Biomicroscopy I - Solutions Exercise Sheet 5

October 8, 2024

1 Geometrical Optics Construction

A. In order to find the focal planes, we use the fact that parallel rays are deviated such that they intersect in the focal plane and rays through the center of a lens are not deviated (see Figure 1). For further explanation see the previous series.

- f_{1i} = 30mm
- f_{2i} = -60mm
- f_{3i} = 30mm
- f_{4i} = 15mm

Furthermore, notice that an intermediate image is formed on lens L_3, which might simplify the ray tracing. When intermediate image is formed you may choose another pair of rays to find the actual image location (e.g. try to use starting from the intermediate image one ray which is again parallel to the optical axis and another one passing through the optical centre of lens L_4 to see that it forms the same image).

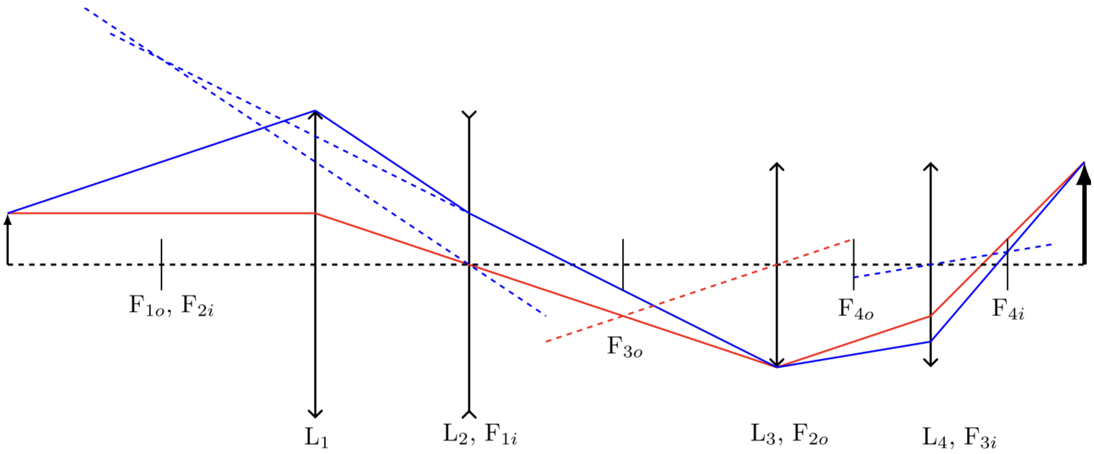

Figure 1: Focal plane construction.

B. The intersection of the last segment of the red ray with the optical axis defines the image side focal plane, since it enters parallel to the optical axis. The image side principal plane is at the position when it reaches the same height as the incoming ray (see Figure 2).

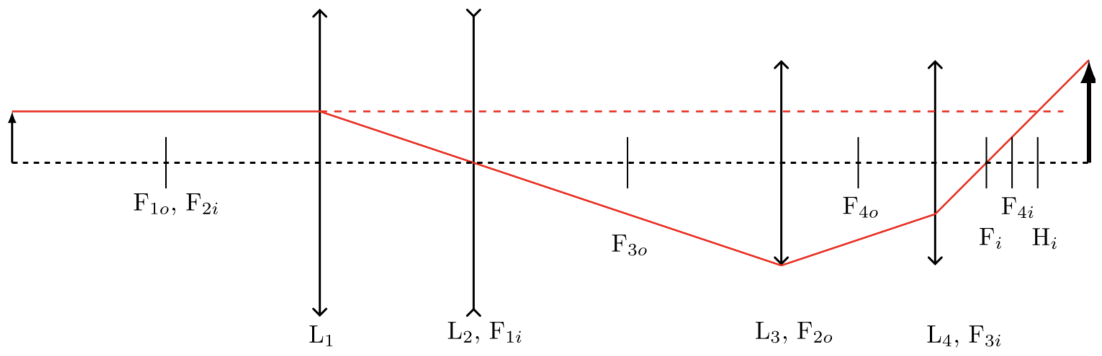

Figure 2: Image side principal plane and focal plane construction.

C. We trace a ray parallel to the optical axis coming from the right propagating to the left. At the same time, we ray trace from right to left: as the ray arrives to the first lens perpendicular it passes through the focus. After that with the aid of helper rays (dashed line) we will know where the ray will cross the next lenses. Then finding the object side focal plane (Fo) and the object side principal plane (Ho) is trivial as it would be checking the cross with the optical axis and with the first horizontal line we have traced at the beginning (see Figure 3).

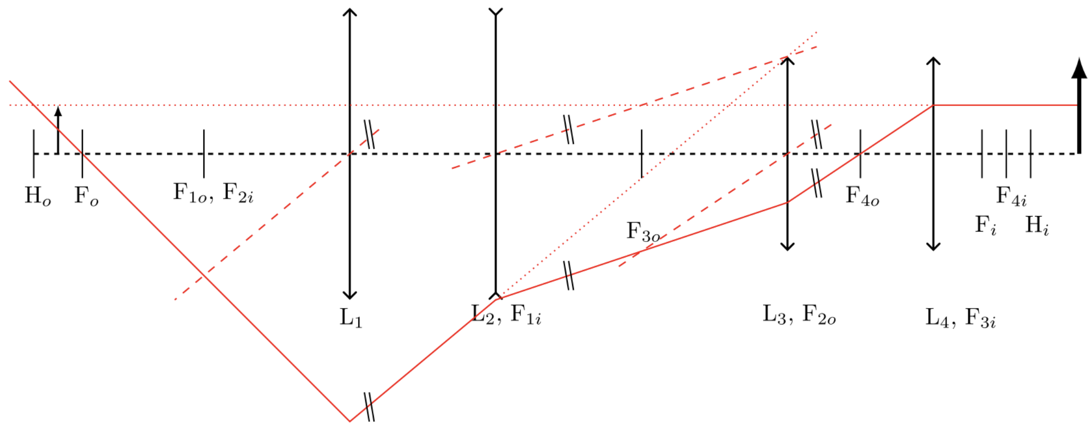

Figure 3: Object side principal plane and focal plane construction.

D. The cardinal points are the focal points and planes, principal planes, nodal points (the intersection of the optical axis with the principal planes for a system in air) and surface vertices (first and last surface points on the optical axis of the system). In paraxial optics they completely describe the system. Omitting the space between the principal planes and superposing them, the system can be considered as a thin lens at the position of the principal planes with the focal length given by the cardinal points:

- The focal length is $f_i = -f_o = \overline{H_iF_i} = -\overline{HoFo} = -1\text{cm}$.
- The object and image distances are $d_o = \overline{H_oO} = 0.5\text{cm}$ and $d_i = \overline{HiI} = 1\text{cm}$.
- The magnification is $m = 2$.

Recall that the transfer matrix for a system from the object through a thin lens to an image is:

$$M = T_i R T_o = \begin{bmatrix} 1 & d_i \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_i} & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -d_o \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 - \frac{d_i}{f_i} & d_i - d_o + \frac{d_i d_o}{f_i} \\ -\frac{1}{f_i} & 1 + \frac{d_o}{f_i} \end{bmatrix}.$$

Plugging in the measured values, we have:

$$M = T_i R T_o = \begin{bmatrix} 1 - \frac{1}{-1} & 1 - 0.5 + \frac{1 \cdot 0.5}{-1} \\ -\frac{1}{-1} & 1 + \frac{0.5}{-1} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 1 & 0.5 \end{bmatrix}.$$

Therefore, the magnification is M_{11}=m=2. Note that we have a correct object-image relation (that is, the imaging condition is satisfied), as $M_{12} = 0$. We further note that, for fast calculations, the imaging condition $M_{12} = 0$ is equivalent to

$$\frac{1}{-d_0} + \frac{1}{d_i} = \frac{1}{f_i}$$

and due to similarity of triangles, the magnification m can be written as

$$m = \frac{d_i}{d_o}$$

E. The marginal ray (green) defines the aperture: the first lens L_1 (see Figure 4). Since it is the first diffracting element it is also the entrance pupil. The image of the aperture through the system to the right is the exit pupil. An image is the intersection of two arbitrary rays starting at the same object point. Since the blue ray was given by the exercise, the image can be found by the intersection of the green and the blue ray.

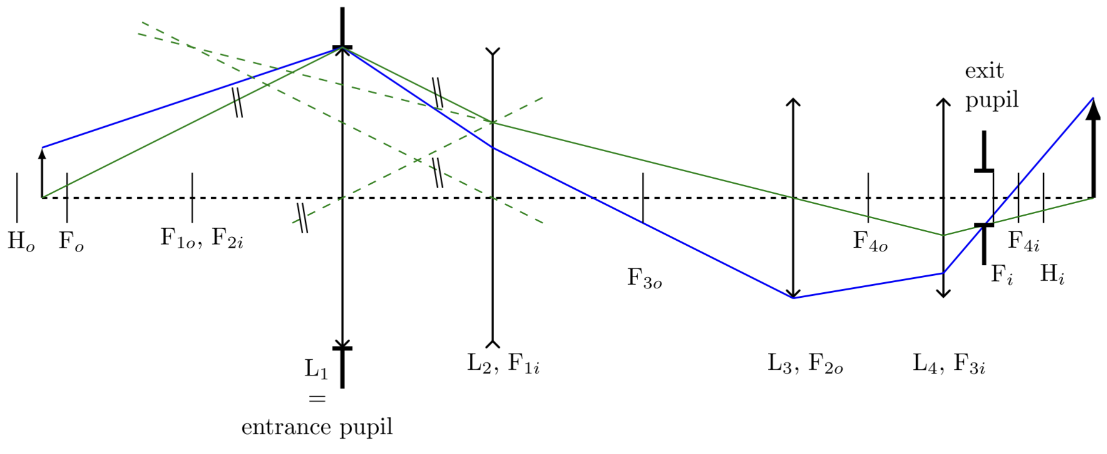

Figure 4: Aperture and the entrance and exit pupils.

F. The field stop is found with the chief ray: The first element that limits a ray going through the center of the entrance pupil when it starts at higher and higher positions in the object plane is the field stop. In this case it is the lens L3, which is located on an intermediate image plane (see Figure 5). Since it is an intermediate image, the field stop located at this position is imaged back and forth to the object and image plane respectively, which defines directly the entrance and the exit windows.

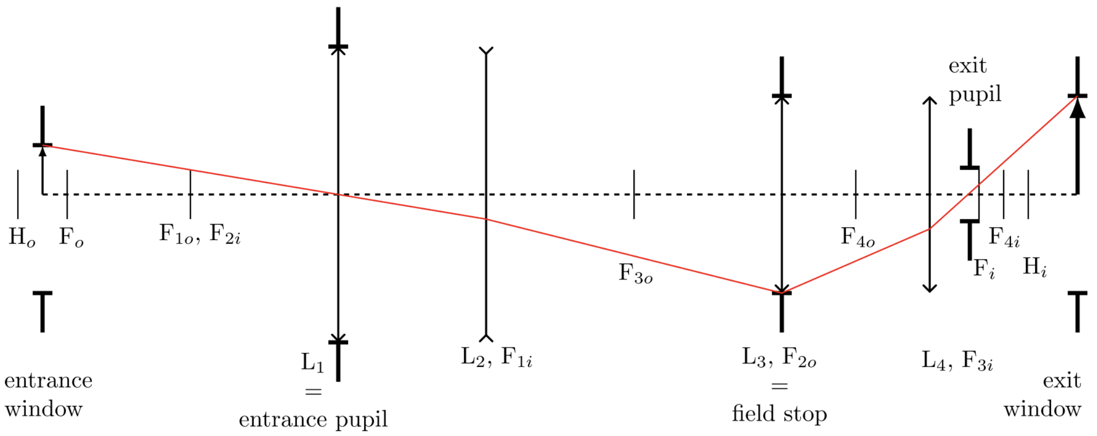

Figure 5: Chief ray, field stop, entrance and exit windows.

G. Since the lens L_3 is located on an intermediate image plane, the final image is the same with or without lens L_3. However, note, that by omitting the lens, the chief ray is now cut by lens L_4, which therefore becomes the field stop (see Figure 6). The entrance window shows that the field of view is much larger with lens L3, and much smaller without it. Therefore this lens is a *field lens*.

The same effect could be attained with a larger lens L_4. However it is very difficult and expensive to fabricate high quality large lenses with a relatively short focal length. In this case a field lens can help.

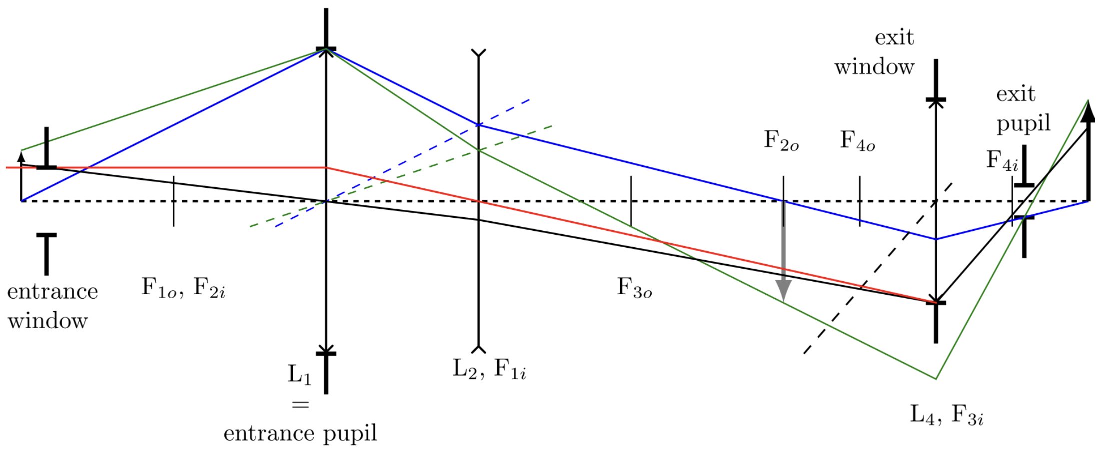

Figure 6: Effect of a field lens.

2 Magnification with a single lens (Loupe)

Given the object size $h = 5\text{mm}$, the relaxed eye condition $w = 25\text{cm}$ and the focal length of the loupe $f_M = 25.4\text{mm}$:

A. The angular magnification M is

$$M = \frac{\alpha_M}{\alpha_{unaid}} = \frac{w}{f_M} = \frac{25\text{cm}}{2.54\text{cm}} = 9.84$$

B. Let h_i denote the image size. It is given by

$$h_i = h \cdot M = 5\text{mm} \cdot 9.84 = 49.2\text{cm}$$

C. The angle subtended by the diamond at the unaided eye when held at near point is

$$\alpha_{unaid} = -\frac{h}{w} = -\frac{5\text{mm}}{250\text{mm}} = 0.02$$

D. The angle subtended at the aided eye is

$$\alpha_{aid} = -\frac{h}{f_M} = -\frac{5\text{mm}}{25.4\text{mm}} = 0.197$$