MICRO-561

Biomicroscopy I

Syllabus (tentative)

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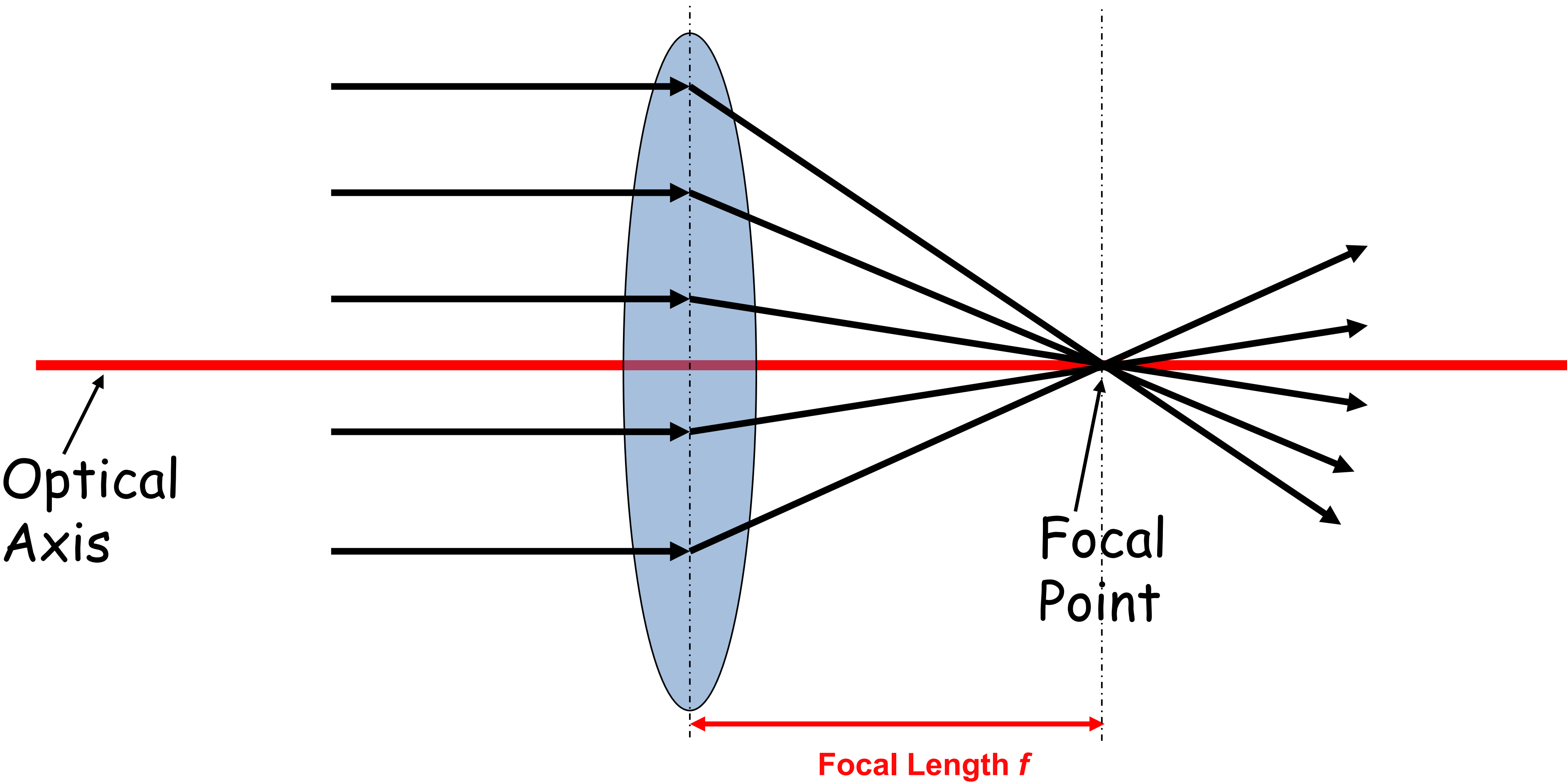
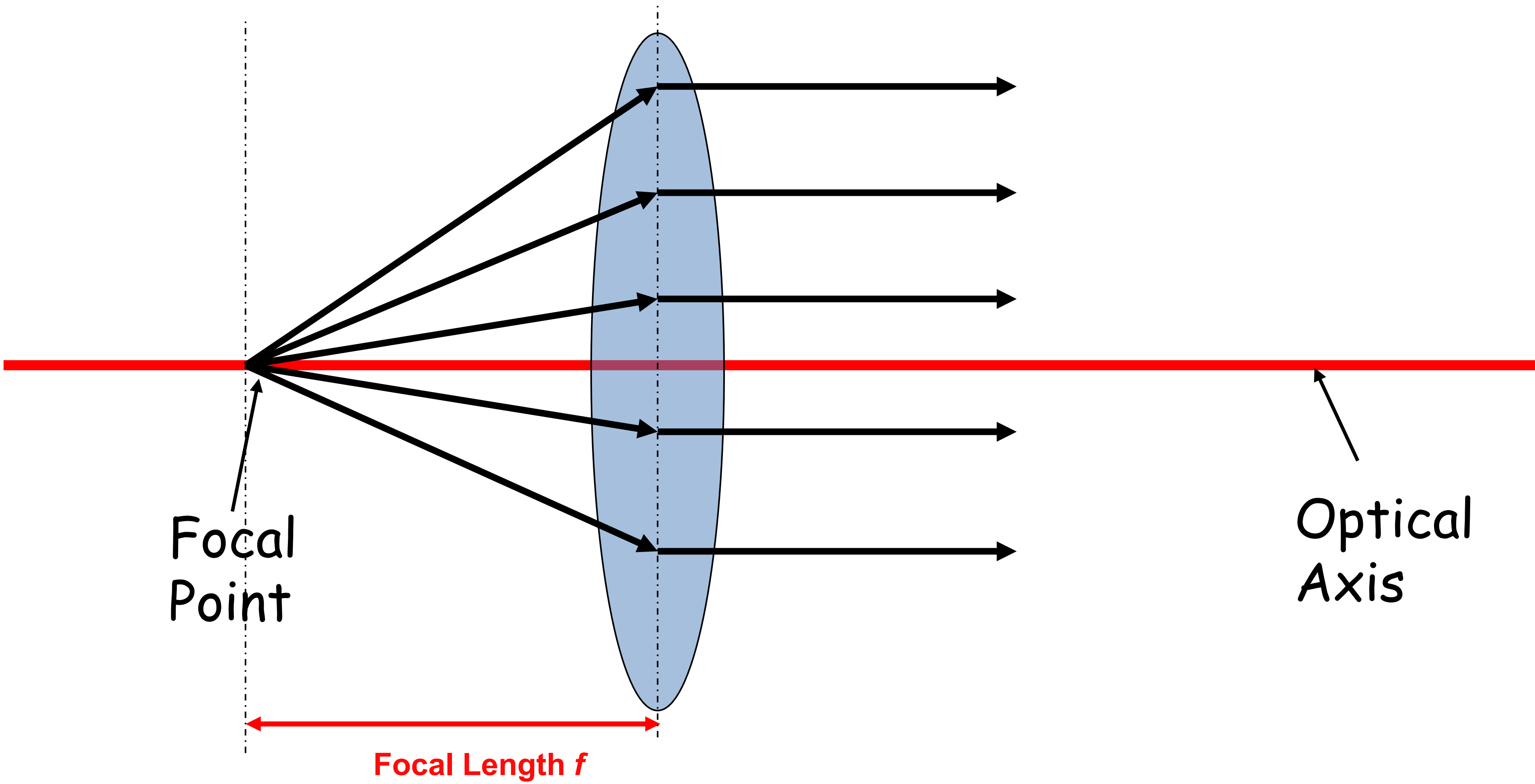
Ray Tracing: Thin Lens

1. **The Parallel Ray:** Light rays that enter the lens parallel to the optical axis leave the lens by passing through <u>Focal Point</u>



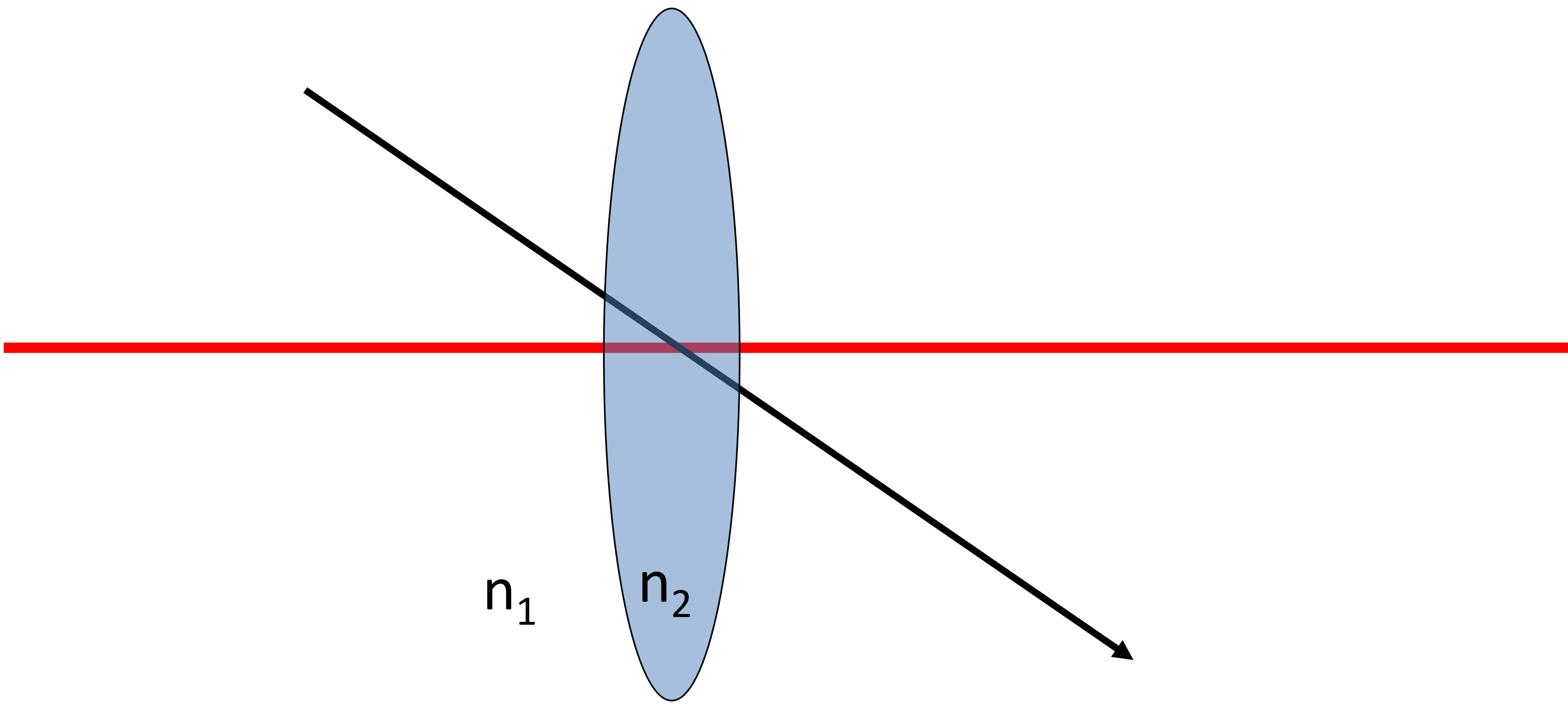
Ray Tracing: Thin Lens

2. The Focal Ray: Light rays that enter the lens from the focal point exit the lens parallel to the optical axis.



Ray Tracing: Thin Lens

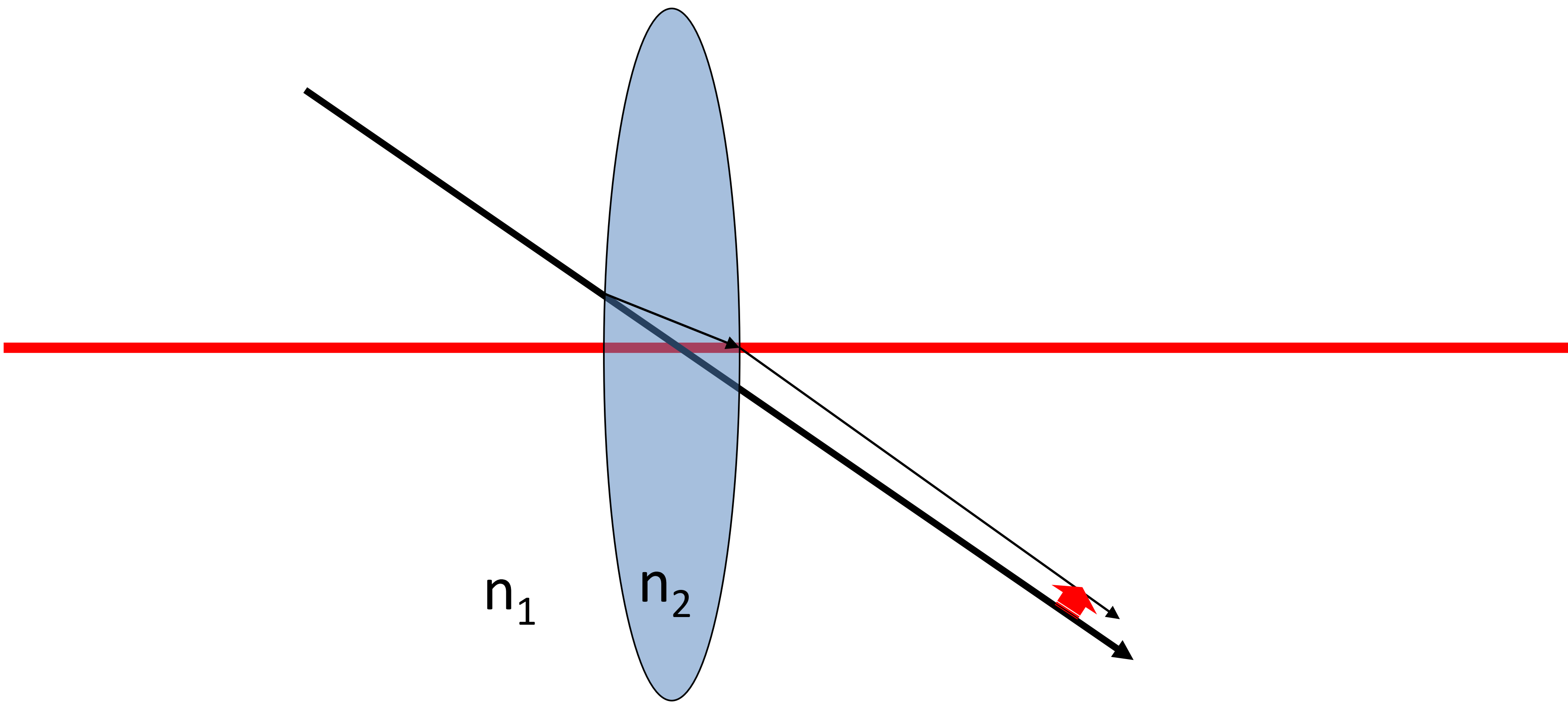
3. **The Chief Ray:** A ray that enters through the center of lens goes straight, without deviation



Ray Tracing: Thin Lens

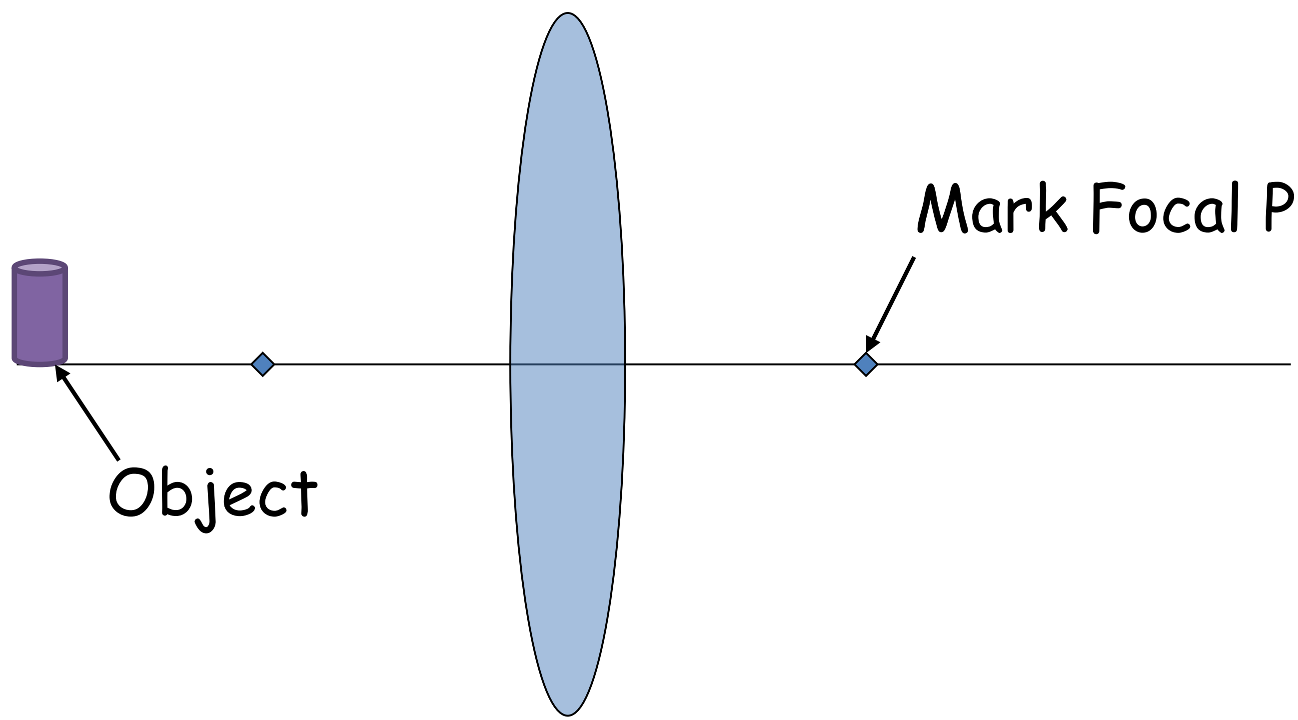
3. **3. The Chief Ray:** A ray that enters through the center of lens goes straight, without deviation

Correction: Note that normally there is a shift, but this shift can be neglected only if the glass is thin.



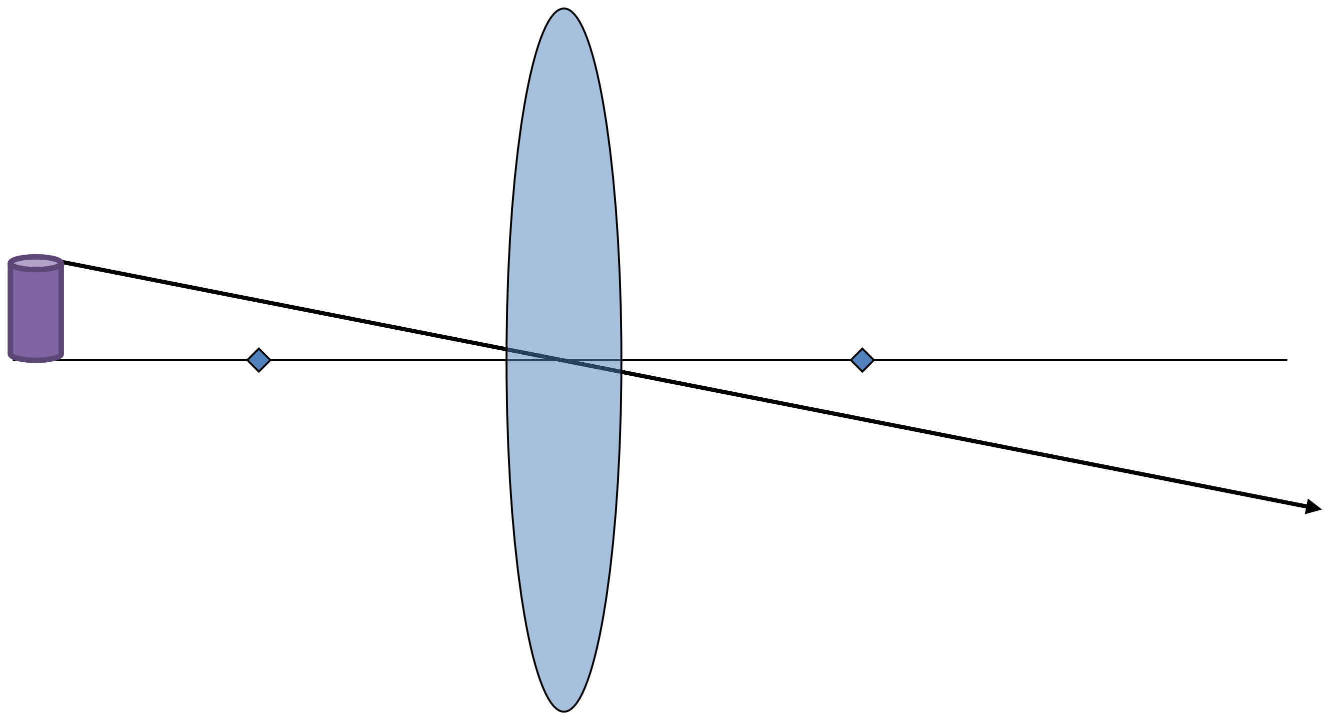
Example: Ray tracing with a positive thin lens

Let's applying ray tracing using **three principle rays** to an extended object, and find its image



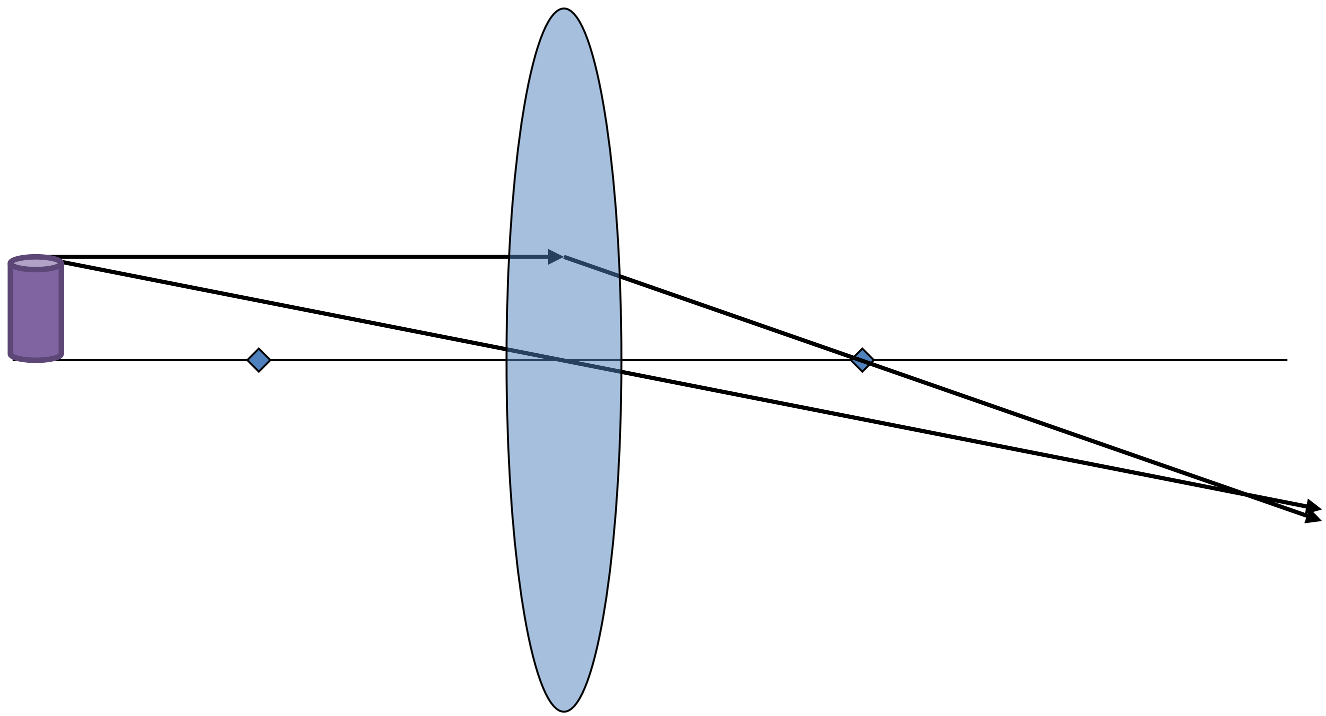
Example: Ray tracing with a positive thin lens

1. **The Chief Ray:** A ray enters through the center of the lens goes straight



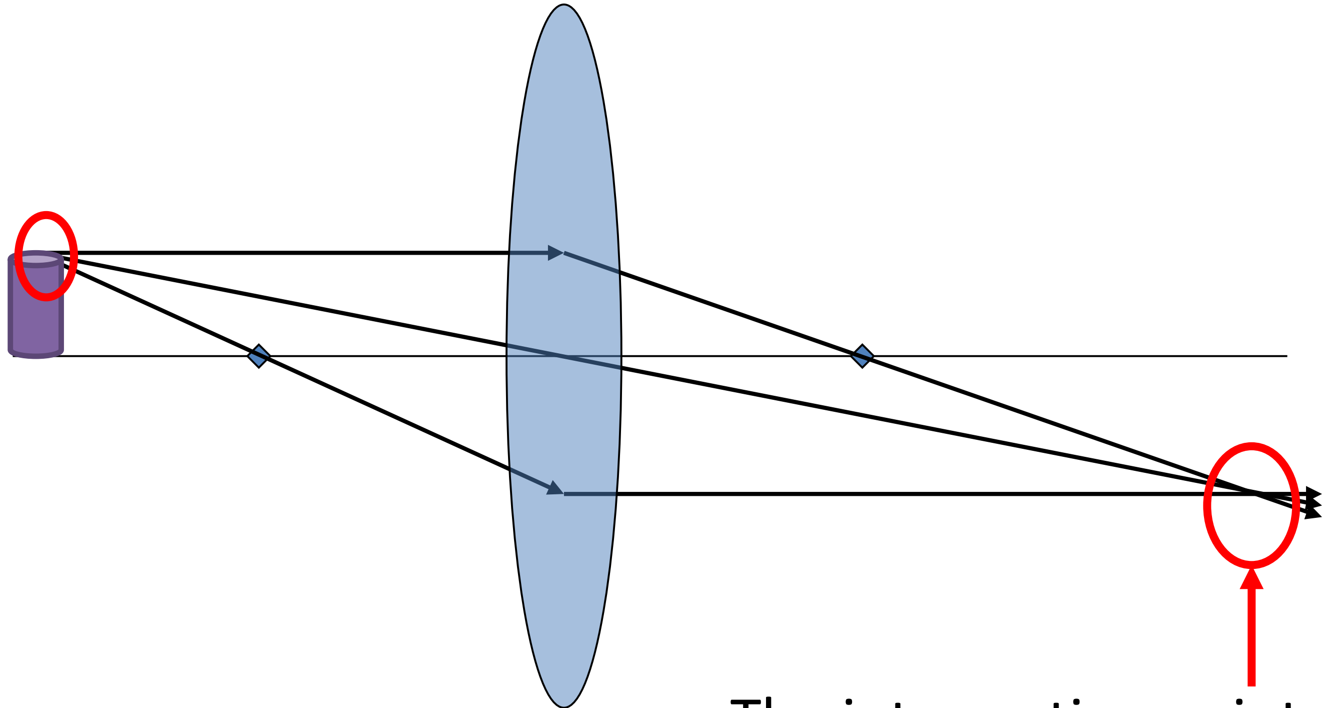
Example: Ray tracing with a positive thin lens

2. **The Parallel Ray:** Light rays that enter the lens parallel to the optical axis leaves through Focal Point



Example: Ray tracing with a positive thin lens

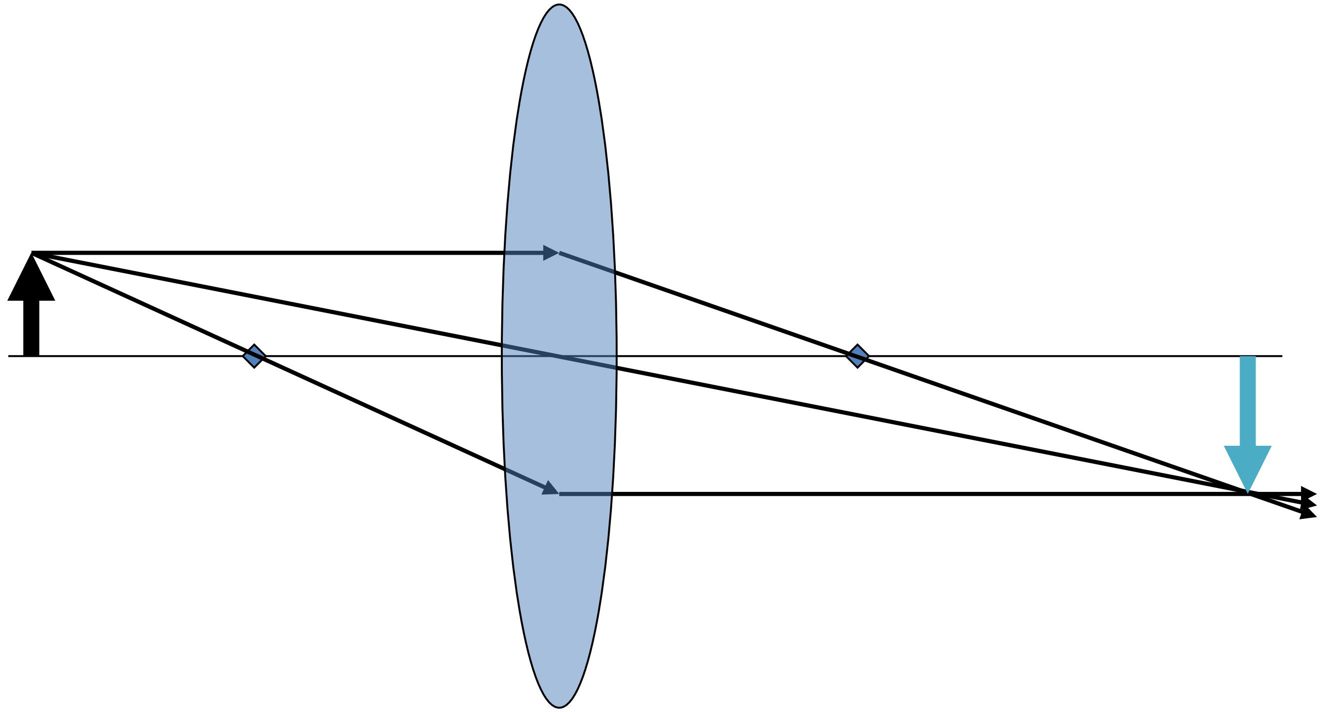
3. **The Focal Ray:** Light rays that enter the lens from the focal point, exit parallel to the optical axis.



The intersection point of these three lines corresponds to the “**image**” of the object.

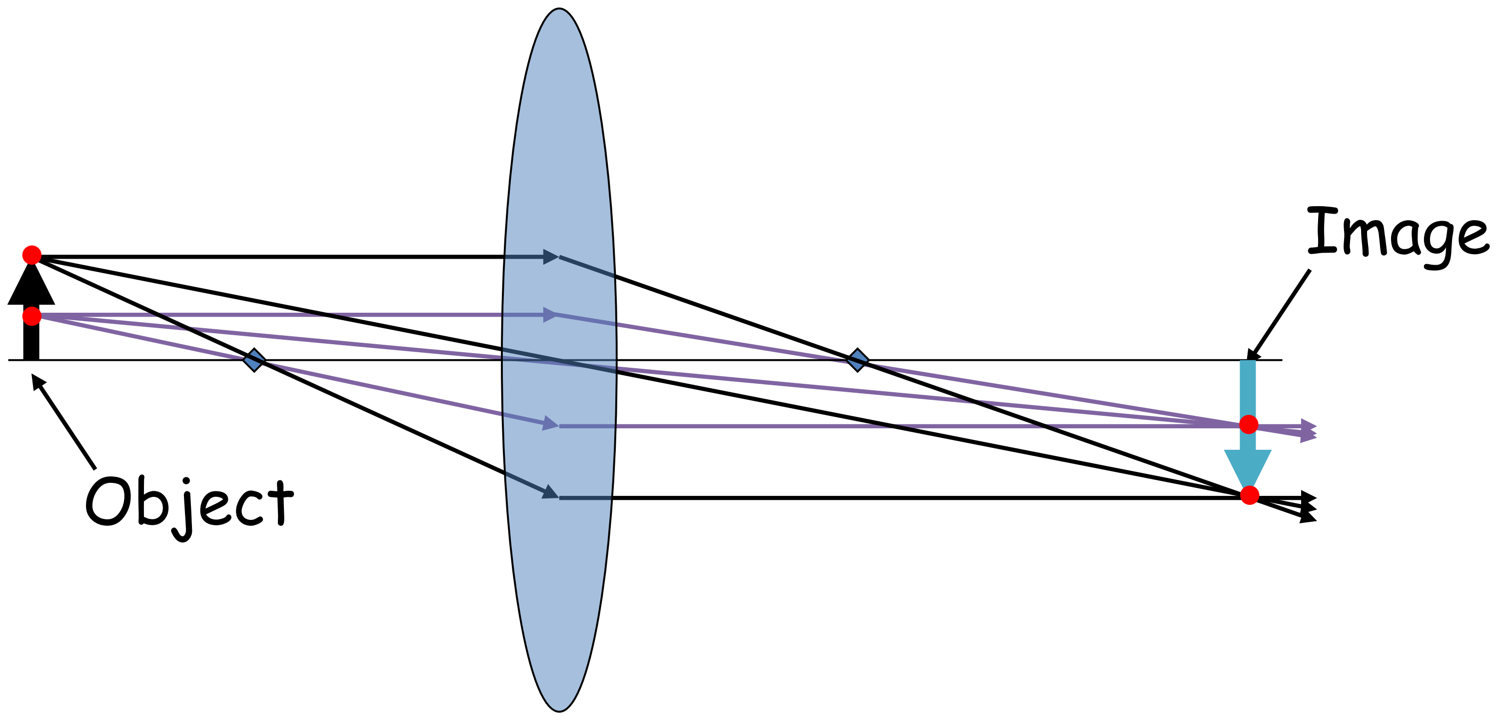
Example: Ray tracing with a positive thin lens

Ray tracing convention generally uses arrows to represent the object.



Example: Ray tracing & imaging with a positive thin lens

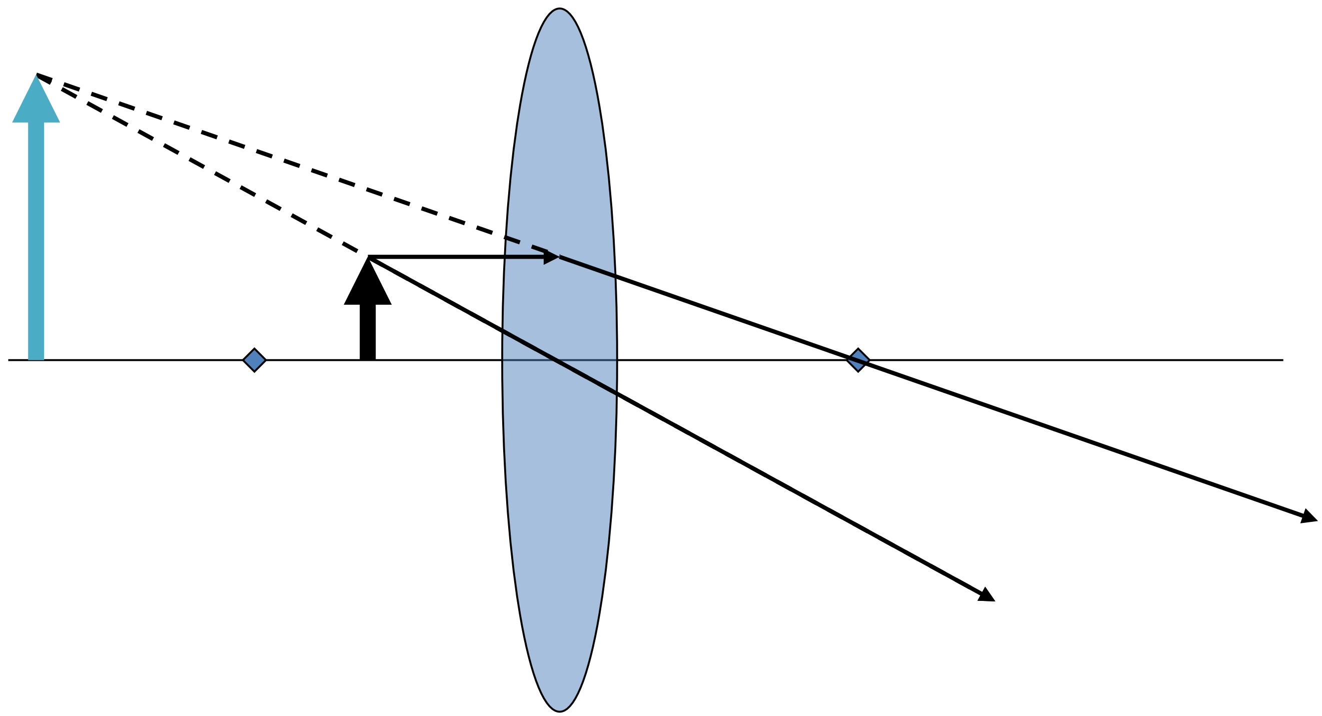
Same three rays can be applied for each point along the object.



Example: Ray tracing & imaging with a positive thin lens

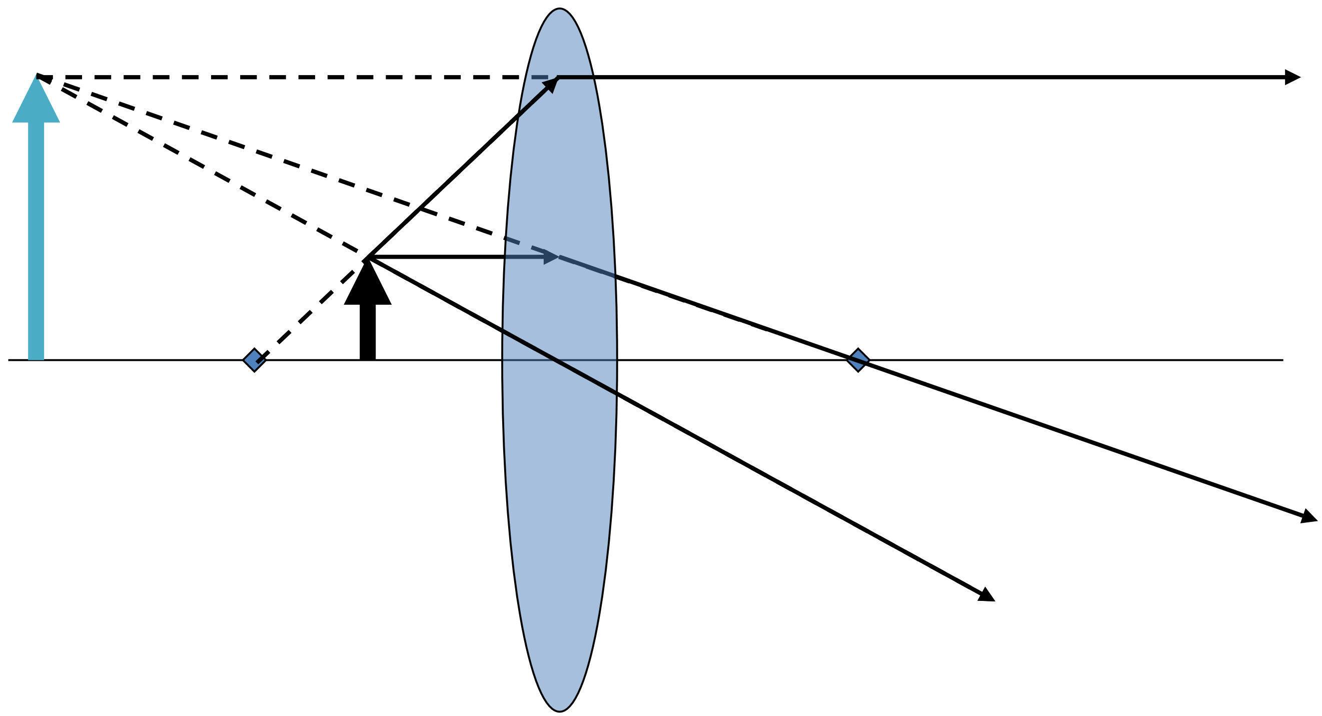
For object within the focal point:

- intersection point occurs for the extension of the rays (dashed lines), and it is at the back side of the object
- ➔ This corresponds to a "virtual image"



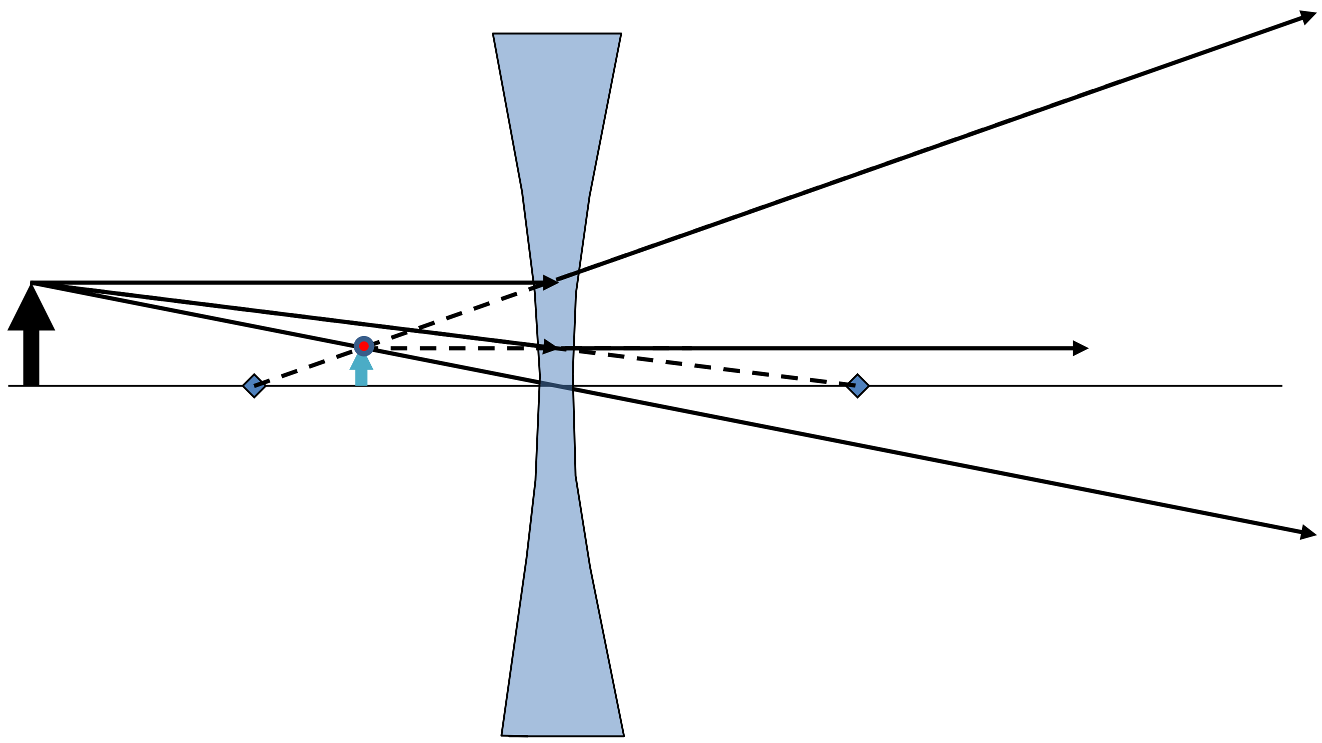
Example: Ray tracing & imaging with a positive thin lens

- Only need two rays are sufficient to locate the image.
- Of course one can use all the rules to trace three (or more) rays, and the intersection point will remain same



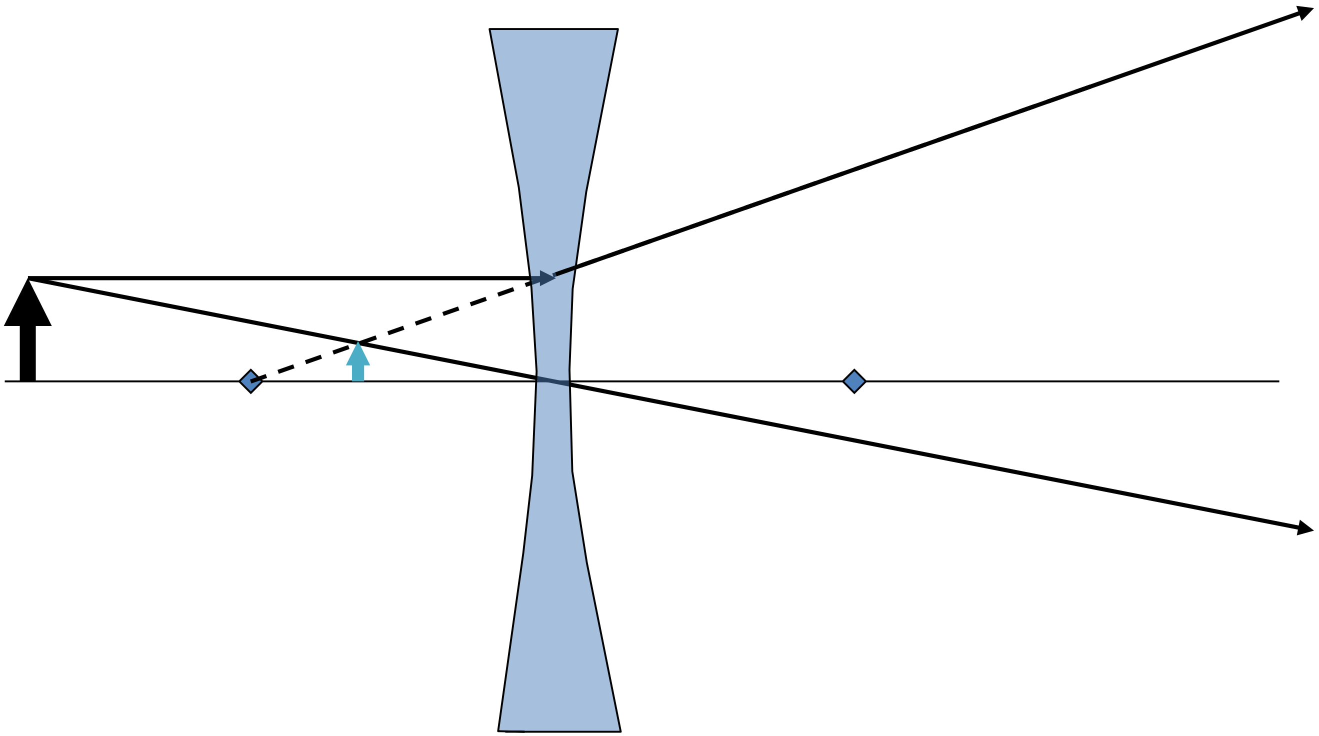
Example: Ray tracing with a negative thin lens

Same three rules can be applied to a concave (diverging) lens.



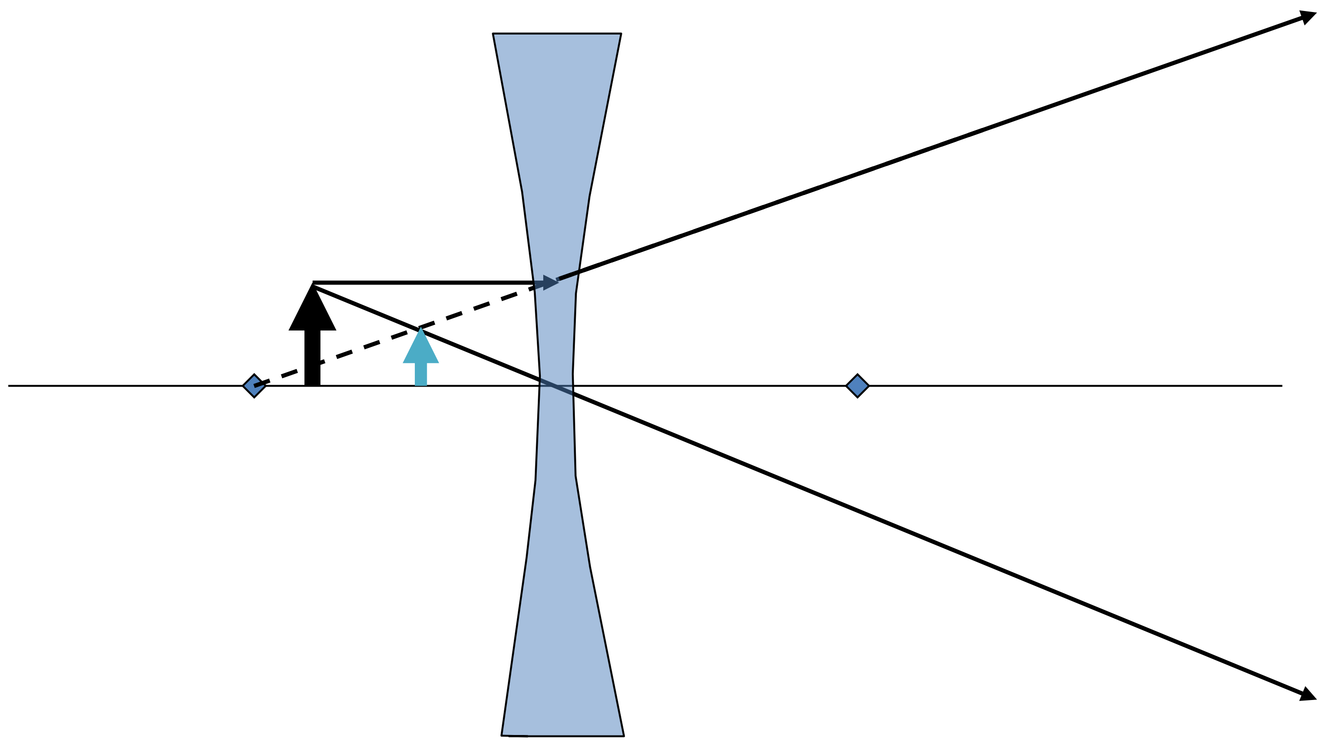
Example: Ray tracing with a negative thin lens

Again, two rays are enough to locate virtual image.

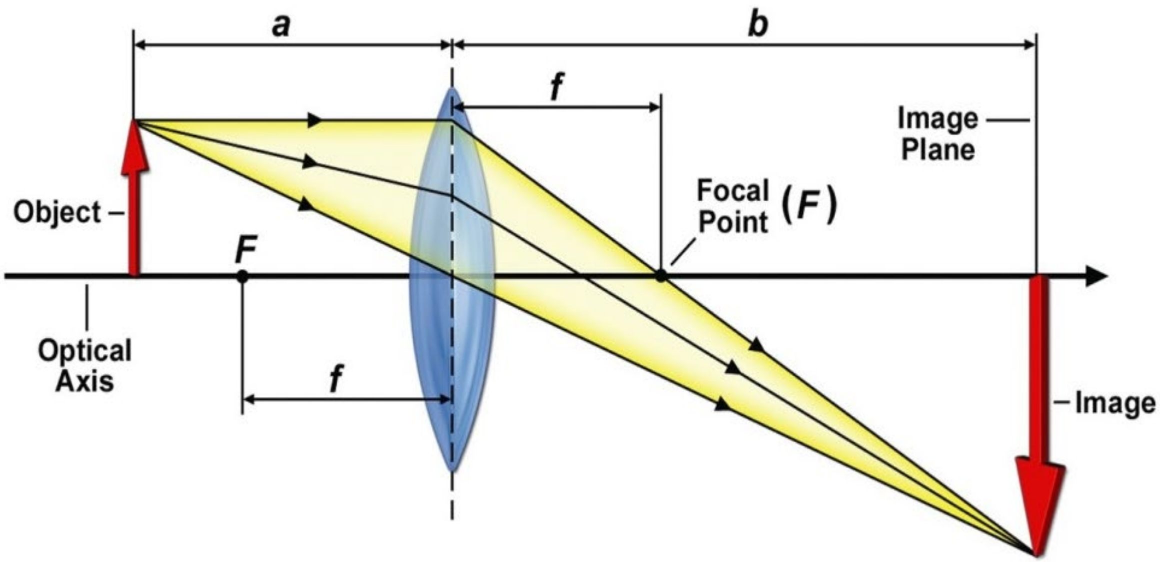


Example: Ray tracing with a negative thin lens

Concave lens makes a **virtual image** that is **smaller than the object**
....no matter where the object is located.



Formulas for thin lens: object-image relationship

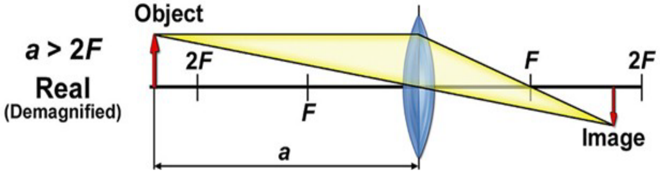


$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$ Focal length (lens maker's formula)

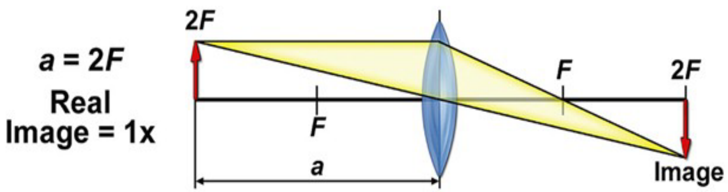
$$\frac{1}{f} = \frac{1}{a} + \frac{1}{b}$$ Imaging condition

$$M = \frac{b}{a}$$ Magnification

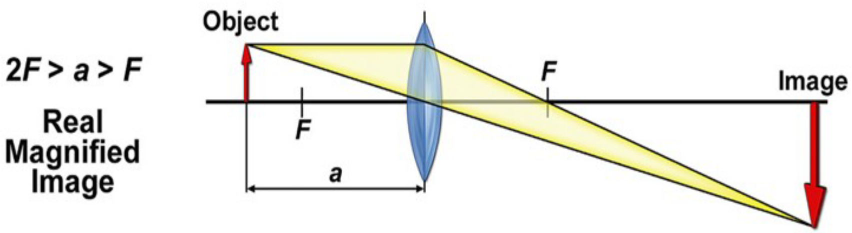
Example: object-image relation for a positive lens



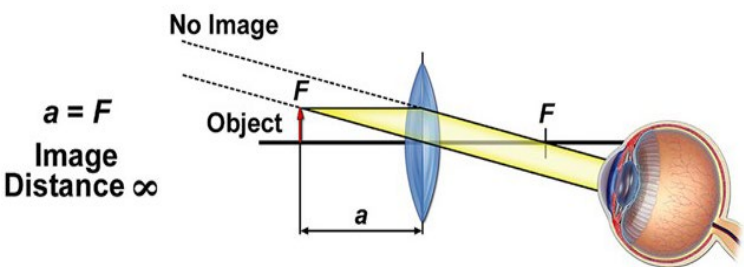
a>2F.
A real demagnified image is formed. M<1



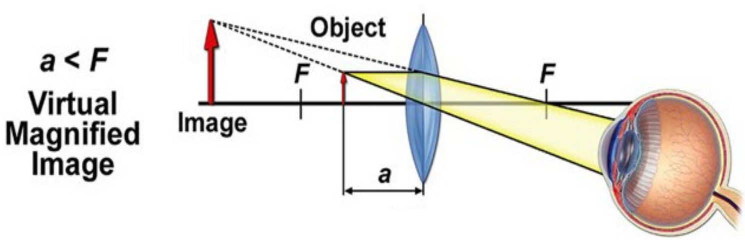
a = 2F.
A real image is formed but with no magnification and M=1
A special case. B=2F also.



2F>a>F
A real magnified image is formed. M>1
This is used for producing the "first" image in a compound microscope



a=F.
Image distance b is infinite.
No image exists that can be projected on a screen

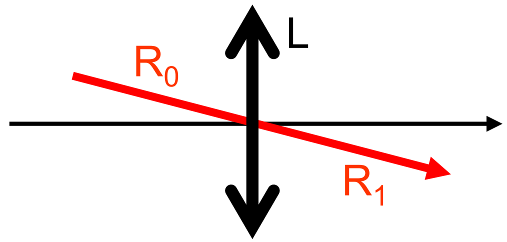


a<F.
No real image exists. M>1
If the eye is placed behind the lens, a virtual image is perceived on the far side of the lens

Ray Tracing - General

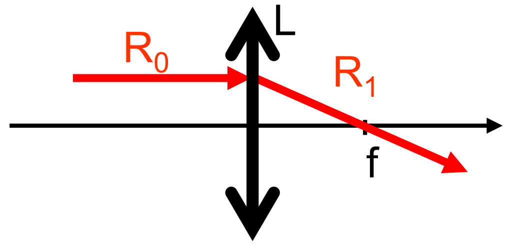
1) The chief ray: rays that pass through optical center

→ not deviated



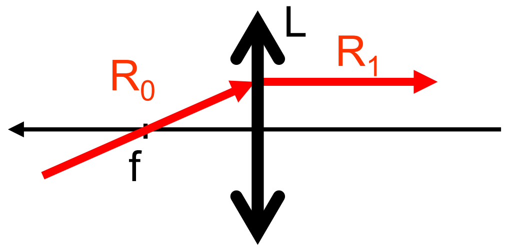
2) The parellel rays: Rays parallel to optical axis

→ through focal point



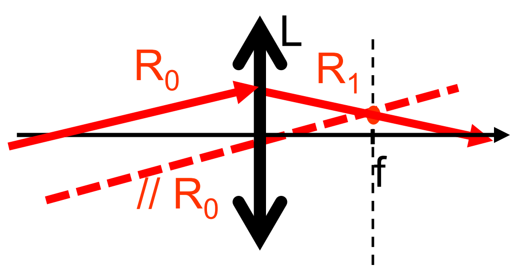
3) The focal rays: Rays pass through focal point

→ parallel to optical axis



4) The general ray: <u>**Ray with any given angle**</u>

- Draw a ray that is parallel to R_0 that passes through the center of the lens, dashed red line.
- This is called helper ray, $//R_0$ ray
- Draw a line vertical to the optical axis in the focal plane, black dashed line
- Find the intersection point of these two dashed lines
- The incoming ray R_0 refracts by the lens and passes through this intersaction point

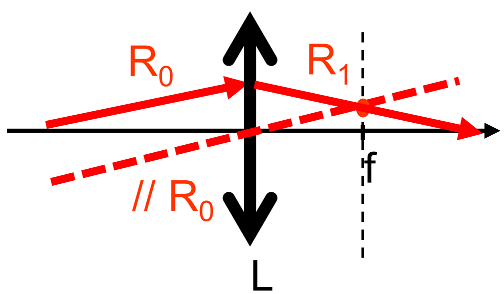


Ray Tracing - General

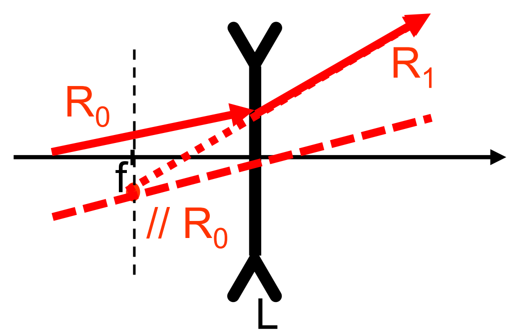
4) General ray: Ray with any given angle

- Draw a ray that is parallel to R_0 that passes through the center of the lens, dashed red line.
- This is called helper ray, $//R_0$ ray
- Draw a line vertical to the optical axis in the focal plane, black dashed line
- Find the intersection point of these two dashed lines
- The original ray refracts by the lens and passes through this intersaction point

Convergent lens:

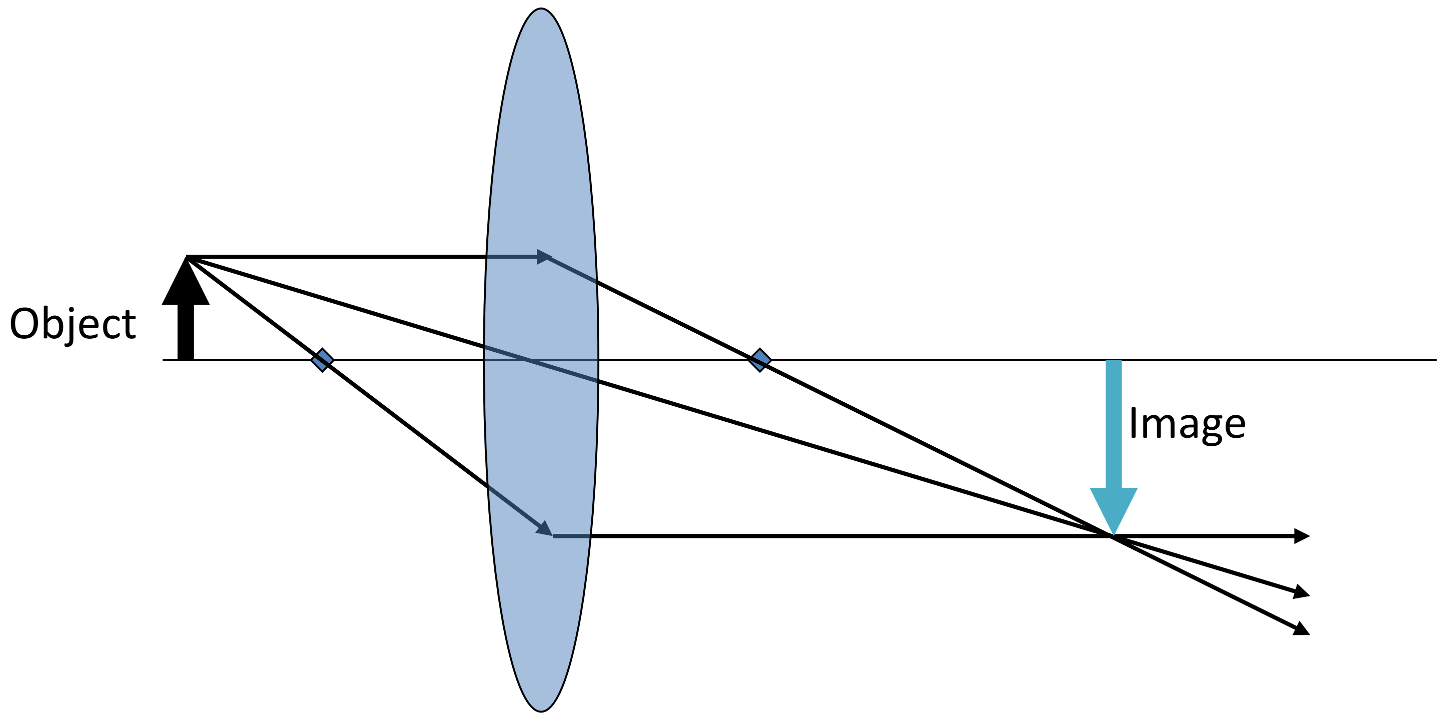


Divergent lens:



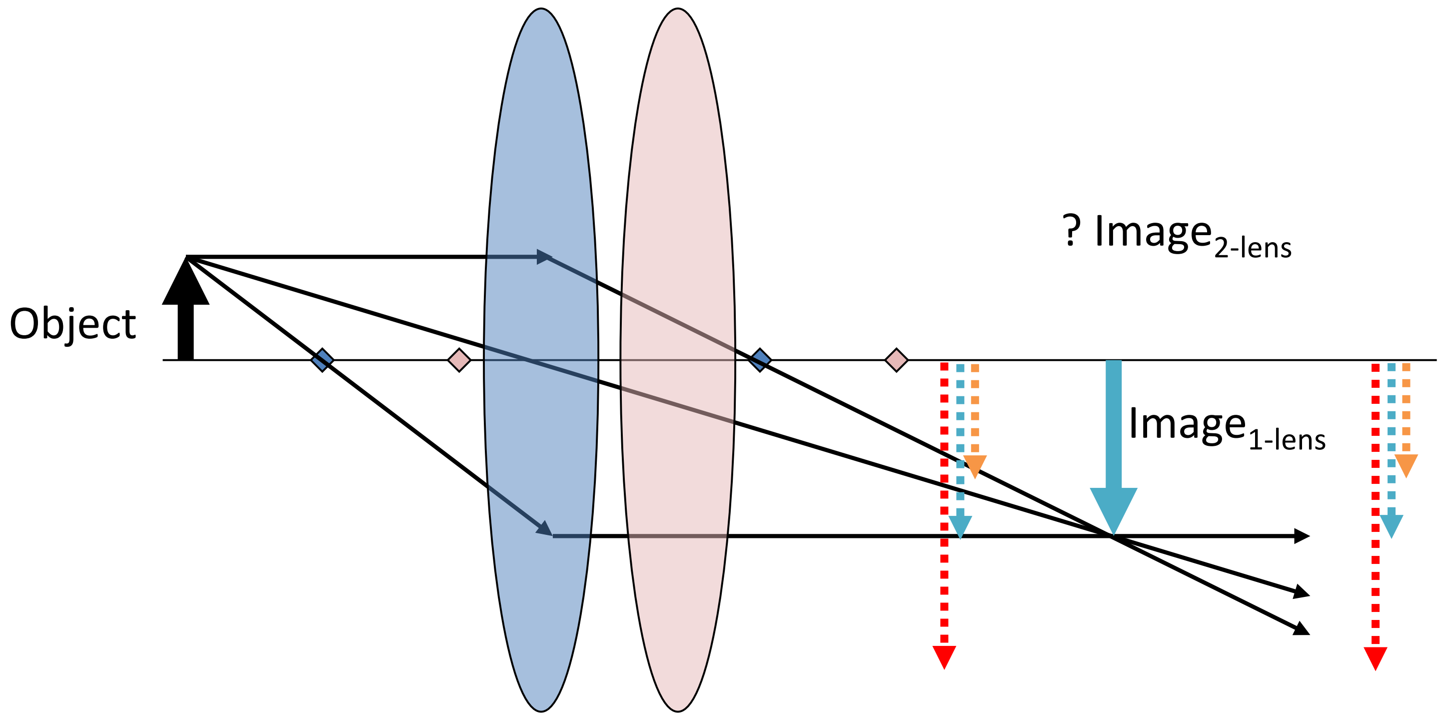
Ray Tracing – Example

Example-1: Let’s begin with one convex lens.



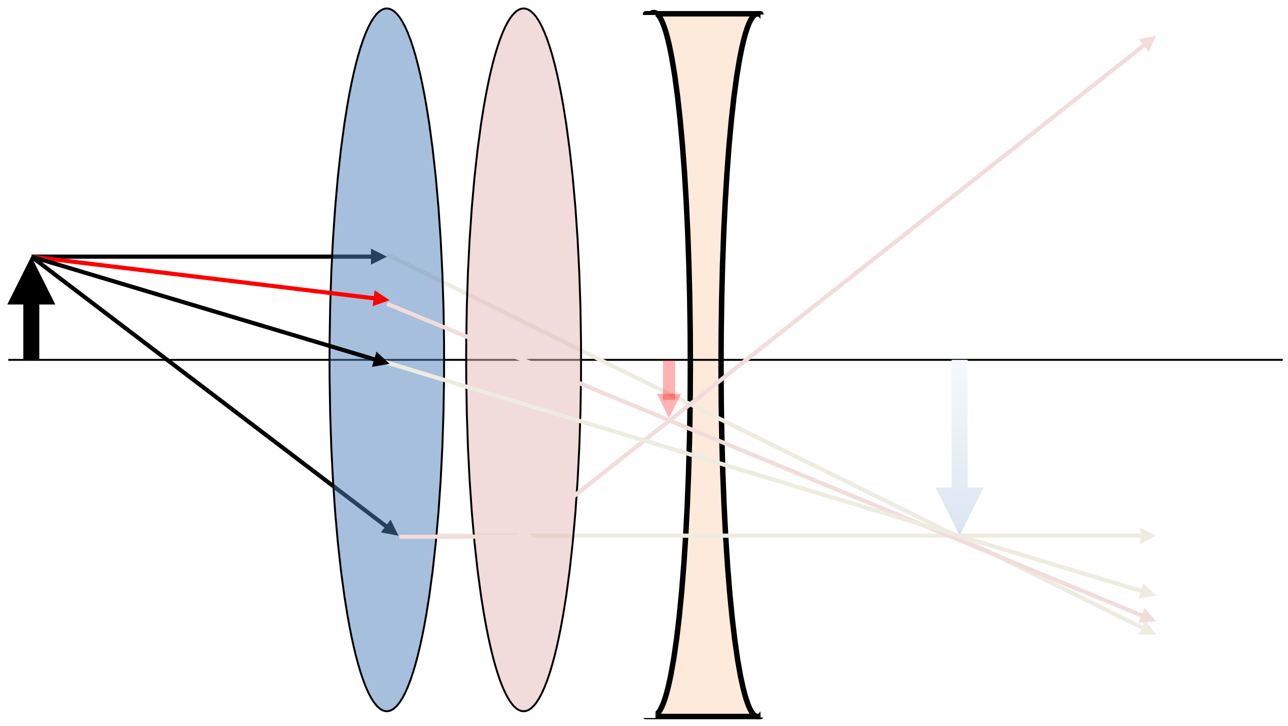
Ray Tracing - Example

Example-2: Add a second convex lens.

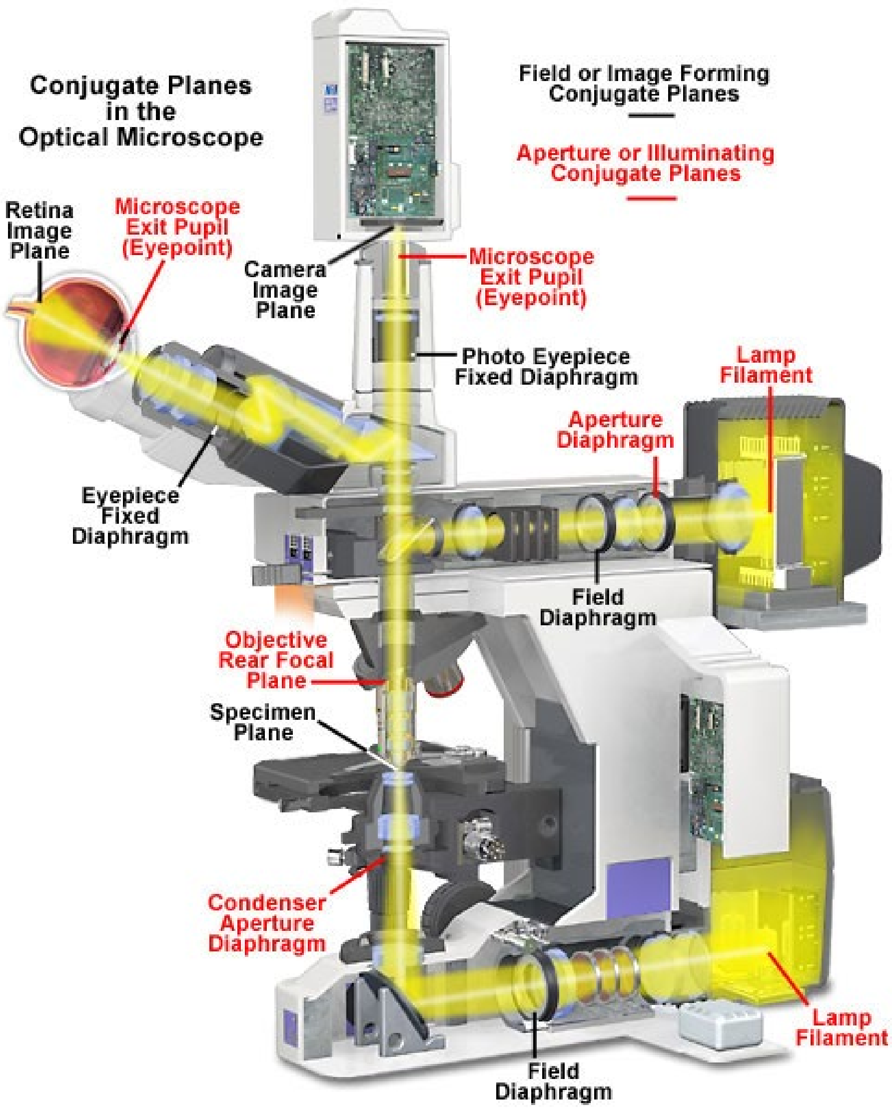


Ray Tracing - Example

.. it will get complicated (and not practical) for multiple lens system:



Microscope – An Optical System



Microscopes (optical imaging systems) contain multiple lenses.

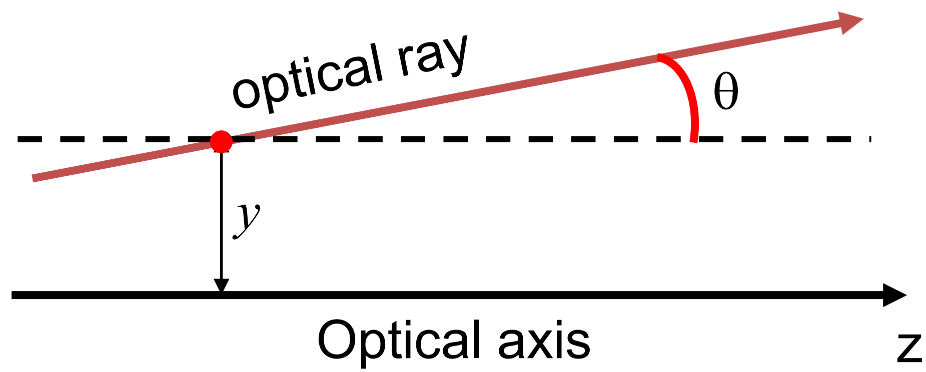
Q: How could we handle ray bending after many refracting/reflecting components?

A: Matrix approach is an alternative option.

The Ray Vector

A light ray can be defined by two co-ordinates:

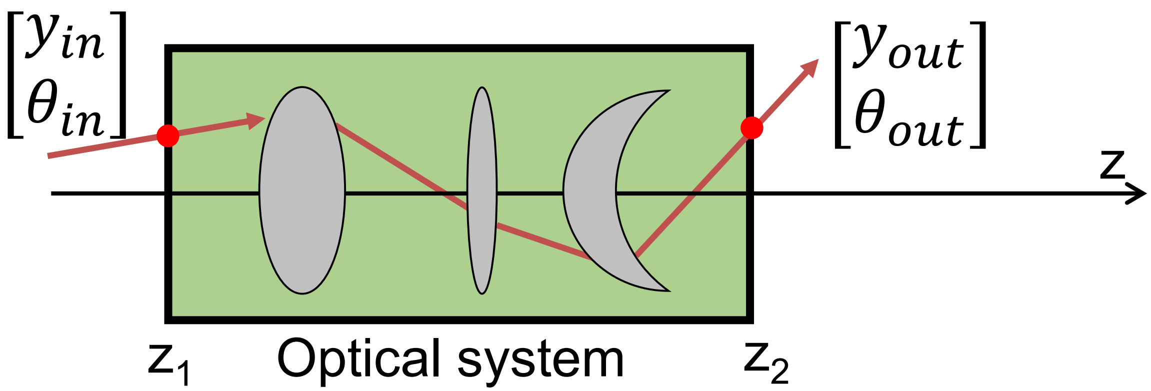
- its position, y
- its slope, θ



These two parameters define a **ray vector**, which will change as the ray propagates or passes through the optical elements.

$$\begin{bmatrix} y \\ \theta \end{bmatrix}$$

Matrix Optics



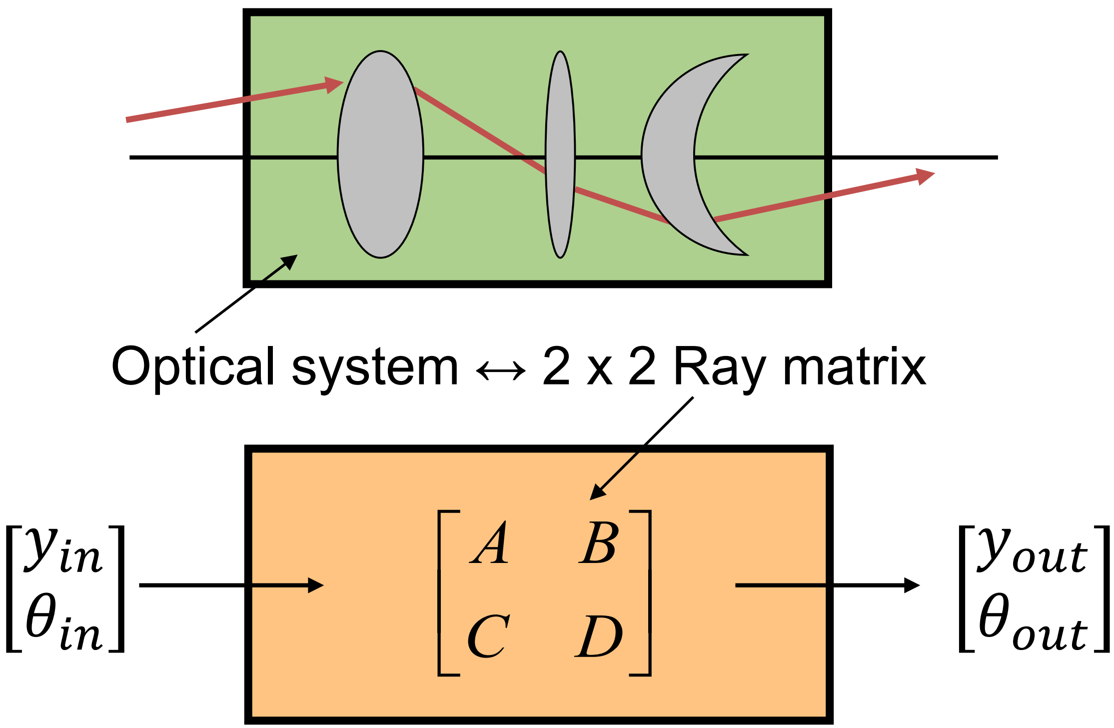
- An optical system is a set of optical components placed between two transverse planes at z_1 and z_2, which are referred as the input plane and the output plane.
- The system is characterized by its effect on an incoming ray of arbitrary position & direction ($y_{in,}$ θ_{in})
- The system steers the ray so that it has a new position and direction at the output plane, ($y_{out,}$ θ_{out})
- In the paraxial approximation, when all angles are sufficiently small (i.e. $\sin(\theta) \approx \theta$) the relationship between ($y_{in,}$ θ_{in}) and ($y_{out,}$ θ_{out}) coordinates is linear and given by:

$$\boxed{\begin{aligned} y_{out} &= Ay_{in} + B\theta_{in} \\ \theta_{out} &= Cy_{in} + D\theta_{in} \end{aligned}}$$

A, B, C and D are real numbers

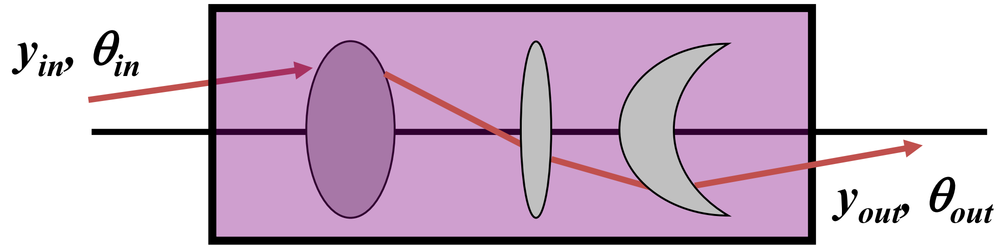
Ray Matrices

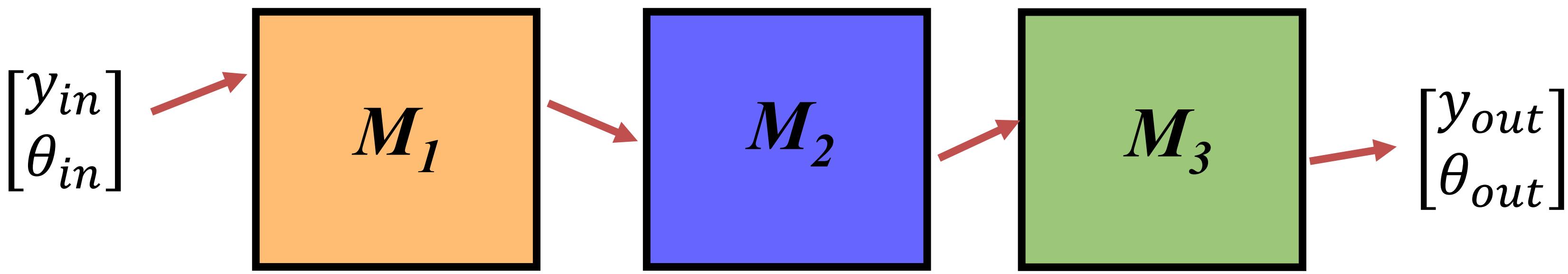
- For optical systems with multiple components, we can define a 2 x 2 **ray matrices**.



These matrices are often called ABCD Matrices.

Cascaded Elements





we simply multiply ray matrices.

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = M_3 \left\{ M_2 \left(M_1 \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix} \right) \right\} = M_3 M_2 M_1 \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix}$$

Notice the order in the multiplication: 3-2-1

Looks like the order is reverse

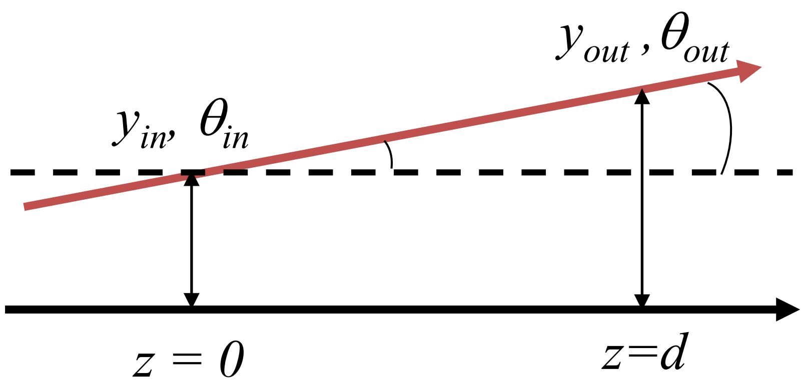
….but it makes sense when you think about it.

PS: Convention follows "Fundamentals of Photonics" by Saleh & Teich. Relevant pages are on Moodle

Ray matrix for propagation in free space or a medium

If y_{in} and θ_{in} are the position and slope upon entering,

let y_{out} and θ_{out} be the position and slope after propagating from $z = 0$ to z.



$$y_{out} = y_{in} + d\theta_{in}$$
$$\theta_{out} = \theta_{in}$$

Rewrite these for matrix notation:

$$y_{out} = 1.\, y_{in} + d.\, \theta_{in}$$
$$\theta_{out} = 0.\, y_{in} + 1.\, \theta_{in}$$

Obtain the terms by comparing the format:

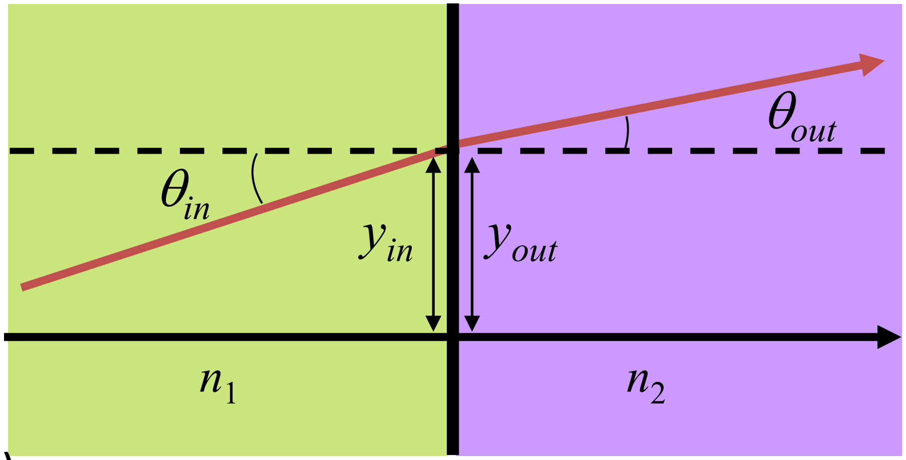
$$y_{out} = Ay_{in} + B\theta_{in}$$
$$\theta_{out} = Cy_{in} + D\theta_{in}$$

$A = 1$; $B = d$; $C = 0$; $D = 1$

$$M_{prop} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix}$$

Ray Matrix **for Planar Interface**



1) At the interface:

$$y_{out} = y_{in}$$

2) Now calculate θ_{out}.

Snell's Law says: $n_1 \sin(\theta_{in}) = n_2 \sin(\theta_{out})$

which becomes for small angles: $n_1\ \theta_{in} = n_2\ \theta_{out}$

Rewrite these for matrix notation:

$$y_{out} = 1.y_{in} + 0.\theta_{in}$$

$$\theta_{out} = 0.y_{in} + \left(\frac{n1}{n2}\right).\theta_{in}$$

Obtain the terms by comparing the format:

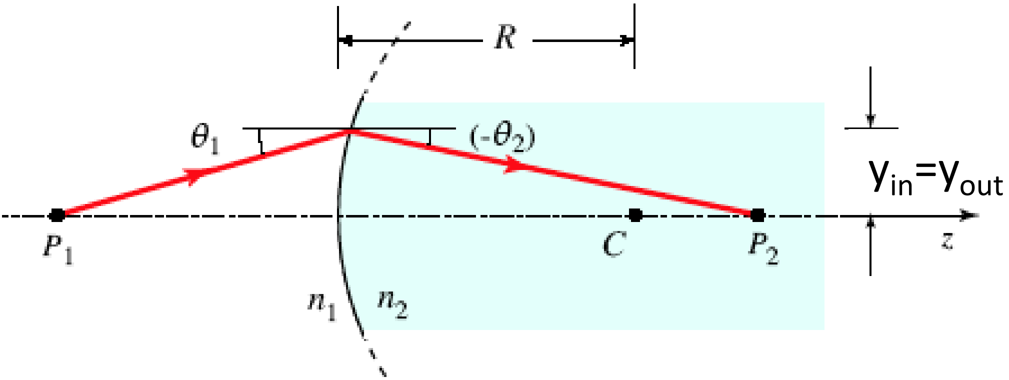
$$y_{out} = Ay_{in} + B\theta_{in}$$

$$\theta_{out} = Cy_{in} + D\theta_{in}$$

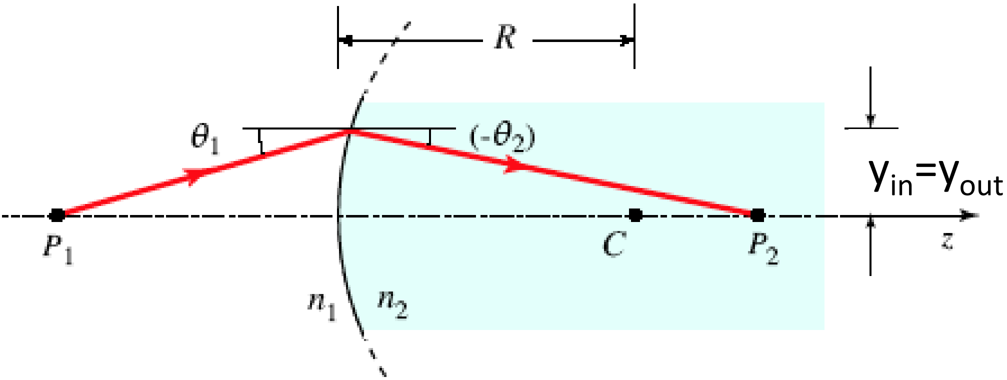
$A = 1$; $B = 0$; $C = 0$; $D = \frac{n1}{n2}$

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{bmatrix} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix} \quad \& \quad M_{planar-interface} = \begin{bmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{bmatrix}$$

Ray Matrix for Curved Interface



Ray Matrix for Curved Interface



1) At the interface: $y_{out} = y_{in}$

2) For θ_{out}, use Snell law :

$$\theta_{out} = \left({}^{n_1}/_{n_2}\right)\theta_{in} + \left({}^{n_1}/_{n_2} - 1\right) y_{in}/R$$

Rewrite these for matrix notation:

$$y_{out} = 1.\, y_{in} + 0.\, \theta_{in}$$

$$\theta_{out} = [\left({}^{n_1}/_{n_2} - 1\right)/R].\, y_{in} + \left({}^{n_1}/_{n_2}\right).\, \theta_{in}$$

Obtain the terms by comparing the format:

$$y_{out} = A y_{in} + B\theta_{in}$$

$$\theta_{out} = C y_{in} + D\theta_{in}$$

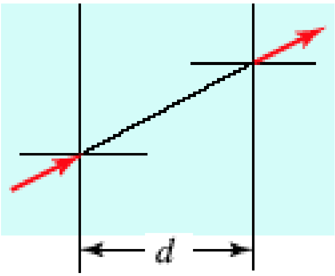
A = 1; B = 0; $C = -\dfrac{(n_2 - n_1)}{n_2 R}$ $D = \dfrac{n_1}{n_2}$

$$M_{curved\ interface} = \begin{bmatrix} 1 & 0 \\ -\dfrac{(n_2 - n_1)}{n_2 R} & \dfrac{n_1}{n_2} \end{bmatrix}$$

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\dfrac{(n_2 - n_1)}{n_2 R} & \dfrac{n_1}{n_2} \end{bmatrix} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix}$$

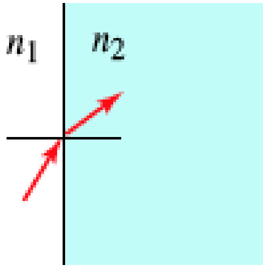
Matrix Optics – Basic Functions

Propagation



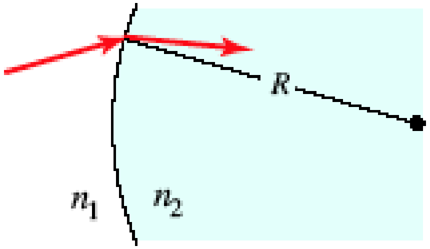
$$M_{prop} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Planar boundary



$$M_{planar-interface} = \begin{bmatrix} 1 & 0 \\ 0 & n_1/n_2 \end{bmatrix}$$

Spherical boundary



Convex, $R>0$; concave, $R<0$

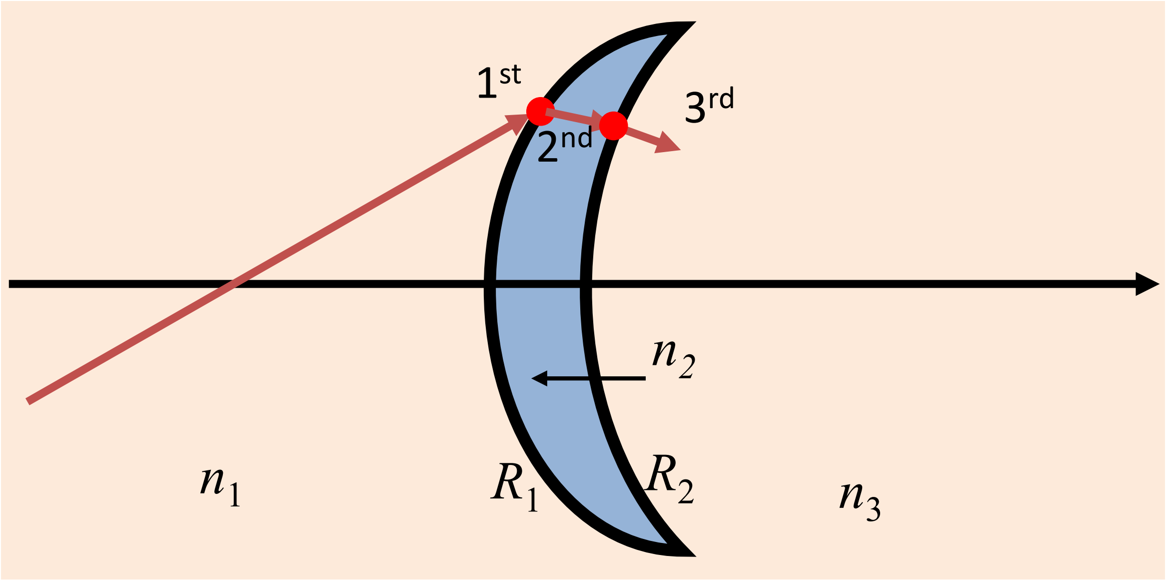
$$M_{curved\ interface} = \begin{bmatrix} 1 & 0 \\ -\dfrac{(n_2-n_1)}{n_2 R} & \dfrac{n_1}{n_2} \end{bmatrix}$$

Note that when R→ ∞, curved surface becomes planar
Thus, C → 0 for M_{curved}

Example: Ray matrix of a lens

When a light ray interacts with a lens, it experiences:

1) The 1st *curved* interface having a curvature of R_1
2) Then, *propagates* in the lens
3) Then, the 2nd *curved* interface having a curvature of R_2



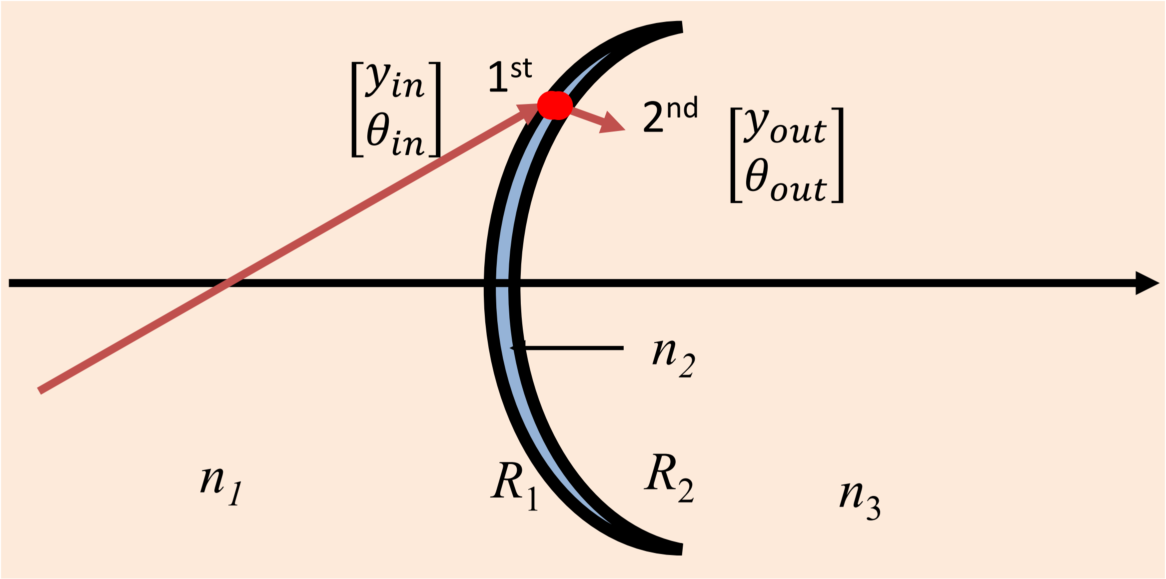
For a thin lens:
- we can neglect propagation inside the lens.
- Thus, in thin lens, light experiences only two curved interfaces.

Example: Ray matrix of a thin lens

With a thin lens, light experiences:

1) First, the curved interface having a curvature of R_1

2) Next, the second curved interface having a curvature of R_2



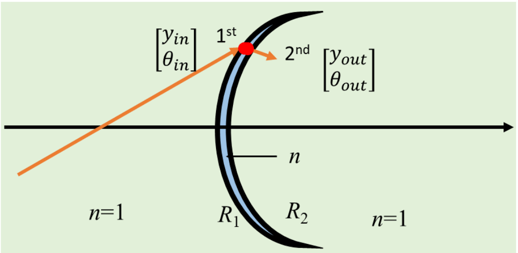
$$\begin{bmatrix} y_{out'} \\ \theta_{out}' \end{bmatrix} = M_{1^{st} curved\ interface} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix} \Rightarrow \begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = M_{2^{nd} curved\ interface} \begin{bmatrix} y_{out'} \\ \theta_{out'} \end{bmatrix}$$

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = M_{2^{nd} curved\ interface} \times M_{1^{st} curved\ interface} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix}$$

Remember reverse order!

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = M_N \ldots M_3 M_2 M_1 \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix}$$

Example: Ray matrix of a thin lens

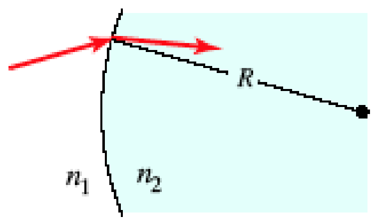


$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \boxed{M_{2^{nd}\ curved\ interface} \times M_{1^{st}\ curved\ interface}} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix}$$

$$\begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = M_{thin\ lens} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix}$$

Remember matrix for curved interface

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{(n_2 - n_1)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$$



Convex, $R > 0$; concave, $R < 0$

$$M_{1^{st} curved\ interface} = \begin{bmatrix} 1 & 0 \\ \frac{-(n-1)}{nR_1} & \frac{1}{n} \end{bmatrix} \qquad \begin{matrix} n_1 = 1 \\ n_2 = n \end{matrix}$$

$$M_{2^{nd} curved\ interface} = \begin{bmatrix} 1 & 0 \\ \frac{-(1-n)}{R_2} & n \end{bmatrix} \qquad \begin{matrix} n_1 = n \\ n_2 = 1 \end{matrix}$$

$$M_{thin\ lens} = M_{2^{nd} curved\ interface} \times M_{1^{st} curved\ interface} = \begin{bmatrix} 1 & 0 \\ \frac{-(1-n)}{R_2} & n \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{-(n-1)}{nR_1} & \frac{1}{n} \end{bmatrix}$$

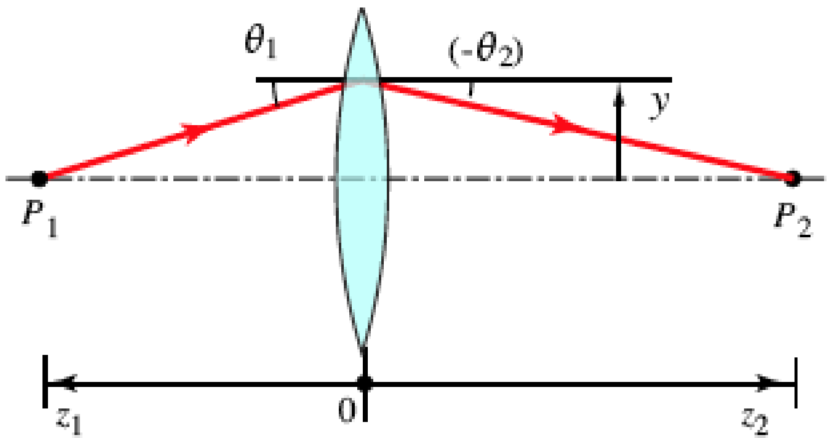
$$M_{thin\ lens} = \begin{bmatrix} 1 & 0 \\ -(n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) & 1 \end{bmatrix}$$

➔ This can be written as: $M_{thin\ lens} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$

Remember Lens-Maker's Formula From Ray Optics ➔ where, $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

Link between ray tracing & matrix representation

THIN LENS:
1) Position is ~nearly same immediately after the ray exits a thin lens
2) Thin lens only deflects the ray (i.e., change the outgoing angle) according to the law of refraction



From Ray Tracing

Ray Deflection :

$$\theta_2 = \theta_1 - \frac{y}{f},$$

Focal length:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Imaging Condition:

$$\frac{1}{f} = \frac{1}{z_{obj}} + \frac{1}{z_{im}}$$

Magnification:

$$\text{mag} = \frac{z_{im}}{z_{obj}}$$

From Matrix Representation

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} y_{out} \\ \theta_{out} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} \begin{bmatrix} y_{in} \\ \theta_{in} \end{bmatrix}$$

Ray Deflection :

$$\rightarrow \theta_{out} = \theta_{in} - \frac{y_{in}}{f}$$

Matrix Representation of a Thin Lens

Lens maker's formula

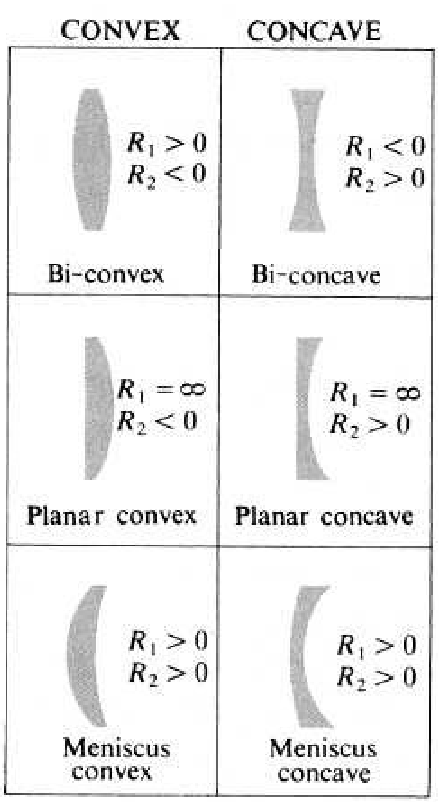
$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

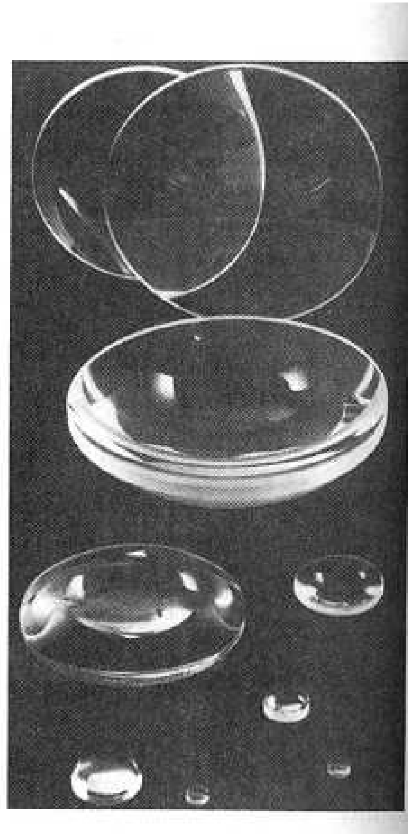
converging

f = +

diverging

f = -



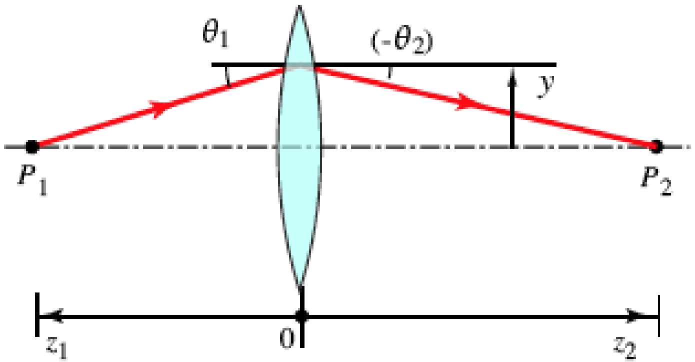


$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

- Positive radii indicate convex surfaces
- Negative radii indicate concave surfaces
- Negative angles point downward from the z-axis in the direction of ray propagation

Matrix Representation of Lenses & Curved Mirrors

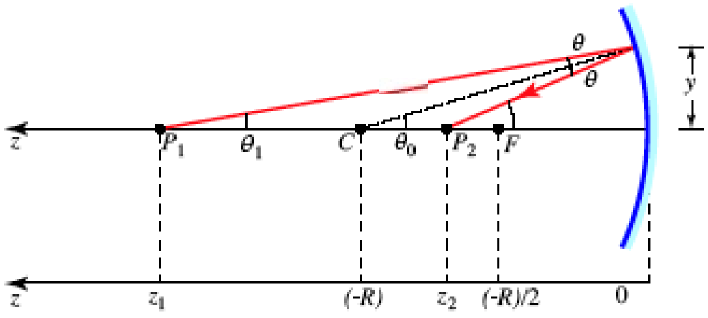
Under Paraxial Approximation ➔ $\sin\theta \approx \tan\theta \approx \theta$



1) Position is ~nearly same immediately after the ray exits a thin lens
2) Thin lens only deflects the ray (i.e., changes the outgoing angle) according to the law of refraction

Ray Deflection : $\theta_2 = \theta_1 - \frac{y}{f},$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$



1) Position is ~same immediately after the ray reflects from a curved mirror
2) Mirror only reflects the ray (i.e., changes the outgoing angle) according to the law of reflection

Ray Deflection : $\boxed{(-\theta_2) + \theta_1 \approx \frac{2y}{-R}}$ (from ray optics)

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix} \qquad R = 2f$$

Lenses & Mirrors

Lens maker's formula

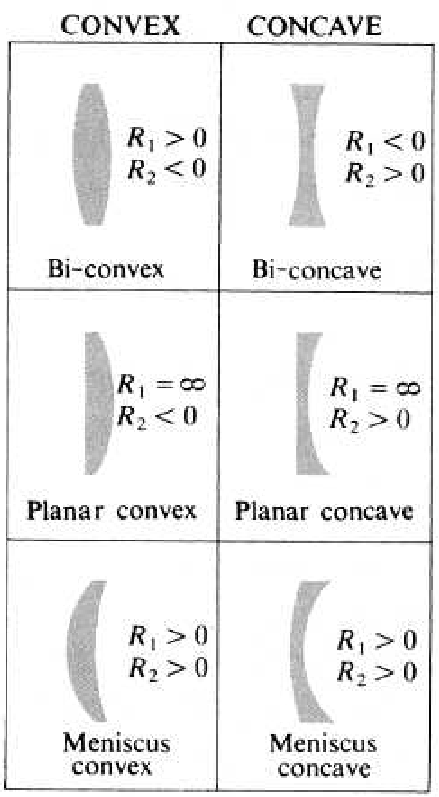
$$\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

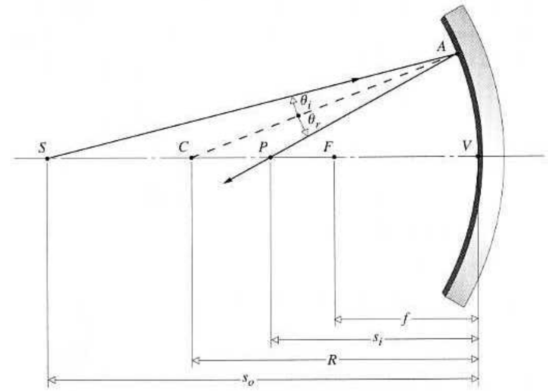
converging

f = +

diverging

f = -





converging

R = +
Convex

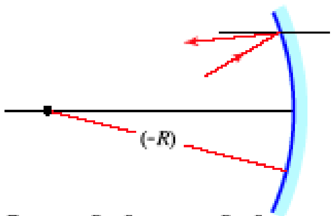
diverging

R = -
Concave

TABLE 5.4 Sign Convention for Spherical Mirrors

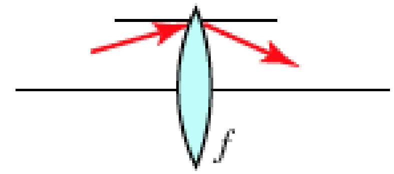
Quantity	Sign +	Sign −
s_o	Left of V, real object	Right of V, virtual object
s_i	Left of V, real image	Right of V, virtual image
f	Concave mirror	Convex mirror
R	C right of V, convex	C left of V, concave
y_o	Above axis, erect object	Below axis, inverted object
y_i	Above axis, erect image	Below axis, inverted image

Lenses & Mirrors



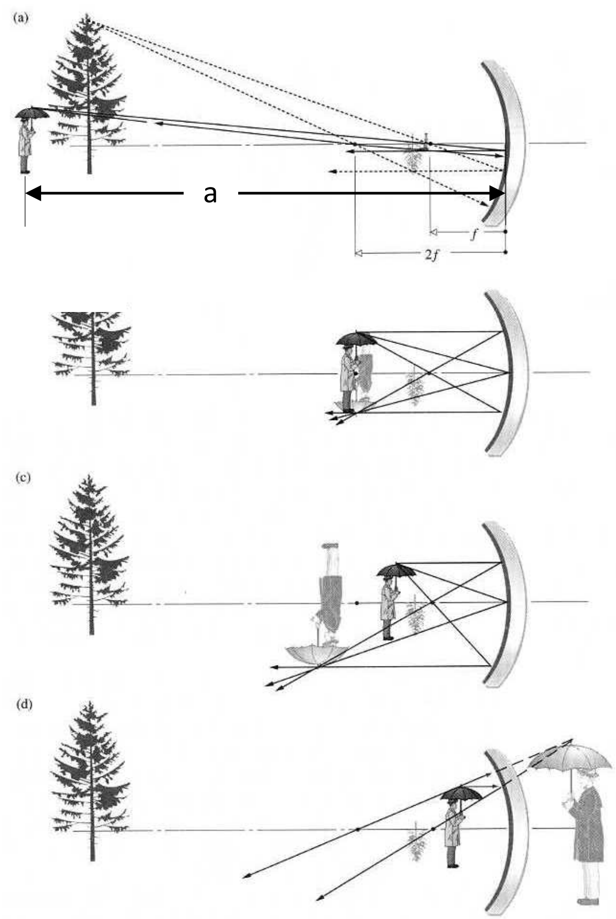
Concave, $R < 0$; convex, $R > 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$



Convex, $f > 0$; concave, $f < 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

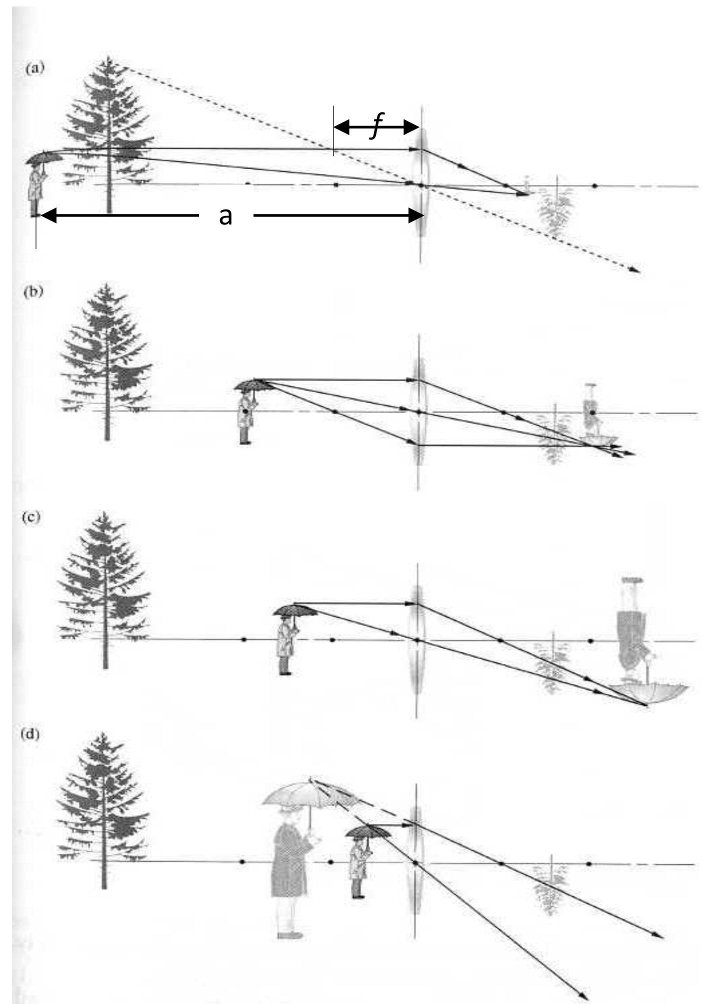


a>2f

a=2f

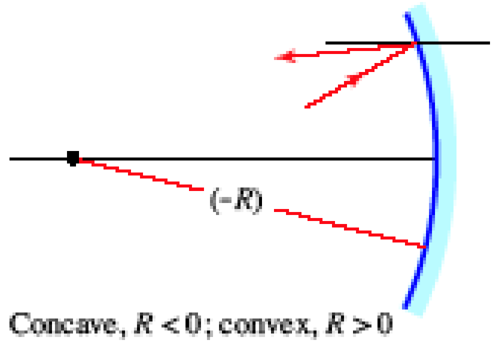
f<a<2f

a<f



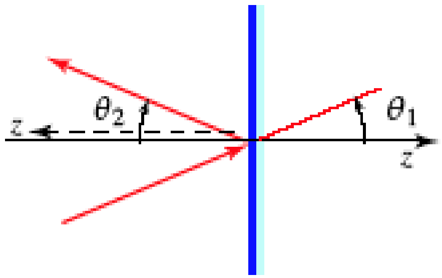
Matrix Representation for Mirrors

Spherical mirror



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$

Planar mirror

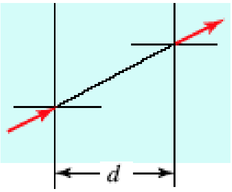


$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Corresponds to the limit case that R→∞
Thus, in ABCD matrix C element becomes 0

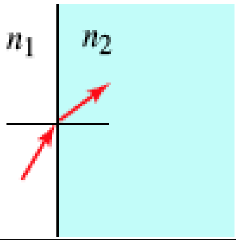
Matrix optics for basic functions & components - Summary

Propagation



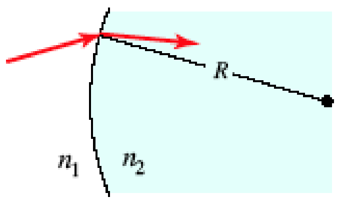
$$\mathbf{M} = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Planar boundary



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{bmatrix}$$

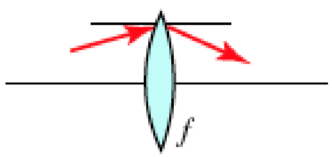
Spherical boundary



Convex, $R > 0$; concave, $R < 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{(n_2 - n_1)}{n_2 R} & \frac{n_1}{n_2} \end{bmatrix}$$

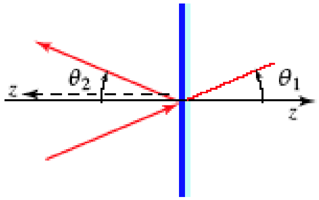
Lens



Convex, $f > 0$; concave, $f < 0$

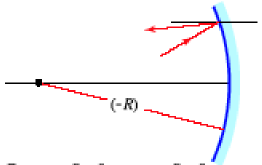
$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{bmatrix}$$

Planar mirror



$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Spherical mirror

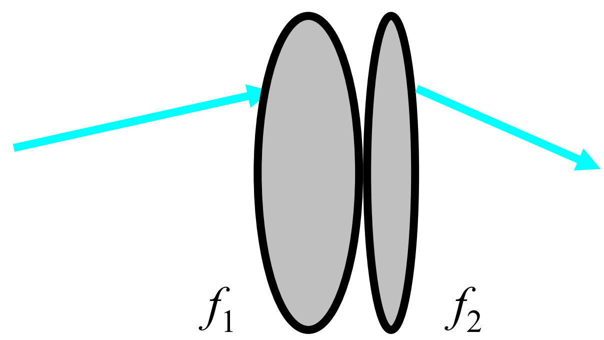


Concave, $R < 0$; convex, $R > 0$

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{bmatrix}$$

Example: Consecutive Lenses

Suppose we have an "**optical system**" consisting of two thin lenses right next to each other with no space in between.



When light enters this system, it experiences:

1) Initially, 1st lens with focal length of f_1
2) Then, 2nd lens with focal length of f_2

$$M_1 = M_{thin\ lens\ with\ f_1}$$

$$M_1 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix}$$

$$M_2 = M_{thin\ lens\ with\ f_2}$$

$$M_2 = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix}$$

$$M_{total} = M_2 \times M_1$$

$$M_{total} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_2} & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} & 1 \end{bmatrix}$$

$$M_{total} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_1} - \frac{1}{f_2} & 1 \end{bmatrix}$$

Two consecutive lenses act as one lens whose focal length is computed by the "resistive sum" ➔

$$\frac{1}{f_{total}} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$M_{total} = \begin{bmatrix} 1 & 0 \\ -\frac{1}{f_{total}} & 1 \end{bmatrix}$$