Biomicroscopy I - Exercise 07

October 29, 2024

1 Intensity pattern for two interfering sources

Intensity pattern for interfering sources in a given point is given by the following equation:

$$I = I_1 + I_2 + 2(I_1 I_2)^{\frac{1}{2}} \cos\varphi$$

where I_1 and I_2 are intensities of the light coming from the first and second source at the point of interest and $\varphi = \varphi_2 - \varphi_1$ is the phase difference between two waves in this point. Derive this formula by using vector diagram formalism.

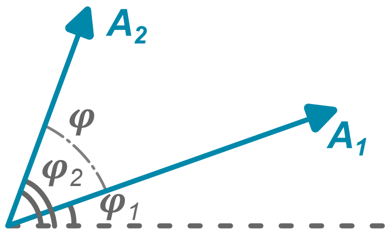

Figure 1: Vector diagram. Two vectors represent field vectors coming from two sources with their own amplitude and phase. Field amplitude squared is proportional to intensity $I \sim |A|^2$.

2 Double slit interference

Two waves of equal intensity I_0 (in plane of interference[1]), originating at the slit points $x = +\frac{d}{2}$ and $x = -\frac{d}{2}$, interfere in the plane $z = L$ as illustrated in Figure 2. This experimental setup is similar to that used by Thomas Young in his double slit experiment in which he demonstrated interference.

A. Assume that the waves radiating at the two-slits are spherical with intensity I_0 (in plane of interference), as shown in Figure 2b. Show that the intensity of the interfering two waves at the plane $z = L$ is:

$$I(x) = 2I_0\left(1 + \cos\left(\frac{2\pi d}{\lambda L}x\right)\right)$$

Hint: consider the path difference between two arms (Figure 2c)

[1]this assumption could be done for spherical waves in case distance to the plane is much bigger than the distance between the sources

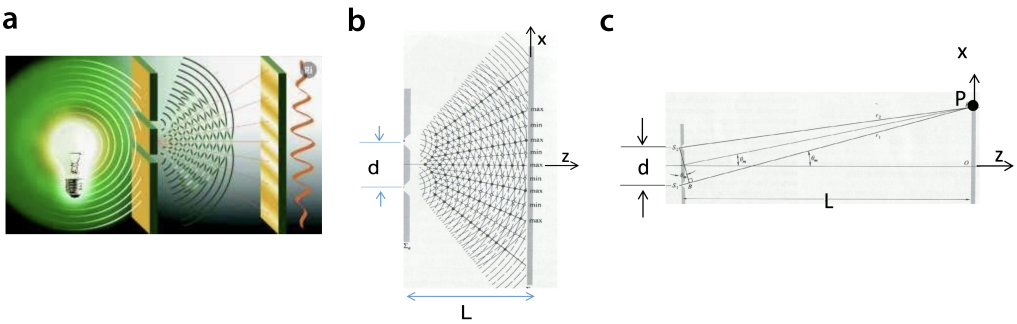

Figure 2: **Double slit interference (a)** Experimental setup. **(b)** Intensity pattern. **(c)** Optical path difference between two arms.

B. What are the values of the maximum and minimum intensity, in terms of I_0?

C. Assume that the spacing between the two slits is 1mm, the screen is 2m away from the double-slit plane, and the wavelength of light is 600nm. Assume the central symmetry axis (indicated as o in Figure 2c) is the origin.

   a. What is the location of the first maximum?
   b. What is the location of the first minimum?
   c. What is the location of the third maximum?

3 Diffraction grating

Consider a linear transparent grating formed by periodic slits as shown in Figure 3. The spacing between the slits is d.

Assume that we launch a parallel monochromatic beam of wavelength λ. The transmitted light after the grating diffracts into multiple diffraction orders, where each order has a specific diffraction angle θ_m.

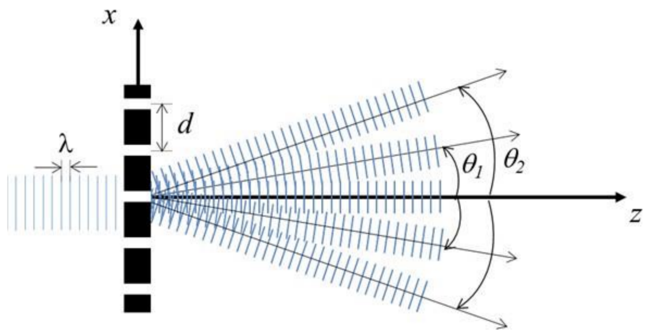

Figure 3: Diffraction grating and diffraction orders

A. Compute the diffraction angle θ_m in terms of λ and d.

B. Assume that the screen is at $z = 1$m and the incident wavelength $\lambda = 0.5\mu$m. If the first maximum measured on the screen along the x-axis is at $x_1 = 5$cm, compute the number of grating slits per mm.

C. Assume you are using the same grating as the above question (**B.**). If you change the wavelength λ to 600nm, what will be the angle and the location (along the x-axis) of the first diffraction order (*i.e.* θ_1 and x_1)?

D. Assume the screen is still at $z = 1$m and the incident wavelength is $\lambda = 0.5\mu$m. Now you use a denser grid, where the spacing between the slits is reduced by half (*i.e.* $d = d_0/2$). By using the values of Question **B.** for d_0 calculate the location of the first diffraction order (x_1) for this new grating. Does this denser grating diffract more or less compared to the one in Question **B.**?

E. Assume that three different plane waves corresponding to red, green and blue color ($\lambda_\mathrm{R} =$ 650 nm, $\lambda_\mathrm{G} = 550$ nm and $\lambda_\mathrm{B} = 450$ nm correspondingly) are incident on the diffraction grating shown in Fig. 3. Sketch the diffraction pattern on the screen (maxima locations) for these three waves relative to each other.

4 1D Fourier transform of periodic functions

The Fourier transform $\mathcal{F}\{f(x)\}$ as a function of the spatial frequency $p_x = \frac{k_x}{2\pi}$ is given by:

$$F(p_x) = \mathcal{F}\{f(x)\} = \int_{-\infty}^{\infty} f(x) e^{-i2\pi p_x x} dx \tag{1}$$

Moreover, recall the following identity:

$$\int_{-\infty}^{\infty} e^{-i2\pi p_x x} dx = \delta(p_x) \tag{2}$$

where $\delta(\cdot)$ denotes the Dirac delta function.

Using Equations 1 and 2, explicitly compute the Fourier transform and sketch the spectral lines of the following harmonic signals:

A. $A\cos(2\pi k_0 x)$

B. $A\sin(2\pi k_0 x)$

C. $Ae^{i2\pi k_0 x}$

Recall Euler's formula: $e^{-ix} = \cos(x) - i\sin(x)$.

5 Fourier transform of a unit pulse

The unit pulse, also known as the rectangular function, rect (or alternatively, Π) is defined as

$$\mathrm{rect}\left(\frac{x}{a}\right) = \Pi\left(\frac{x}{a}\right) = \begin{cases} 1, & \text{if } \quad |x| \leq \frac{a}{2} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

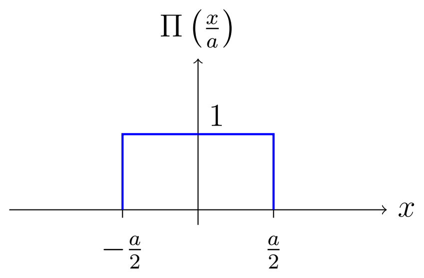

Figure 4: Rectangular function

Explicitly compute its Fourier transform (see Equation 1). Express the result in terms of the sinc function:

$$\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}. \tag{4}$$