Highly Dynamic Quadruped Locomotion via Whole-Body Impulse Control and Model Predictive Control

- ***Authors**: Donghyun Kim, Jared Di Carlo, Benjamin Katz, Gerardo Bledt, and Sangbae Kim*

- *Rayan Bouchallouf*
- *Christian Doimo*
- *Javier De Ramon Murillo*

Mini Cheetah

- 12 proprioceptive actuators (3 on each leg)

Integrated with hall-effect encoders for precise position sensing

- Central IMU for body orientation and balance

Abstract

- Dynamic legged locomotion is a challenging topic because of the lack of established control schemes which can handle aerial phases, short stance times, and high-speed leg swings.

- Our controller combines whole body control (WBC) and model predictive control (MPC).

Development of the hybrid control

Proposed solution: Combining MPC and WBIC to get the strengths of both techniques

MPC

- **Strength**: Long-term planning ⇒ Excellent for anticipation
- **Weakness**: Low update frequency (40 Hz) ⇒ Cannot handle sudden perturbations effectively

WBIC

- **Strength**: High-frequency, precise control (500 Hz) ⇒ Excellent for managing perturbations in real-time
- **Weakness**: Lacks long-term planning ⇒ Limited in anticipation

Previous existing methods

- Existing WBCs focus more on following the CoM

- Idea of reaction force tracking originates from the impulse planning used in Cheetah 2

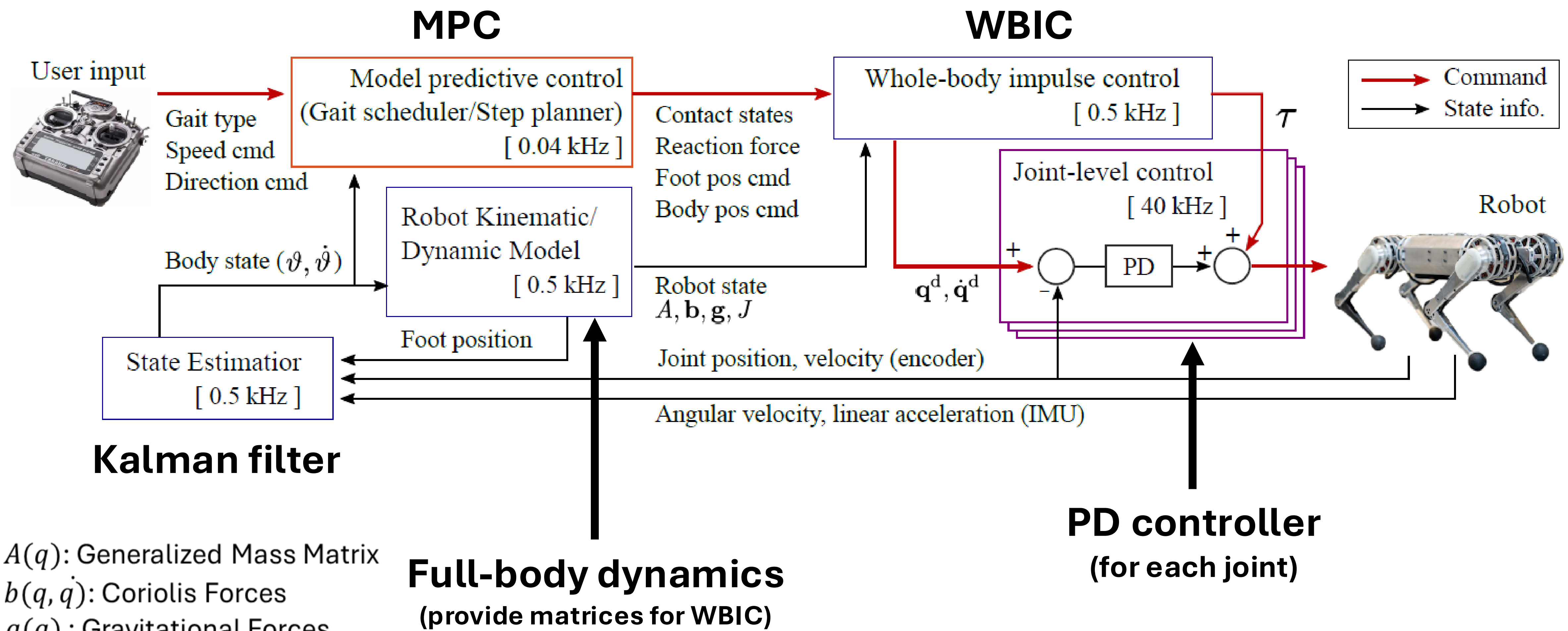
“High-speed bounding with the MIT Cheetah 2: Control design and experiments” *H.-W. Park, P. M. Wensing, and S. Kim*

- Convex MPC can perform various dynamic gaits at high speed on both Cheetah 3 [1] and Mini-Cheetah [2].

“Dynamic Locomotion in the MIT Cheetah 3 Through Convex Model-Predictive Control”, *J. Di Carlo, P. M. Wensing, B. Katz, G. Bledt, and S. Kim*

“Mini Cheetah- A Platform for Pushing the Limits of Dynamic Quadruped Control.” B. Katz, J. Di Carlo, and S. Kim,

Architecture



$A(q)$: Generalized Mass Matrix

$b(q, \dot{q})$: Coriolis Forces

$g(q)$: Gravitational Forces

$J(q)$: Jacobian

MPC

- **Simplified model** ⇒ Enables faster computation at 40 Hz
- **Provides optimal reaction forces over a long horizon** ⇒ Enhances planning and anticipation of dynamic gaits
- **Predefined contact sequence and gait patterns** ⇒ Keeps the formulation convex (fast computation and unique global minimum solution)

Model equation : $\mathrm{x}(k+1) = A_k \mathrm{x}(k) + B_k \hat{\mathrm{f}}(k) + \hat{\mathrm{g}}$

Minimize : $\min_{\mathrm{x},\mathrm{f}} \sum_{k=0}^{m} ||\mathrm{x}(k+1) - \mathrm{x}^{\mathrm{ref}}(k+1)||_{\boldsymbol{Q}} + ||\mathrm{f}(k)||_{\boldsymbol{R}}$

Constraint : $| f_x | \leq \mu f_z, \quad | f_y | \leq \mu f_z, \quad f_z > 0 \longleftarrow$ Friction force

Final output provided to the WBIC: $\boldsymbol{x}^{\boldsymbol{des}}$ and $\boldsymbol{f}_{\boldsymbol{r}}^{\boldsymbol{MPC}}$

$$\mathrm{x} = \begin{bmatrix} \Theta^\top & \mathrm{p}^\top & \omega^\top & \dot{\mathrm{p}}^\top \end{bmatrix}^\top$$

$$\hat{\mathrm{f}} = \begin{bmatrix} \mathrm{f}_1 & \cdots & \mathrm{f}_n \end{bmatrix}^\top$$

WBIC

1. Computes joint positions and velocities through task prioritization (null-space projection of the Jacobian) using MPC body and foot pos:

$$\mathrm{q}_j^{\mathrm{cmd}} = \mathrm{q}_j + \Delta \mathrm{q}_{i-1} + {J_{i|pre}}^{\dagger}\left(\mathrm{e}_i - J_i \Delta \mathrm{q}_{i-1}\right)$$

$$\dot{\mathrm{q}}_i^{\mathrm{cmd}} = \dot{\mathrm{q}}_{i-1}^{\mathrm{cmd}} + {J_{i|pre}}^{\dagger}\left(\dot{\mathrm{x}}_i^{\mathrm{des}} - J_i \dot{\mathrm{q}}_{i-1}^{\mathrm{cmd}}\right)$$

$$\boxed{\ddot{\mathrm{q}}_i^{\mathrm{cmd}}} = \ddot{\mathrm{q}}_{i-1}^{\mathrm{cmd}} + \overline{J_{i|pre}^{\mathrm{dyn}}}\left(\ddot{\mathrm{x}}_i^{\mathrm{cmd}} - \dot{J}_i \dot{\mathrm{q}} - J_i \ddot{\mathrm{q}}_{i-1}^{\mathrm{cmd}}\right)$$

2. Quadratic Programming $\Rightarrow$ Use f_r^{MPC} and $\ddot{q}^{cmd}$ to minimize $f_r - f_r^{MPC}$ and $\ddot{q} - \ddot{q}^{cmd}$:

$$\min_{\delta_{\mathrm{f}_r}, \delta_f} \quad \delta_{\mathrm{f}_r}^{\top} Q_1 \delta_{\mathrm{f}_r} + \delta_f^{\top} Q_2 \delta_f$$

Constraint :

$$\begin{aligned}
&S_f\left(A\ddot{\mathrm{q}} + \mathrm{b} + \mathrm{g}\right) = S_f J_c^{\top} \mathrm{f}_r && \text{(floating base dyn.)}\\
&\boxed{\ddot{\mathrm{q}}} = \boxed{\ddot{\mathrm{q}}^{\mathrm{cmd}}} + \begin{bmatrix} \delta_f \\ 0_{n_j} \end{bmatrix} && \text{(acceleration)}\\
&\boxed{\mathrm{f}_r} = \mathrm{f}_r^{\mathrm{MPC}} + \delta_{\mathrm{f}_r} && \text{(reaction forces)}\\
&W \mathrm{f}_r \geq 0, && \text{(contact force constraints)}
\end{aligned}$$

3. Computes joints torque using f_r and $\ddot{q}$ from quadratic programming :

$$\begin{bmatrix} \tau_f \\ \tau_j \end{bmatrix} = A\boxed{\ddot{\mathrm{q}}} + \mathrm{b} + \mathrm{g} - J_c^{\top} \boxed{\mathrm{f}_r}$$

Final output provided to the PD controllers : $\boldsymbol{\tau}, \boldsymbol{q_j^{cmd}}, \boldsymbol{\dot{q}_j^{cmd}}$

Experimental Results

Task	$\mathbf{k}_p$ (s^{-2})	$\mathbf{k}_d$ (s^{-1})
Body Orientation	$[100 \quad 100 \quad 100]^\top$	$[10 \quad 10 \quad 10]^\top$
Body Position	$[100 \quad 100 \quad 100]^\top$	$[10 \quad 10 \quad 10]^\top$
Foot Position	$[100 \quad 100 \quad 100]^\top$	$[10 \quad 10 \quad 10]^\top$

TABLE I

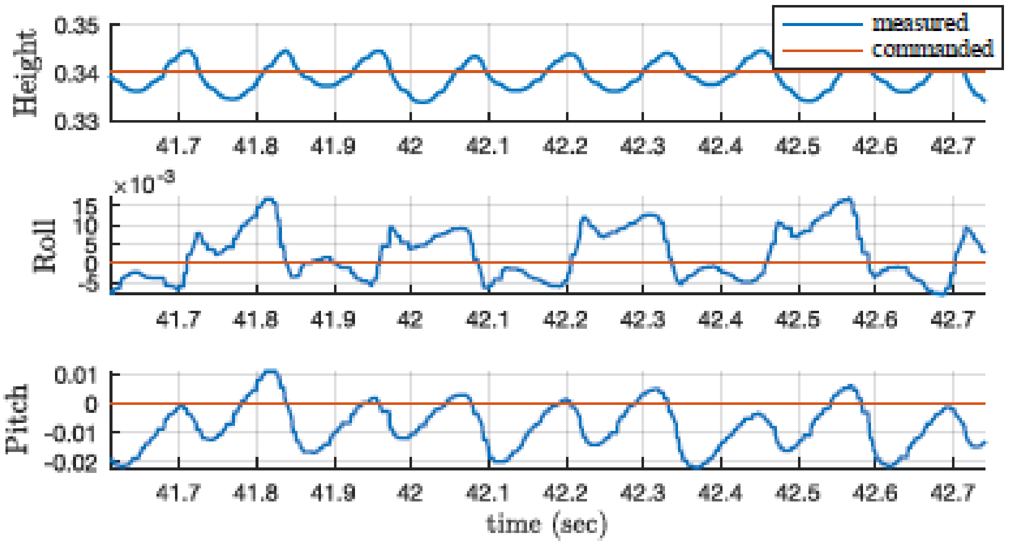
TASK AND GAIN SETUP

Table summarizing task setup and feedback gains of WBIC used in the experiments.

For the QP problem, $Q_1 = 1, Q_2 = 0.1$.

Joint feedback controllers:
$K_p = 3Nm/rad$ and
$K_d = 0.3Nms/rad$

- Maximum forward speed of **3.7m/s** (**one of the fastest** untethered quadruped robots running speed)
- The **hardware capability is fully utilized** in the test.
  - Max commanded joint velocity = 34rad/s
  - Max commanded torque at the hip joints = 25,5 Nm.
  - Max joint velocity=40rad/s
  - Max torque= 17 Nm



(d) Body height and orientation

Outdoor tests

- Various gaits with different speeds over 1m/s:gallop, bounding, pronking,trot,pacing…
- Robustness of the controller over different types of terrain(slippery/rough…)

Strong and weak points

+Robust navigation even over hard terrain
+Flexible and efficient execution of complexly dynamic gaits

-Computationally heavy which might limit it utilization of integrated systems
-Might require modifications if visual inputs are considered

Conclusion

- **New Control Scheme for quadrupled robots**

- Research's where the article was cited:
  1. ‘Whole-body MPC for highly redundant legged manipulators: experimental evaluation with a 37 DoF dual-arm quadruped’(2023)
  2. Quadrupedal Locomotion via Event-Based Predictive Control and QP-Based Virtual Constraints" (2020)
  3. "Whole-Body Nonlinear Model Predictive Control Through Contacts for Quadrupeds" (2017):

Questions for exam

1. What is the advantage of combining MPC and WBIC ?
   - Slide 4
2. What techniques the article uses to allow the MPC computed in real-time ?
   - Slide 7

Thanks for your attention