Image Processing 1, Exercise 2

We expect students to be able to: 1) compute basic Fourier transforms; 2) characterize specific image processing systems. These exercises will help you develop these skills.

1 Fourier transform

[basic] Computation and manipulation of Fourier transforms. You are encouraged to use the Fourier tables provided on Moodle.

(a) Compute the Fourier transform $\mathcal{F}\{f\}$ of the following functions:

  i. $f(x,y) = \text{rect}\left(\frac{x}{5} - \frac{1}{2}\right)\text{rect}\left(y - \frac{3}{2}\right)$

  ii. $f(\boldsymbol{x}) = e^{\mathrm{j}\langle \boldsymbol{\omega_0}, \boldsymbol{x}\rangle}\text{sinc}\left(\frac{\boldsymbol{x}}{2\pi}\right)$, $\boldsymbol{x} \in \mathbb{R}^2$

(b) Compute the inverse Fourier transform $\mathcal{F}^{-1}\{\hat{f}\}$ of the following functions:

  i. $\hat{f}(\omega) = \frac{4}{4+\omega^2}$

  ii. $\hat{f}(\boldsymbol{\omega}) = e^{\mathrm{j}\,\langle \boldsymbol{\omega}, \boldsymbol{x}_0\rangle}\ |\det(\boldsymbol{A})|^{-1}\ \hat{g}(\boldsymbol{A}^{-T}\,\boldsymbol{\omega})$, where $\mathbf{A}$ is a (2×2) invertible matrix and $\mathbf{x}_0 \in \mathbb{R}^2$

(c) Use the definition of Fourier transform to show that $\int_{\mathbb{R}} \text{sinc}(x)\mathrm{d}x = 1$.

2 Characterization of linear shift-invariant systems

[basic] Practical examples of the characterization of linear shift invariant systems.

(a) A two-dimensional system has the impulse response $h(x,y) = \text{rect}(x)e^{\mathrm{j}\omega_0 x}(\text{rect} * \text{rect})(y)$. Give the associated transfer function H.

(b) A linear shift-invariant system $\mathcal{T}$ is such that

$$\mathcal{T}\{\sin(\omega\cdot)\}(x) = \sin\left(\omega\left(x - \frac{\pi}{2}\right)\right),\ \text{and}\ \mathcal{T}\{\cos(\omega\cdot)\}(x) = \cos\left(\omega\left(x - \frac{\pi}{2}\right)\right)$$

for every $\omega \in \mathbb{R}$. Give its transfer function.

(c) The transfer function H of a linear shift-invariant system $\mathcal{T}$ is known to be $H(\omega) = e^{-j\omega x_0}$. What is the impulse response h of the system $\mathcal{T}$?