Biomicroscopy I - Exercise 08

November 5, 2024

1 Fourier transform with a 2-lens, 4*f* system

When using two lenses, we can do the Fourier transform two times such that we can reconstruct the image of our object in the final image plane, as shown in Figure 1. If the two lenses are identical, we call this a $4f$ configuration.

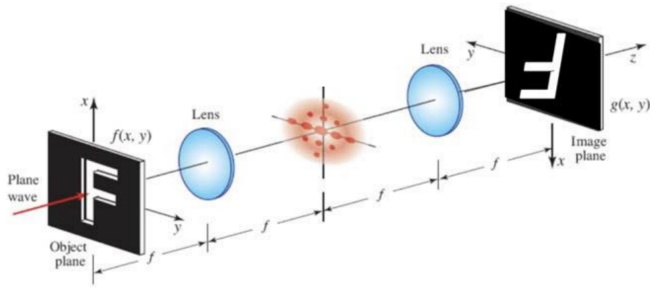

Figure 1: $4f$ system.

A. In Figure 1, the object is a letter 'F'. Using ray tracing, show that the image and the object are identical to each other apart from an inversion (*i.e.* magnification $m = -1$).

B. In this $4f$ system, you can observe the Fourier transform of the object in the back-focal plane of the first lens. Find this plane in Figure 1.

C. Assume a $4f$ system as shown in Figure 2 in which the object is a cosine function (sinusoidal along the x-axis). Sketch the expected pattern at the back focal plane of the first lens and second lens.

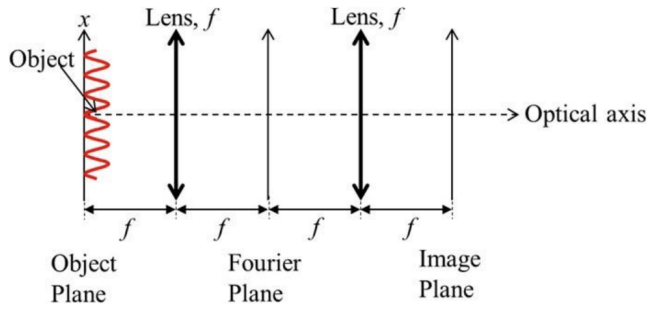

Figure 2: $4f$ system with a sinusoidal object

D. Assume a $4f$ system as shown in Figure 3 in which the object is a centered rectangular function along the x-axis. Sketch the expected patterns at the back focal plane of the first lens and second lens. How will your sketches change if the width of the rectangular function increases twice? How will your result change if the width of the rectangular function decreases by half?

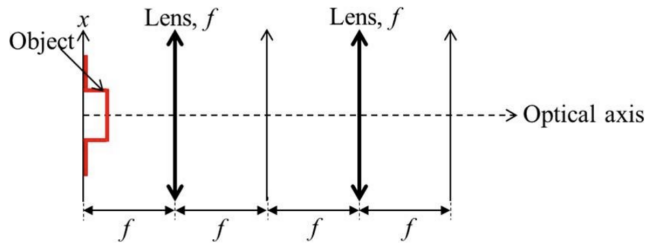

Figure 3: $4f$ system with a rectangular object

2 'Fourier house' with a 2-lens, $4f$ system

Consider a 2-lens, $4f$ imaging system as shown in the figures from last week's exercise (Ex. 8, 1, 2, 3). Imagine an object as pictured on Figure 4, which is a house with different sinusoidal shadings.

Figure 4: House with different sinusoidal shadings

A. If this object is placed in the object plane of a $4f$ configuration, what would you observe in the back-focal plane of the first lens (*i.e.* at the Fourier transformation plane)?

B. Describe what you would see in the back-focal plane of the second lens (*i.e.* at the image plane).

C. Imagine you place a filter in the back-focal plane of the first lens. The filter is an opaque plate containing a transparent hole (or multiple holes) of a certain shape and size. What is the filter to see only the roof of the house in the image plane?

D. What filter would you use in order to see only the sky?

Hint: assume the sinusoidal shadings in the house have a period T with an offset of 1.

3 Point spread function (PSF) in a $4f$ configuration

A. Imagine a $4f$ system in which the object is a Dirac delta function (representing a point source) as shown in Figure 5. Assume that the lenses are infinitely large and there is no aperture in the optical pathway. Sketch what you expect to observe at the Fourier and image planes.

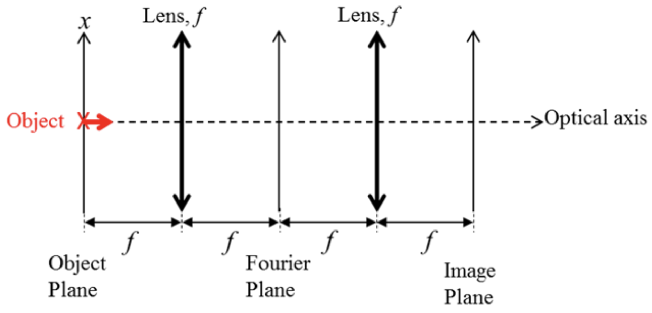

Figure 5: House with different sinusoidal shadings

B. Assume you have an aperture in your $4f$ system in the back-focal plane, as shown in Figure 6, which limits the amount of light transmitted through your system. Suppose your aperture is described by a rectangular function: $f(x) = \text{rect}\left(\frac{x}{a}\right)$.

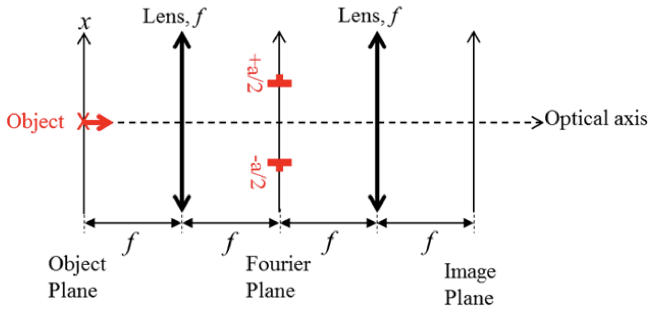

Figure 6: House with different sinusoidal shadings

What would you observe in the image plane? What is the physical meaning of this result?