Image Processing 1, Exercise 4

1 Acquisition System

[basic] In this exercise, we show you how to model a real acquisition system using basic modules from the course. You will learn how to express an acquisition system in three different manners: a block diagram, a space-domain formula, and a frequency-domain formula.

The acquisition system of some camera is blurring an otherwise perfect image $f \in \mathbb{R}^2$. The blurring is modeled by an ideal filter with impulse response $h_{\mathrm{blr}}(x_1, x_2) = \mathrm{sinc}(x_1)\mathrm{sinc}(x_2)$. The blurred image is sampled by the sampling function $p(x) = \sum_{\boldsymbol{k}\in\mathbb{Z}^2} \delta(\boldsymbol{x} - \boldsymbol{k})$. Finally, the sampled image is displayed on an LCD display which has $h_{\mathrm{LCD}}(x_1, x_2) = \mathrm{rect}(x_1)\,\mathrm{rect}(x_2)$ as its impulse response. The displayed image is $g \in \mathbb{R}^2$.

(a) Draw a block diagram of the whole system, you can refer to page 2-20 of the lecture slides of Chapter 2a for an example of a block diagram.

(b) Express g in terms of f.

(c) Express $\hat{g}$ in terms of $\hat{f}$.

2 Lloyd-Max

[intermediate] In the lecture, you have learned that the Lloyd-Max quantizer achieves minimum error. In this exercise, you will prove that it is also unbiased.

Let $\tilde{f} = Q(f)$ be a quantized version of the image f, where Q is the Lloyd-Max quantizer. Show that $E\{f\} = E\{\tilde{f}\}$; that is, applying Lloyd-Max quantization does not change the mean of an image.

3 Difference Equations

[intermediate] We recall the 1D z-transform from signals and systems and then use the 2D z-transform to analyze a 2D difference equation. Difference equations express the current output of a system recursively as a weighted sum of previous inputs and outputs. They are convenient for designing fast algorithms. Inverting the z-transform then let you find the underlying impulse response.

(a) Let f be defined recursively as $f[k] = \delta[k] + a\,f[k-1]$. Use the z-transform to compute an explicit expression for $f[k]$ assuming that it is causal (ROC $= \{z : |z| > |a|\}$).

(b) Let $f[\boldsymbol{k}] = u[k_1 + 3]u[4 - k_2]e^{2k_2 - k_1}$. Give its $\boldsymbol{z}$-transform.

(c) Let $f[\boldsymbol{k}] = \delta[k_1, k_2] + af[k_1 - 1, k_2] + bf[k_1, k_2 - 1] - abf[k_1 - 1, k_2 - 1]$. Use the z-transform to compute an explicit expression for $f[\boldsymbol{k}]$, again assuming that it is causal.