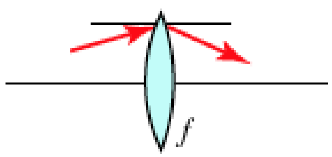
Spherical boundary



Convex, $R > 0$; concave, $R < 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{(n_2 - n_1)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$$

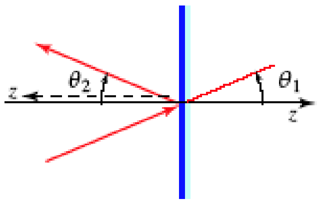
Lens



Convex, $f > 0$; concave, $f < 0$

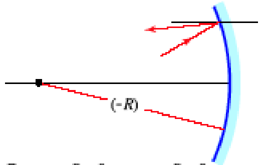
$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

Planar mirror



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

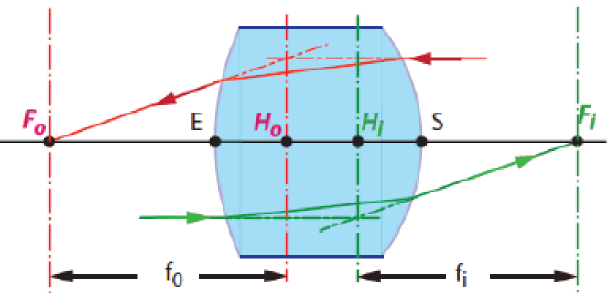
Spherical mirror



Concave, $R < 0$; convex, $R > 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$

Example: Cardinal Points & Planes of a Thick lens

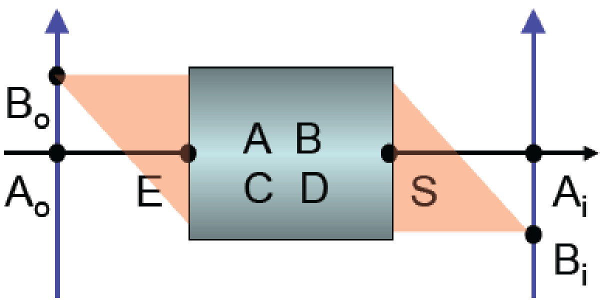


Lens thickness = $\overline{ES} = e$

Lens index = n Outside index =1

Opt. Power of 1st Surface $\Phi_1 = (n-1)/nR_1$

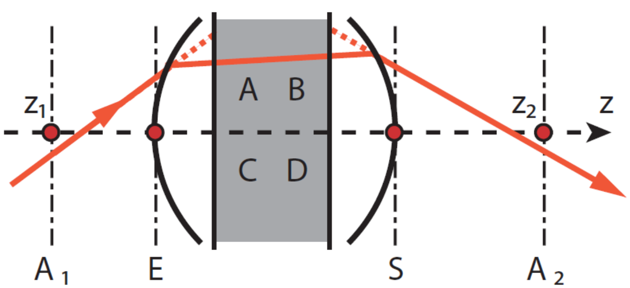
Opt. Power of 2nd Surface $\Phi_2 = (1-n)/R_2$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} =$$

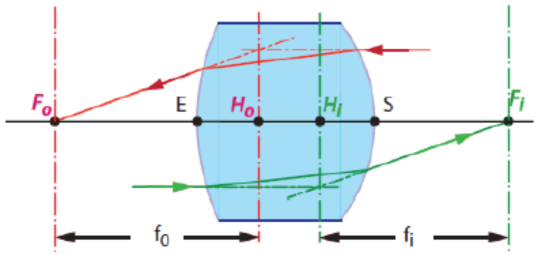
$$= \begin{bmatrix} 1 & 0 \\ -\Phi_2 & n \end{bmatrix} \times \begin{bmatrix} 1 & e \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -\Phi_1 & 1/n \end{bmatrix}$$

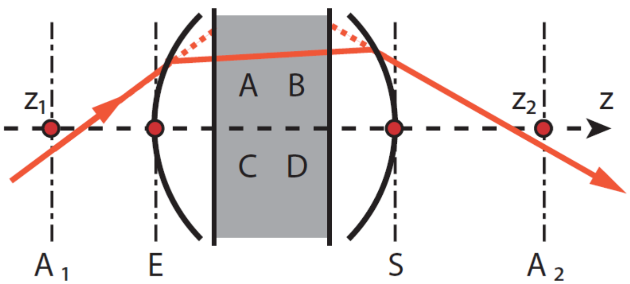
$$= \begin{bmatrix} 1 - e\Phi_1 & e/n \\ -\Phi_{tot} & 1 - e\Phi_2/n \end{bmatrix}$$



$$\Phi_{tot} = n\Phi_1 + \Phi_2 - e\Phi_1\Phi_2$$
$$= (\mathrm{n}-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{n-1}{n}\frac{e}{R_1R_2}\right)$$

Example: Cardinal Points & Planes of a Thick lens





Distances:	Notation:	Directed Distances:	ABCD elements:
Object Focal Point	F_o	$\overline{EF_o}$	D/C
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$
Object Principle Plane	H_o	$\overline{EH_o}$	$(D-1)/C$
Image Focal Point	F_i	$\overline{SF_i}$	$-A/C$
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$
Image Principle Plane	H_i	$\overline{SH_i}$	$(1-A)/C$

$$A = 1 - e\Phi_1$$

$$B = e/n$$

$$C = -\Phi_{tot}$$

$$D = 1 - e\Phi_2/n$$

Object Focal Point Position

$$F_o = \overline{EF_o} = \frac{D}{C} = \frac{\frac{e\Phi_2}{n} - 1}{\Phi_{tot}}$$

Image Focal Point Position

$$F_i = \overline{SF_i} = \frac{-A}{C} = \frac{e\Phi_1 - 1}{\Phi_{tot}}$$

Object Principle Plane Position

$$H_o = \overline{EH_o} = \frac{1}{C}(D-1) = \frac{1}{-\Phi_{tot}}\left(1 - \frac{e\Phi_2}{n} - 1\right) = \frac{e\Phi_1}{n\Phi_{tot}}$$

Object Focal & Image Focal Lengths

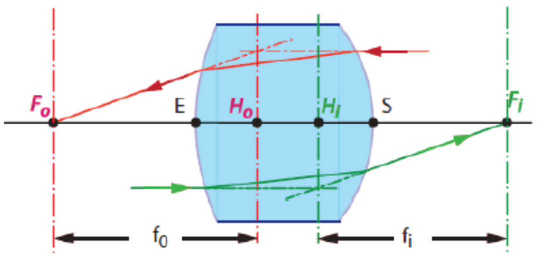
$$f_i = -f_o = -\frac{1}{C} = \frac{1}{\Phi_{tot}}$$

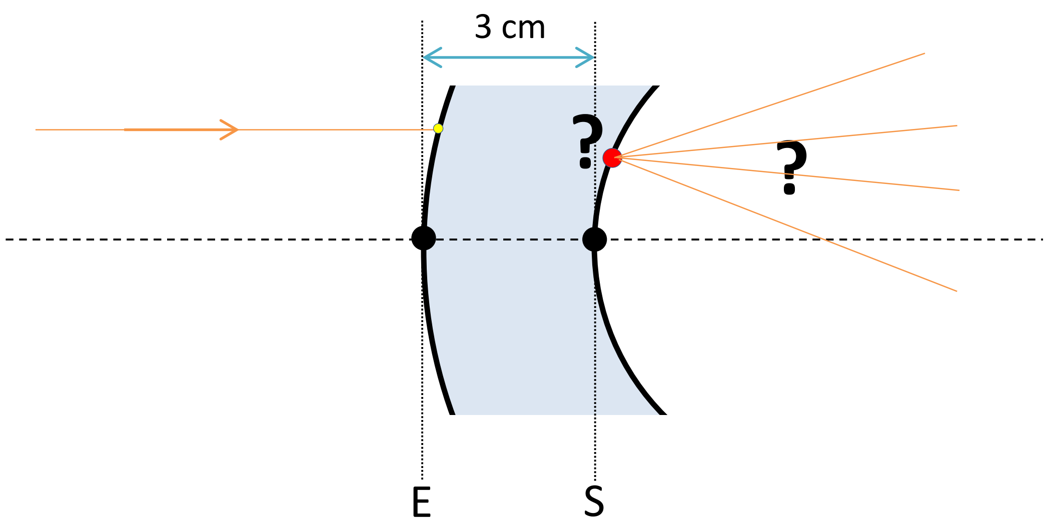
Image Principle Plane Position

$$H_i = \overline{SH_i} = \frac{1}{C}(1-A) = \frac{1}{-\Phi_{tot}}(1 - 1 + e\Phi_1) = -\frac{e\Phi_1}{\Phi_{tot}}$$

Example: if we use ray tracing for a thick lens

Lens parameters	Num. value
1^{st} interface curvature (R_1)	+5 cm
2^{nd} interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5





Can you determine the path of the ray after the thick lens?

This means, identifying:

- The point that the ray exits the lens?
- The exit angle?

➔ **Instead of ray optics, lets use the approach of cardinal points and planes!**

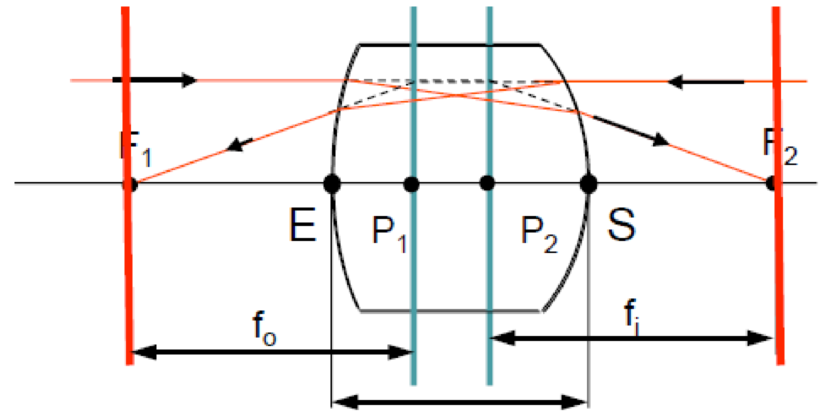
Example: if we use cardinal planes for a thick lens

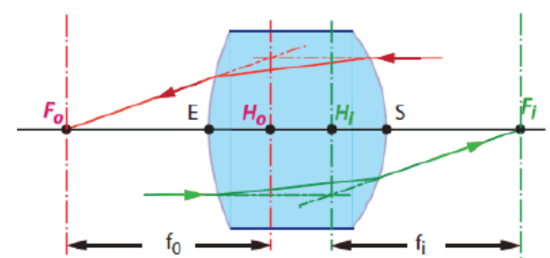
Lens thickness = $\overline{ES} = e$

Lens index = n Outside index =1

Optical Power of 1st curvature $\Phi_1 = (n-1)/nR_1$

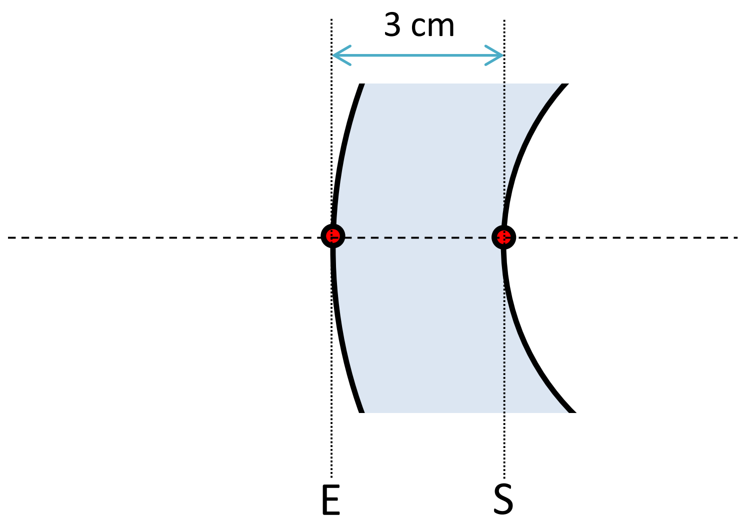
Optical Power of 2nd curvature $\Phi_2 = (1-n)/R_2$

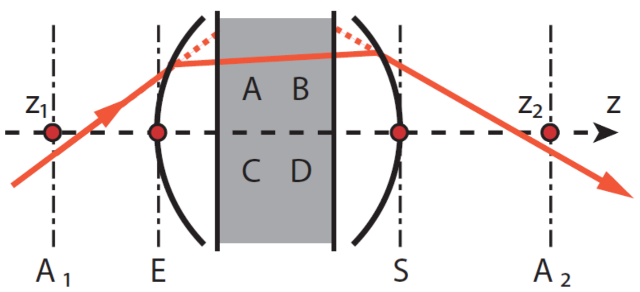




Example: Ray Tracing a Thick lens

Lens parameters	Num. value
1^{st} interface curvature (R_1)	+5 cm
2^{nd} interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5

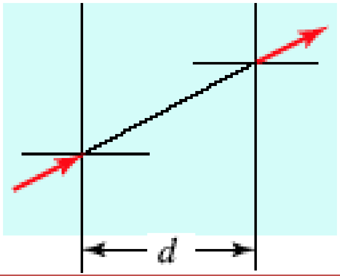




$$T(E \rightarrow S) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = T_{2nd\ spherical\ curvature} \times T_{propagation} \times T_{1st\ spherical\ curvature}$$

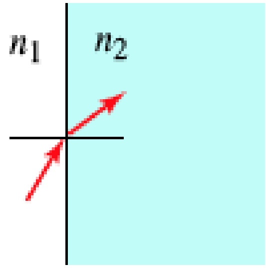
Reminder: Simple Optical Components

Propagation



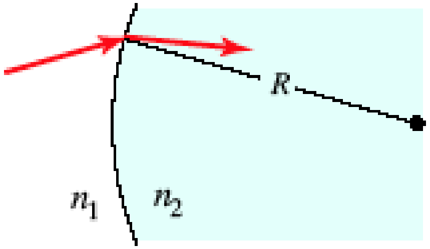
$$\mathbf{M} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Planar boundary



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

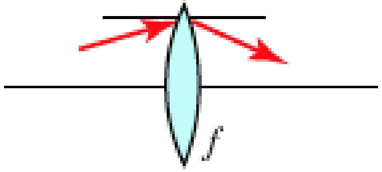
Spherical boundary



Convex, $R > 0$; concave, $R < 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{(n_2 - n_1)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$$

Thin Lens



Convex, $f > 0$; concave, $f < 0$

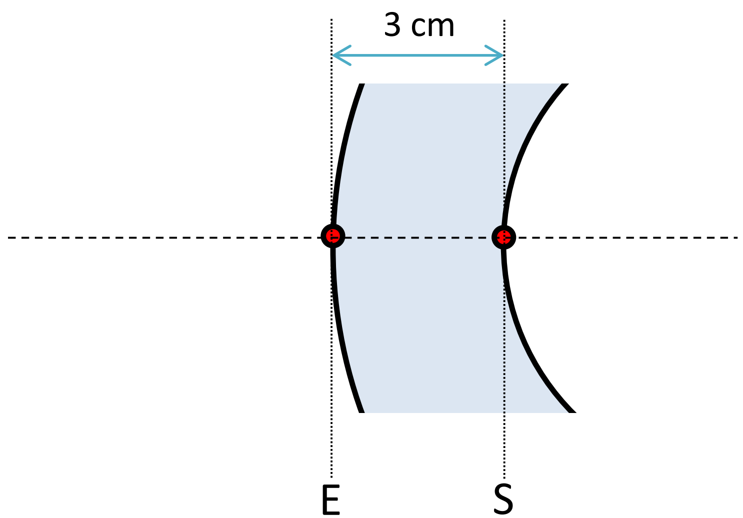
$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

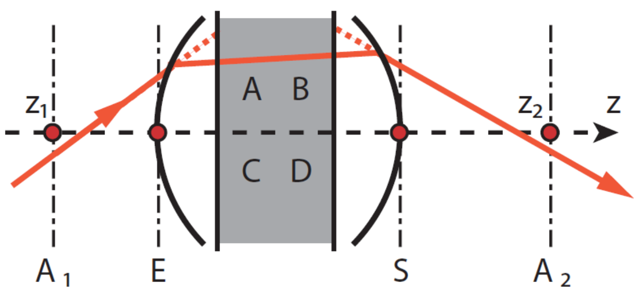
Example: Ray Tracing Thick lens

Lens parameters	Num. value
1st interface curvature (R_1)	+5 cm
2nd interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5

$$\Phi_1 = (n-1)/nR_1 = (1.5-1)/1.5x5 = 2/30$$

$$\Phi_2 = (1-n)/R_2 = (1-1.5)/2 = -1/4$$





$$T(ES) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = T_{2nd\ spherical\ curvature} \times T_{propagation} \times T_{1st\ spherical\ curvature}$$

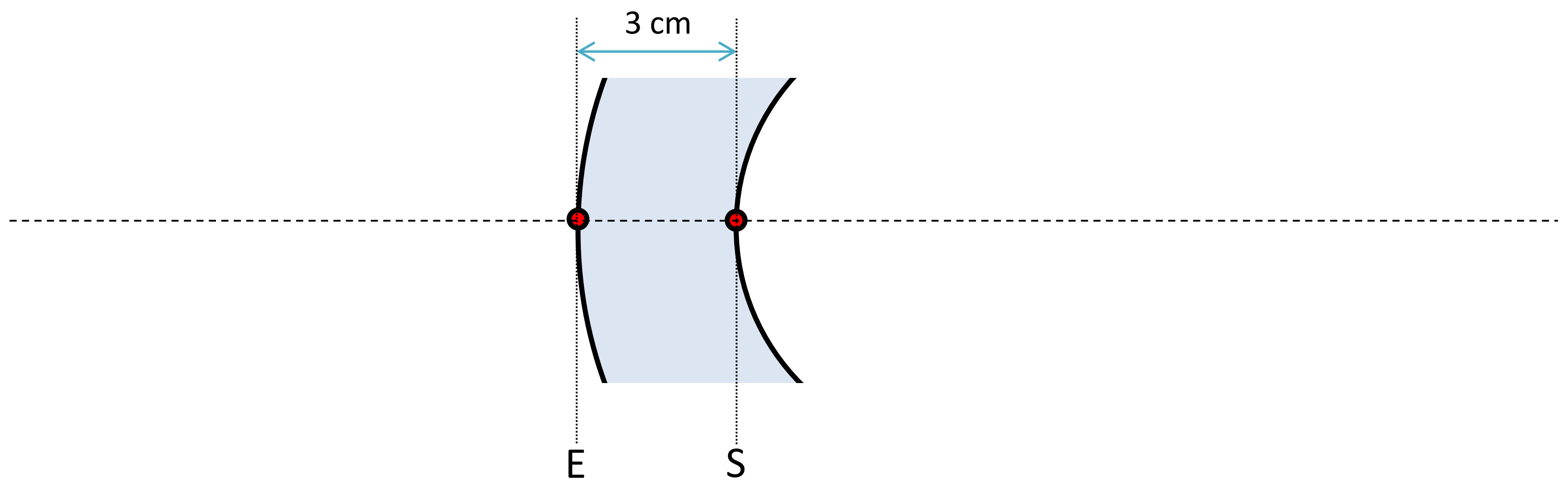
$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\Phi_2 & n \end{bmatrix} \times \begin{bmatrix} 1 & e \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -\Phi_1 & 1/n \end{bmatrix}$$

Example: Thick lens

Lens parameters	Num. value
1st interface curvature (R_1)	+5 cm
2nd interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5

$$T(ES) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.8 & 2 \\ 0.1 & 1.5 \end{bmatrix}$$

Distances	Notation	Directed Distances	ABCD elements
Object Focal Point	F_o	$\overline{EF_o}$	D/C
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$
Object Principle Plane	H_o	$\overline{EH_o}$	$(D-1)/C$
Image Focal Point	F_i	$\overline{SF_i}$	$-A/C$
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$
Image Principle Plane	H_i	$\overline{SH_i}$	$(1-A)/C$

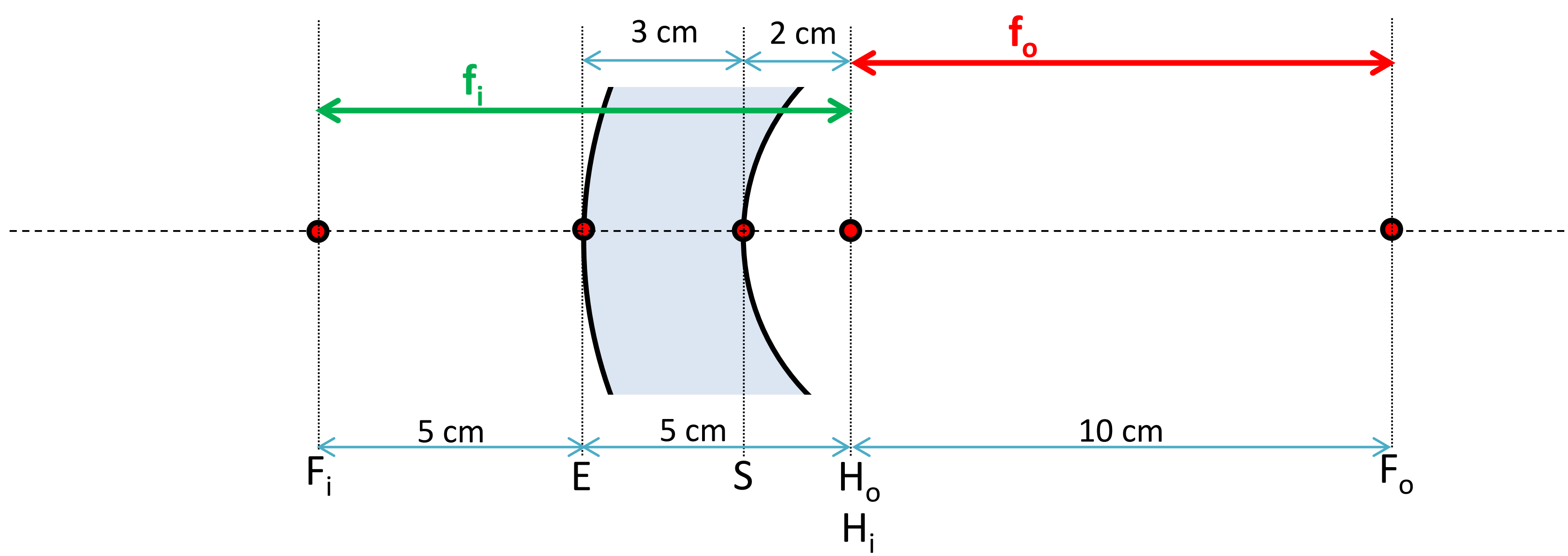


Example: Thick lens ➔ Object Focal Length f_o

Lens parameters	Num. value
1st interface curvature (R_1)	+5 cm
2nd interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5

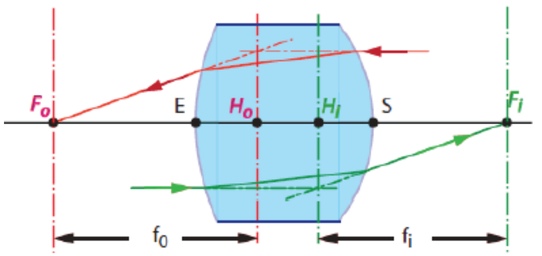
$$T(ES) = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0.8 & 2 \\ 0.1 & 1.5 \end{bmatrix}$$

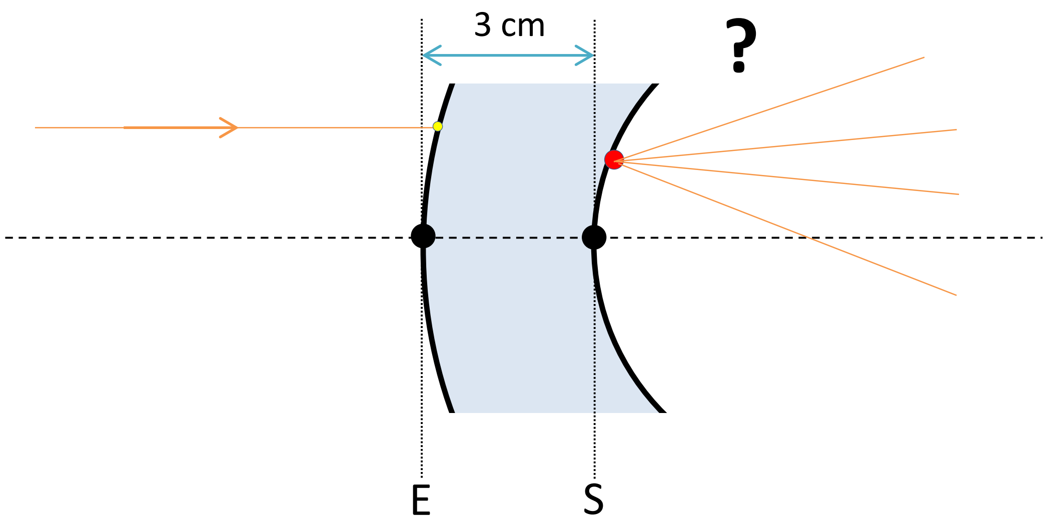
Distances	Notation	Directed Distances	ABCD elements	Calculated Value
Object Focal Point	F_o	$\overline{EF_o}$	D/C	+15 cm
Object Focal Length	f_o	$\overline{H_oF_o}$	$1/C$	+10 cm
Object Principle Plane	H_o	$\overline{EH_o}$	$(D-1)/C$	+5 cm
Image Focal Point	F_i	$\overline{SF_i}$	$-A/C$	-8 cm
Image Focal Length	f_i	$\overline{H_iF_i}$	$-1/C$	-10 cm
Image Principle Plane	H_i	$\overline{SH_i}$	$(1-A)/C$	+2 cm



Remember the question:

Lens parameters	Num. value
1st interface curvature (R_1)	+5 cm
2nd interface curvature (R_2)	+2 cm
Central thickness, e	3 cm
Lens index (glass)	1.5



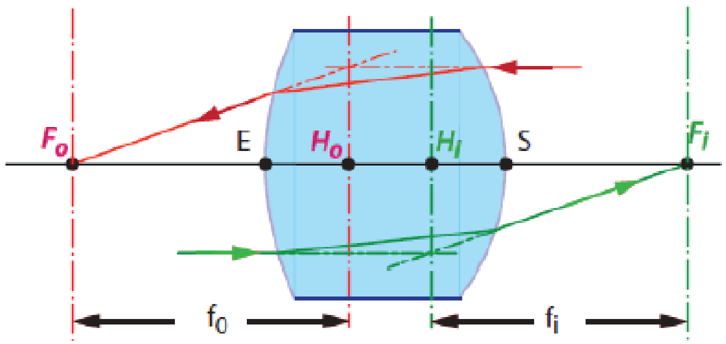


Can you determine the path of the ray after the thick lens?

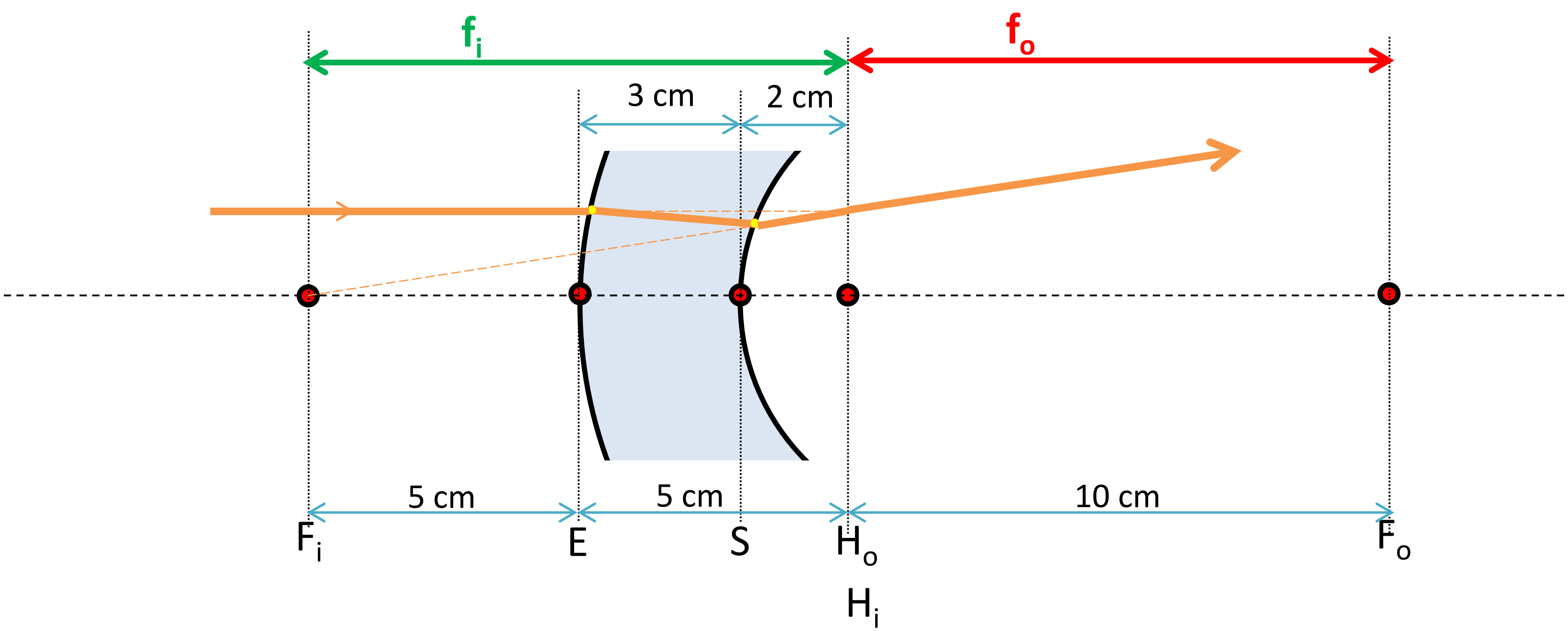
This means, finding :

- the point that the ray exits the lens?
- the exit angle?

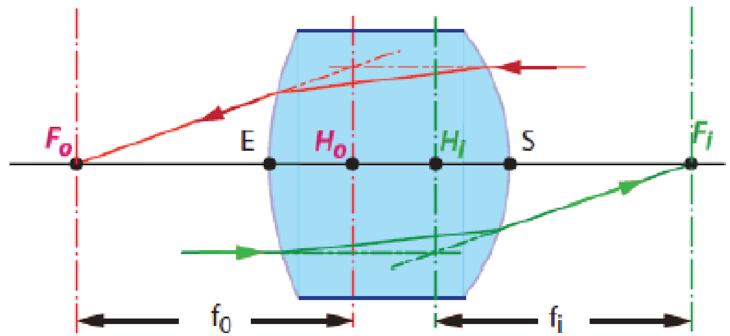
Thick lens example: ray tracing with cardinal planes



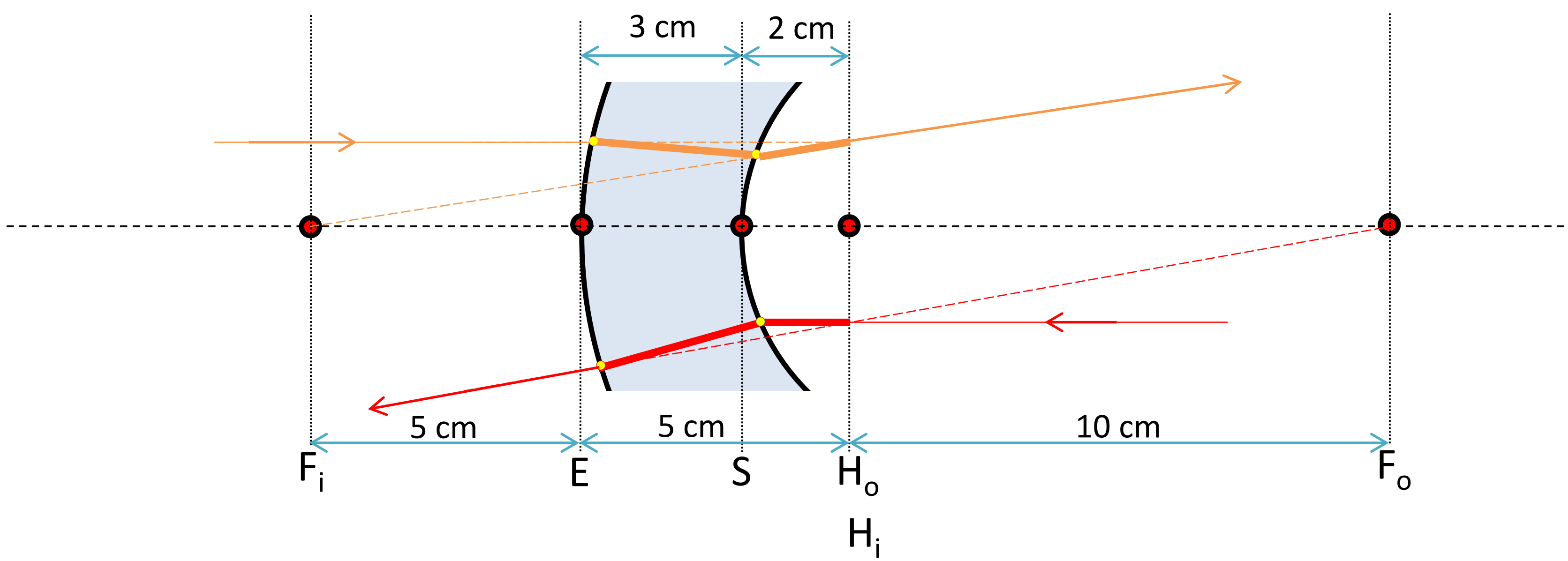
Ray tracing is simpler to implement with the cardinal points & planes:



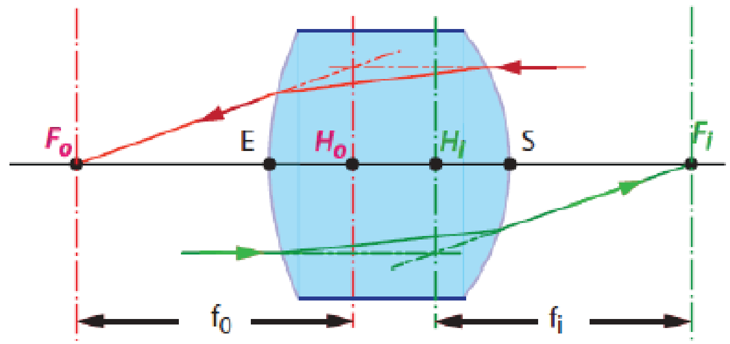
Thick Lens Example: Ray Tracing with Cardinal Points



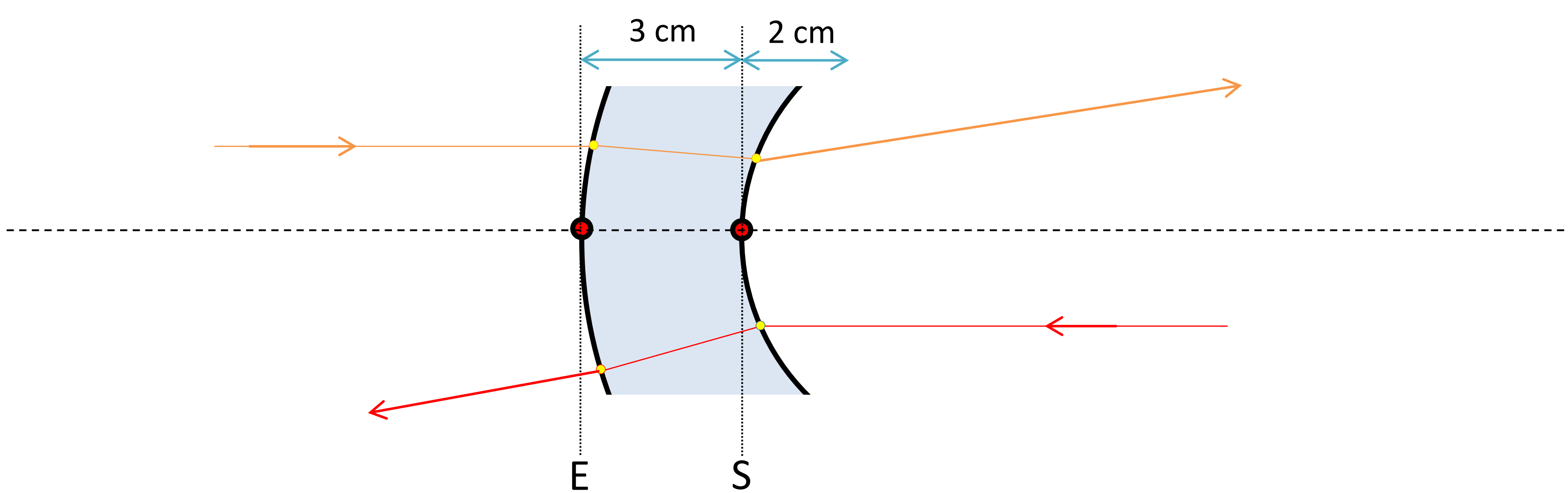
Ray tracing is simpler to implement with the cardinal points & planes:



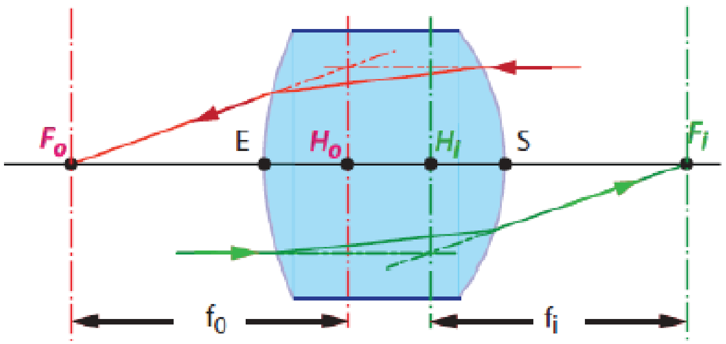
Thick Lens Example Summary: Ray Tracing With Cardinal Points



Ray tracing is simpler to implement with the cardinal points & planes.



Example summary: Thick lens



Cardinal plane(s)

- *Principal planes* H_o, H_i
- *Not always inside the thick lens*
- *The refraction happens at the virtual/fictious principal plane*
- *Always magnification M = 1 between principal planes*
- *Focal plane(s)* F_o, F_i
- *Common point for rays parallel to axis*