EPFL

Biped Walking Pattern Generation by using Preview Control of Zero-Moment Point

Kajita et al.

Group 7:

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Legged robot

Introduction

Main Contribution:

- Development of a method for generating stable biped walking patterns base on **Zero-Moment Point (ZMP) criteria**

- **Key Aspects:**
  - **Robot Type**:
    - Biped humanoid robots
  - **Control Type**:
    - Position control
    - Ensures dynamic stability by tracking the ZMP trajectory
  - **Design Method**:
    - Mathematically driven (Optimization through MPC)
  - **Gait Type**:
    - Stable and dynamic bipedal walking

Equations

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- Assumptions:
  - The hips of the robot stay approximately at the same height z_c during walking
  - Sin(θ) ≈ θ and θ ≈ y/z_c
- System equations:

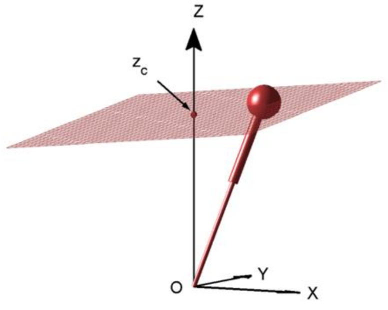

Figure 2: A pendulum under constraint

$$\sum \vec{F}_{\text{ext}} = m\vec{a}$$

$$\boxed{\ddot{x} = \frac{g}{z_c}x + \frac{1}{mz_c}\tau_y} \quad (1)$$

$$\boxed{\ddot{y} = \frac{g}{z_c}y - \frac{1}{mz_c}\tau_x} \quad (2)$$

Equations

- We can then find the ZMP by setting the accelerations to zero, where ρx and ρy are the coordinates of the ZMP on the floor.

$$\boxed{\begin{aligned} p_x &= -\frac{\tau_y}{mg} \\ p_y &= \frac{\tau_x}{mg} \end{aligned}} \qquad (3)$$

- By substituting Eqs. (3) to the 3D-LIPM (1) and (2) we obtain :

$$\boxed{\ddot{x} = \frac{g}{z_c}(x - p_x)} \qquad \boxed{\ddot{y} = \frac{g}{z_c}(y - p_y)}$$

Intuition

- Transform previous equations to have ZMP as outputs:

$$p_x = y - \frac{z_c}{g}\ddot{y}$$

$$p_y = x - \frac{z_c}{g}\ddot{x}$$

$$\tau_{zmp} = mg(x - p_x) - m\ddot{x}z_c = 0$$

- Intuition of the cart-table model
  - ZMP need to stay in the support zone
  - Simplification equilibrium control biped walk

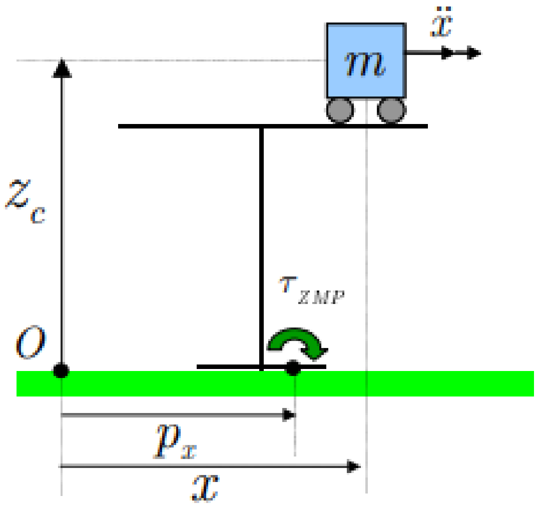

Figure 3: A cart-table model

Walking pattern generation for given ZMP (1/4)

- **Why calculate the movement of the centre of mass (CoM) as a function of the ZMP?**
  - Ensuring stability: Starting from a target trajectory for the ZMP (determined by the support points and the step period), we calculate the trajectory of the centre of mass (CoM) to ensure that the ZMP remains in the support zone, preventing the robot from losing its balance
- **Two Methods to resolve it:**
  - The Fourier transform (FFT) solves the equation in the frequency domain, simplifying the calculation
  - The discrete-time method, which divides time into small units for rapid calculation

Walking pattern generation for given ZMP (2/4)

ZMP control

- Creation of u_x & u_y to control the acceleration

$$\frac{d}{dt}\ddot{x} = u_x \ \& \ \frac{d}{dt}\ddot{y} = u_y$$

- Servo controller adjust u_x & u_y to minimize $\text{ZMP}_{\text{reference}}$ with ZMP_{real}
- When changing legs, the ZMP must move quickly to the new support zone, risking destabilising the robot
- Gradually adjusting the centre of mass (CoM) ensures a smoother transition for the ZMP
- Preview Control anticipates the change and adjusts the CoM in advance to ensure the ZMP moves smoothly and stably.

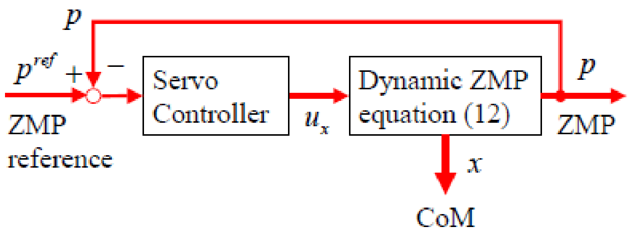

Figure 4: Pattern generation as ZMP tracking control

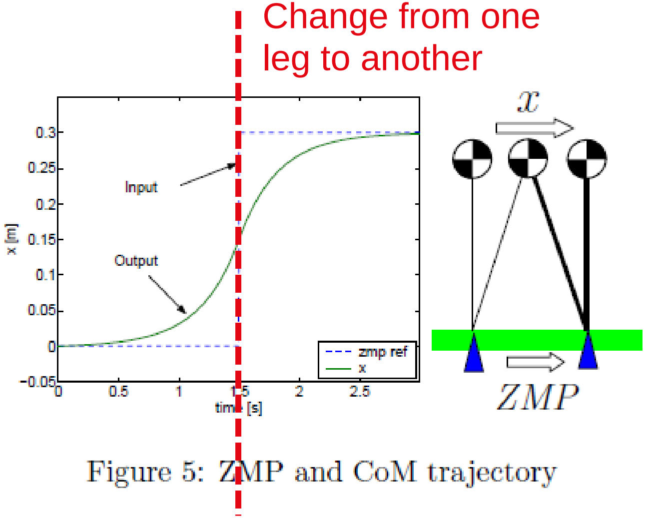

Figure 5: ZMP and CoM trajectory

Walking pattern generation for given ZMP (3/4)

Preview control / MPC

- We look forward in states to anticipate the ZMP change to move smoothly the CoM.
- We discretize time, and use MPC theory to find the best controller u(x) that will minimize the performance index by looking at N steps in the future.
- The associated preview Gains decrease quickly to be almost 0 after 2 seconds (useless to look more than 2 seconds further).
- The ZMP follow the reference with a good accuracy for 1.6 s but struggle for 0.8s because of the G(i) that still impact the optimized controller.

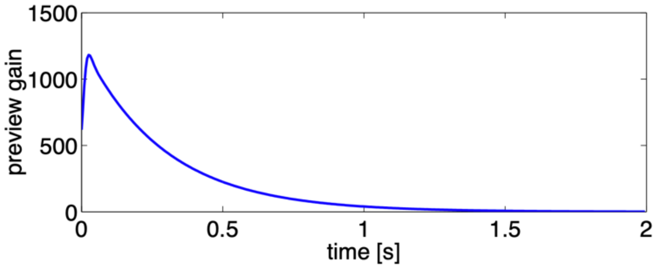

Figure 6: Preview controller gain G_p ($T = 5$[ms], $z_c = 0.814$[m], $Q_e = 1.0$, $Q_x = 0$, $R = 1.0 \times 10^{-6}$)

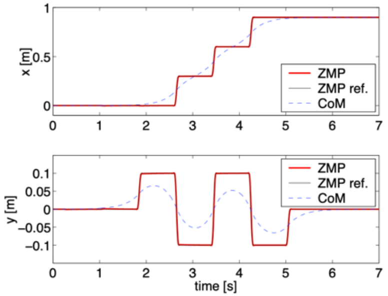

Figure 7: Body trajectory obtained by preview control, previewing period $T * N_L = 1.6(s)$

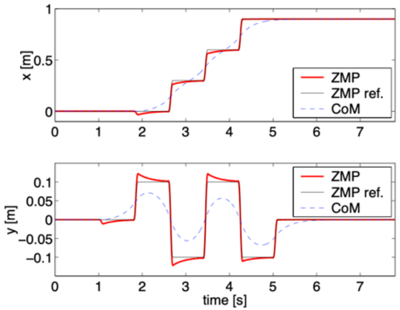

Figure 8: With shorter previewing period $T * N_L = 0.8(s)$

Walking pattern generation for given ZMP (4/4): Real test

- Application model in simulation with a real robot characterization (HRP-2)
- Model:
  - **ZMP Cart-Table Model**: Approximate, based on a single mass point. Simple and fast, but less accurate
  - **ZMP Multibody Model**: Detailed modelling of all parts of the robot More accurate but more computationally expensive
- Use buffer memory to decrease the error in ZMP

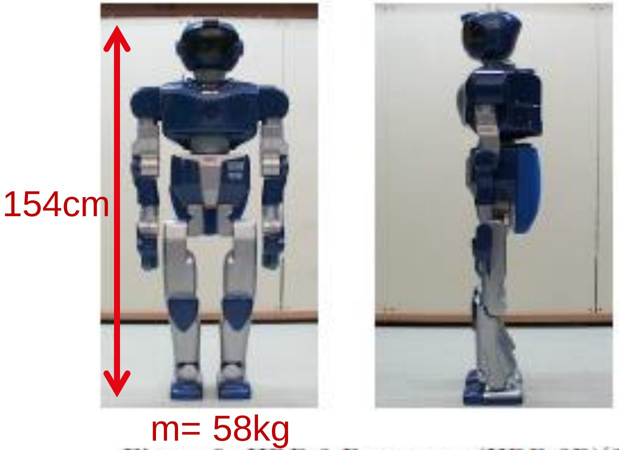

Figure 9: HRP-2 Prototype (HRP-2P)[22]

Without Preview control T×NL of 0,75s

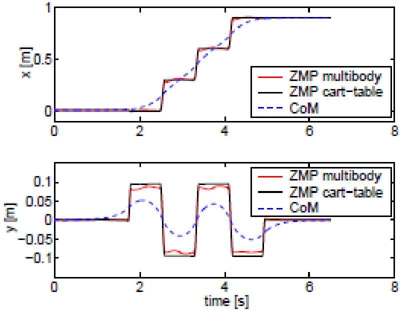

Figure 10: ZMP calculated by table-cart model and multibody model

Error in x: 2,3 cm
Error in y: 1,6 cm

With Preview control T×NL of 0,75s

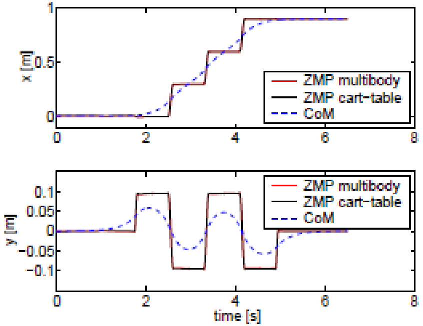

Figure 11: Modified ZMP of multibody model

Error in x: 1,2 cm
Error in y: 0,4 cm

Simulation

- **Objective**: To test the ability of the tread pattern generator to handle spiral staircases
- **Foot placement**: Foot position specified to avoid step edges
- **CoM trajectory**: Horizontal part generated with the previous method; elevation calculated according to the geometric constraint of the stairs
- **Staircase configuration**: Step height 0.1 m; 24° rotation per step; inside radius 0.7 m; outside radius 1.3 m
- **Result**: Successful simulation with OpenHRP; the HRP-2P robot walks stably on the spiral staircase

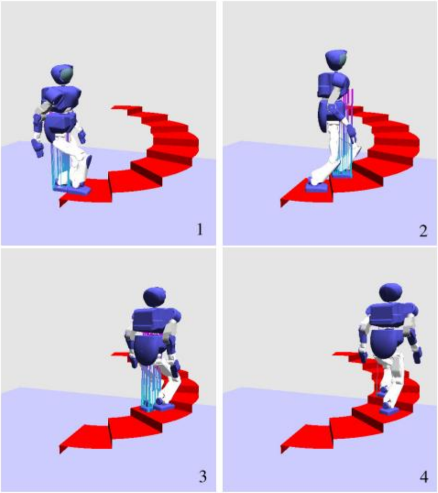

Figure 13: Snapshots of walking on spiral stairs (simulation)

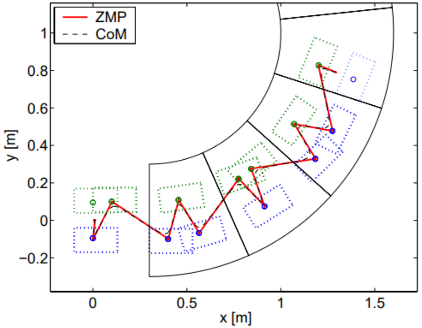

Figure 12: Planned trajectory for a walk on spiral stairs :ZMP and CoM projected on the horizontal plane

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Pros and Cons

PROS

- **Accurate ZMP Tracking**: Preview Control enables the system to anticipate trajectory changes, keeping the ZMP stable within the support area, even during complex movements like spiral stair climbing.
- **Adaptability to Terrain Changes**: The model adapts to geometric constraints, like stairs, by adjusting the CoM's slope and elevation, making it versatile for various environments.
- **Use of Future Information**: By leveraging future errors, Preview Control allows proactive CoM adjustments, compensating for differences between the simplified model (cart-on-table) and the detailed multibody model, reducing imbalance risks.

CONS

- **Computational Complexity**: The multibody model and Preview Control require high computational power, which can be challenging to handle in real-time, especially for robots with many joints.
- **Sensitivity to Modeling Errors**: If the cart-on-table model is oversimplified, CoM adjustments may be insufficient to compensate for errors, potentially compromising the robot's stability.

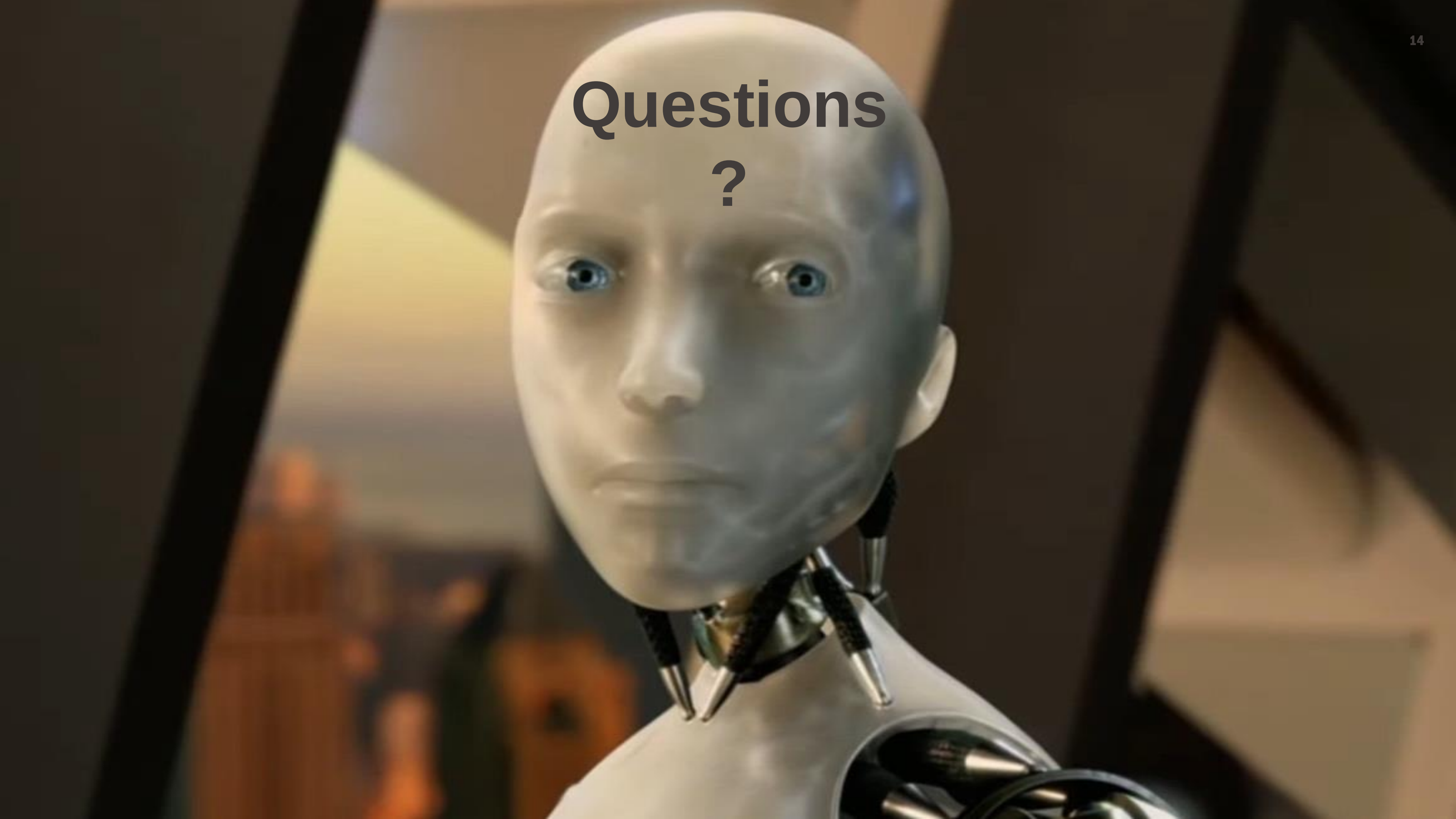
Conclusion

- **New biped gait generation method**: Introduction of a table-top trolley model to design a ZMP tracking controller
- **ZMP controller with Preview Control**: Use of anticipation to compensate for ZMP errors between the simplified table-top trolley model and the detailed multi-body model
- **Demonstration**: Gait trajectory generated for spiral staircases and validated by dynamic simulation
- **Used in several other studies**: Used by ExoRecovery to implement a stepping strategy for push recovery in a lower-limb exoskeleton. The work of Kajita et al. (2003) on ZMP-based walking pattern generation with preview control serves as a foundation for stabilizing the exoskeleton’s center of mass after external perturbations, enabling rapid response to maintain balance. (by M.Bouri and I.Auke)
- **Next objective**: Implementation on a physical robot for real tests

Possible Exam questions

- What are the assumptions to find equations (1) and (2) ?
  - Slide 3
- Why do we use preview control ?
  - Slide 7-8

Questions ?



MPC

First, we discretize the system of Eq. (12) with sampling time of T as

$$\begin{aligned} \boldsymbol{x}(k+1) &= A\boldsymbol{x}(k) + Bu(k), \\ p(k) &= C\boldsymbol{x}(k), \end{aligned} \tag{13}$$

where

$$\begin{aligned} \boldsymbol{x}(k) &\equiv \begin{bmatrix} x(kT) & \dot{x}(kT) & \ddot{x}(kT) \end{bmatrix}^T, \\ u(k) &\equiv u_x(kT), \\ p(k) &\equiv p_x(kT), \\ A &\equiv \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \\ B &\equiv \begin{bmatrix} T^3/6 \\ T^2/2 \\ T \end{bmatrix}, \\ C &\equiv \begin{bmatrix} 1 & 0 & -z_c/g \end{bmatrix}. \end{aligned}$$

With the given reference of ZMP $p^{ref}(k)$, the performance index is specified as

$$J = \sum_{i=k}^{\infty} \{Q_e e(i)^2 + \Delta\boldsymbol{x}^T(i) Q_x \Delta\boldsymbol{x}(i) + R\Delta u^2(i)\}, \tag{14}$$

where $e(i) \equiv p(i) - p^{ref}(i)$ is servo error, $Q_e, R > 0$ and Q_x is a 3×3 symmetric non-negative definite matrix. $\Delta\boldsymbol{x}(k) \equiv \boldsymbol{x}(k) - \boldsymbol{x}(k-1)$ is the incremental state vector and $\Delta u(k) \equiv u(k) - u(k-1)$ is the incremental input.

When the ZMP reference can be previewed for N_L step future at every sampling time, the optimal controller which minimizes the performance index (14) is given by

$$u(k) = -G_i \sum_{i=0}^{k} e(k) - G_x \boldsymbol{x}(k) - \sum_{j=1}^{N_L} G_p(j) p^{ref}(k+j), \tag{15}$$

where G_i, G_x and $G_p(j)$ are the gains calculated from the weights Q_e, Q_x, R and the system parameter of Eq. (13).

ZMP control supplementary

$$
\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u_x
$$

$$
p_x = \begin{bmatrix} 1 & 0 & -z_c/g \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix}. \qquad (12)
$$

Samuel

ZMP vs Inverted pendulum

ZMP

- Requires **precise knowledge of the dynamics** (mass, centre of mass, inertia of each link)
- Based on ZMP to **control balance and generate walking patterns**

Advantages:

- **High** model **accuracy**
- Suitable for situations requiring **precise foot placement** (e.g. walking on ford stones)

Disadvantages:

- Highly **dependent on the accuracy of the** dynamic **model**

Inverted pendulum

- Uses **limited knowledge of dynamics** (position of the total centre of mass, angular momentum).
- Is based on an **inverted pendulum** model and **feedback control**

Advantages:

- Less complex, suitable for **simple dynamic movements**

Disadvantages:

- **Less accurate** foot placement
- **Limitations in situations requiring specific foot positioning**