Image Processing 1, Exercise 6

1 Linear Filters

[refresher] This exercise reviews the basic relation between an input and output signal and a transfer function by using the z-transform. It will help you understand the structure of a recursive filter which has an infinite impulse response.

(a) Let a filter be implicitly given by the recursive equation $y[k] = x[k] + a\,y[k-1]$, where x is the input sequence, y the output sequence with $k \in \mathbb{Z}$, $a \in \mathbb{C}$, and $|a| < 1$. Establish the transfer function H of the filter—as opposed to its frequency response.

(b) Use H to determine the explicit causal impulse response $h[k]$ in terms of $k \in \mathbb{Z}$.

2 Discrete Images and Linear Filtering

[intermediate] Characterization and application of a simple linear filter. You are expected to master this type of analysis. When possible, exploit the separability property and use the z-transform to make life easier.

Consider the following frequency response of a system:

$$H(e^{j\omega_1}, e^{j\omega_2}) = -4j[\sin(\omega_1 + \omega_2) - \sin(\omega_1 - \omega_2) - 3\sin(\omega_2)]$$

(a) Give the corresponding transfer function.

(b) Give the impulse response as a sum of unit impulses.

(c) Give the impulse response as an image.

(d) Give the mask as an image.

(e) It turns out that this filter is separable. Let $H_2(e^{j\omega}) = -8\mathrm{j}\sin(\omega)$, give $h_1[k]$ as a sum of unit impulses such that $h[k_1, k_2] = h_1[k_1]h_2[k_2]$.

(f) Consider the 2D, zero-padded image $f = \begin{bmatrix} \boxed{1} & 2 \end{bmatrix}$ and the 2D version of h_1, which we define as $g_1[k_1, k_2] = h_1[k_1]\delta[k_2]$. Compute the 2D convolution $(g_1 * f)$ and give the result as an image.

(g) Let the 2D version of h_2 be defined as $g_2[k_1, k_2] = \delta[k_1]h_2[k_2]$. Compute $(g_2 * g_1) * f$ and give the result as an image.

(h) Compute $h * f$ and give the result as an image.

(i) Compute the modulus of $H(e^{j\omega_1}, e^{j\omega_2})$. Express the result using trigonometric functions.

(j) Indicate the type (i.e. lowpass, highpass, bandpass, allpass, bandcut, allcut) of the filter h_1 and of h_2.