MICRO-507: Legged Robots

Gait and Trajectory Optimization for Legged Systems Through Phase-Based End-Effector Parameterization

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ETH Zurich, Robotic Systems Lab, "ETH Zurich Testing ANYmal C's Advanced Locomotion Capabilities in Unstructured Natural Terrain," ANYbotics, last modified June 3, 2020, https://www.anybotics.com/news/advancing-legged-robotics-research/.

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26 November 2024

Motivation

Dynamic Models	Advantages	Limitations
Linear Inverted Pendulum (LIP)	- Simplifies CoM dynamics - Fast for predefined tasks	- Restricted base motions - No flight phases or uneven terrain
Full Rigid-Body Dynamics	- High accuracy - Captures vertical and complex motions	- Computationally expensive - Complex to model
Centroidal Dynamics	- Balances simplicity and feasibility - Handles flight phases	- Assumes minimal limb mass deviations - Limited on uneven terrain

➢ Require predefining some aspects of the motion (gait sequence, foothold placement, or step timings)

Contact Schedule Optimization	Advantages	Limitations
Integer Programming	- Optimizes contact schedules - Useful for predefined tasks	- Relies on heuristics - Computationally costly for large variables
Soft-Contact Models	- Models smooth force transitions - Handles contact dynamics	- Stiffness affects convergence - Unsuitable for highly uneven terrain
Linear Complementary Problem (LCP)	- Ensures feasible constraints - Dynamic motion compatibility	- High computational cost - Scaling issues with multi-legged systems

➢ Struggle to balance computational efficiency and adaptability

New Approach

- A **single optimization formulation** that determines:
  - Gait sequence (e.g., walk, trot, jump)
  - Step timings
  - Footholds
  - Body motion in 6 dimensions (position, orientation)
  - Forces at each foot

- Requires only the **start & goal position**, nb of steps, and terrain data
- Generates motion plans for arbitrary terrains and robot morphologies
- Maintains computational efficiency for practical online deployment

Demo video


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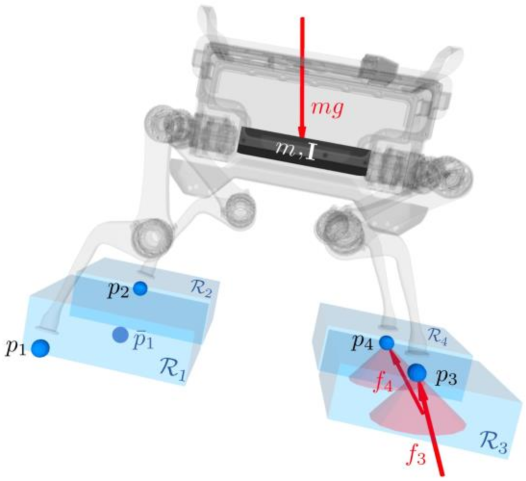
Robotics and Automation Letters (RA-L), 2018
A D R L
RSL
Robotic Systems Lab

Methodology

- Optimization Model
  - Nonlinear Programming (NLP)
  - Decision variables: body motion, feet trajectories, and contact forces
  - Constraints: friction cone, kinematic restrictions, and centroidal dynamics



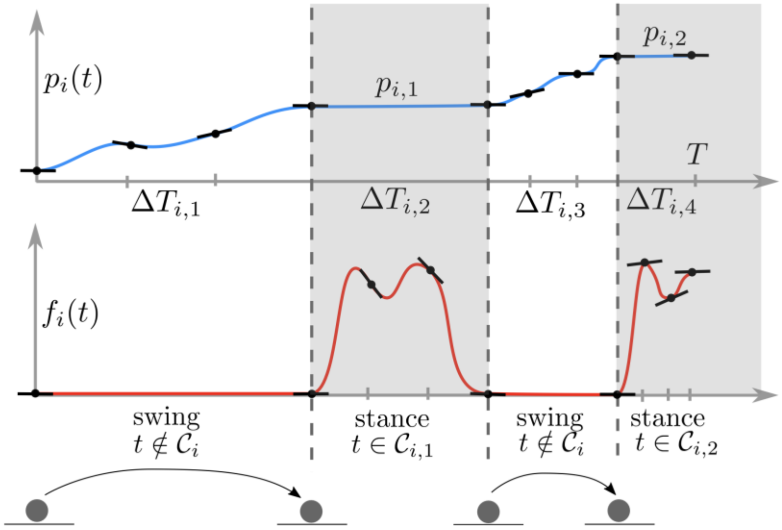
$$
\begin{array}{lll}
\text{find} & \mathbf{r}(t) \in \mathbb{R}^3 & \text{(CoM linear position)} \\
 & \boldsymbol{\theta}(t) \in \mathbb{R}^3 & \text{(base euler angles)} \\
 & \text{for every foot } i: & \\
 & \quad \Delta T_{i,1} \ldots, \Delta T_{i,2n_{s,i}} \in \mathbb{R} & \text{(phase durations)} \\
 & \quad \mathbf{p}_i(t, \Delta T_{i,1}, \ldots) \in \mathbb{R}^3 & \text{(foot position)} \\
 & \quad \mathbf{f}_i(t, \Delta T_{i,1}, \ldots) \in \mathbb{R}^3 & \text{(force at foot)} \\
\text{s.t.} & [\mathbf{r}, \boldsymbol{\theta}](t{=}0) = [\mathbf{r}_0, \boldsymbol{\theta}_0] & \text{(initial state)} \\
 & \mathbf{r}(t{=}T) = \mathbf{r}_g & \text{(desired goal)} \\
 & [\ddot{\mathbf{r}}, \dot{\boldsymbol{\omega}}]^T = \mathbf{f}_d(\mathbf{r}, \mathbf{p}_1, \ldots, \mathbf{f}_1, \ldots) & \text{(dynamic model)} \\
 & \text{for every foot } i: & \\
 & \quad \mathbf{p}_i(t) \in \mathcal{R}_i(\mathbf{r}, \boldsymbol{\theta}), & \text{(kinematic model)} \\
 & \quad \text{if foot } i \text{ in contact}: & \\
 & \quad\quad \dot{\mathbf{p}}_i(t \in \mathcal{C}_i) = \mathbf{0} & \text{(no slip)} \\
 & \quad\quad p_i^z(t \in \mathcal{C}_i) = h_{terrain}(\mathbf{p}_i^{xy}) & \text{(terrain height)} \\
 & \quad\quad \mathbf{f}_i(t \in \mathcal{C}_i) \cdot \mathbf{n}(\mathbf{p}_i^{xy}) \geq 0 & \text{(pushing force)} \\
 & \quad\quad \mathbf{f}_i(t \in \mathcal{C}_i) \in \mathcal{F}(\mu, \mathbf{n}, \mathbf{p}_i^{xy}) & \text{(friction cone)} \\
 & \quad \text{if foot } i \text{ in air}: & \\
 & \quad\quad \mathbf{f}_i(t \notin \mathcal{C}_i) = \mathbf{0} & \text{(no force in air)} \\
 & \quad \sum_{j=1}^{2n_{s,i}} \Delta T_{i,j} = T & \text{(total duration)}
\end{array}
$$

Phase-Based Parameterization

Challenge: Traditional methods require predefined gaits and foot sequences alternating between **Swing** phase and **Stance** phase

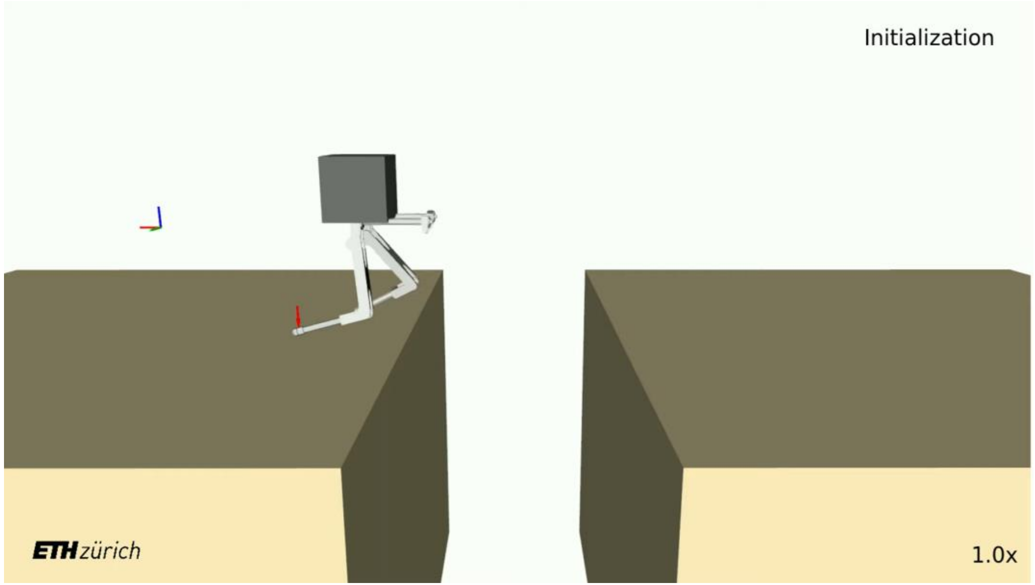
Solution:

- These phases are represented using **polynomials**
- Position and forces during each phase are described by cubic functions
- The algorithm optimizes the phases durations (ΔTi,j) and sequence to generate arbitrary gaits



- ➢ Automatically adapts the gait to the terrain:
  - Inserts flight phases when needed (e.g., for jumping gaps)
  - Adjusts timing for rough terrain or steep inclines

Biped Gap Crossing



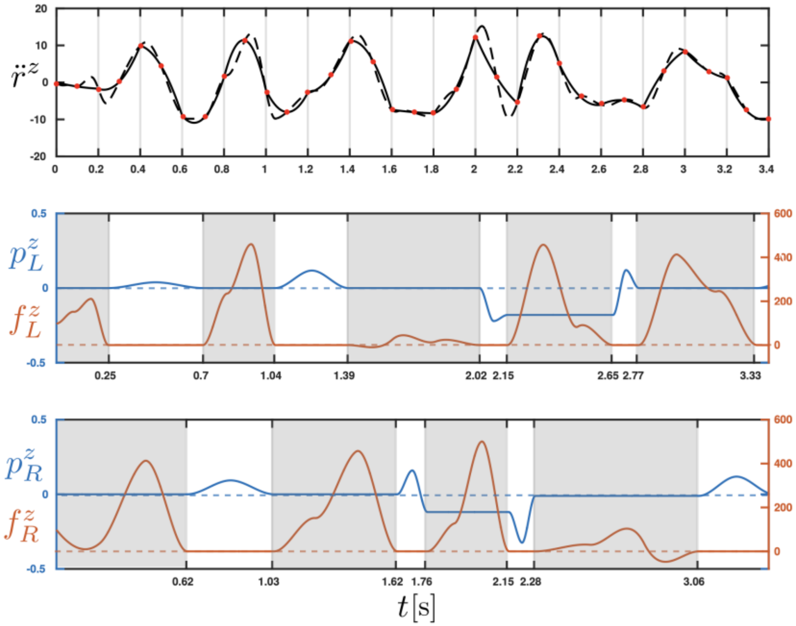


Figure: A generated motion plan for bipedal robot crossing a wide gap

Limitations

- Trade-offs between speed and solver efficiency

- **Simplified Dynamics Model**
  - Model assumes light, immobile limbs
  - May lead to inaccuracies for heavy or highly dynamic limbs
- **Terrain Constraint Gaps**
  - Only applied at specific points along the trajectory
  - May lead to collisions on irregular or complex surfaces
- **Local Minima in Optimization**
  - The solver can get stuck in suboptimal solutions
  - Can limit motion quality

- **Possible improvements**
  - Refining the dynamics model
  - Increasing the frequency of terrain constraint checks
  - Fixing step timings to reduce the problem’s complexity for the solver

Exam Questions

Question 1: What are the main advantages of using a centroidal dynamics model in this work compared to other dynamic models like the Linear Inverted Pendulum (LIP)?

Answer 1:

The centroidal dynamics model offers a middle ground between simplicity and complexity. Unlike the LIP model, which restricts vertical movement and assumes fixed contact points, the centroidal dynamics model:

- Accounts for vertical movements and changes in body orientation, enabling a wider range of dynamic motions.
- Explicitly models individual contact forces, which allows enforcement of friction cone constraints and ensures physically feasible motion.
- Supports the generation of motions with full-flight phases, essential for highly dynamic tasks such as jumping.

Question 2: How does the optimization framework handle transitions between contact and flight phases?

Answer 2:

The framework uses a continuous parameterization of foot forces and positions, where forces are automatically set to zero during flight phases and non-zero during contact. This approach avoids the complexity of integer programming and enables seamless transitions between contact and flight.