**Date:** 18 janvier 2022 **Durée:** 2:30 hours

# Version A

First name		
2 <sup>nd</sup> name		
SCIPER	 Signature :	

Wait for the exam to start before turning this page! The document is printed in two-sided mode, 12 pages. Do not remove clips.

- Leave your student card CAMIPRO visible in front of you on the desk
- Only allowed documents 3 two sided A4 pages of your personal notes.
- Any electronic device and any pocket calculator, with internet access or storage capability, is strictly prohibited. If you have one with you, give it to the attendants during the exam.

Multiple choice questions are counted as defined in the additional sheet, as follows:

+3 points, /+2 points, /+1 point

0 for all other cases. There is always **only one correct answer**!

- The true/false questions give one point for a correct mark, 0 in all other cases:
- Use a pencil, so you can erase properly in case of change of mind.
- If a flaw is detected in a question, the examiners may cancel it.
- For the other problems, use these handed out sheets (10 pages in total).
- Reading of the multiple choice questions is automated, to avoid mistakes mark your replies as shown below:





# Exercise. 1 : True or false ? (15 pts)

Please answer as **True** or **False** on the answer sheet for the following statements.

1.1	Parallel robots are characterized by a closed kinematic loop.			
1.2	In general, a parallel robot is less rigid than a serial one.			
1.3	The role of the derivative gain of a PD controller is to introduce damping in the closed loop behaviour.			
1.4	The proportional gain of a PD controller reduces the static error.			
1.5	The dynamic model of a robot relates joint positions and joint torques.			
1.6	The optimal reduction gear ratio (corresponding to optimal fitting of motor + reduction gear + load), does not allow to minimize energy losses			
1.7	The optimal reduction gear ratio (corresponding to optimal fitting of motor + reduction gear + load) allows maximum output velocity			
1.8	Redundant industrial robots have more actuators than their number of degrees of freedom			
1.9	The Jacobian of a robot relates the applied force at the tool with joint torques.			
1.10	The Jacobian of a robot relates joint positions with joint angles.			
1.11	Forward kinematics of a robot gives position & orientation of the end-effector in function of joint angles.			
1.12	Stepper motors are more precise than brushless DC motors			
1.13	Stepper motors are faster than brushless DC motors			
1.14	The trace of the direction cosine matrix is always = 1			
1.15	Calculation of an orientation quaternion gives $\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4} \}$ Can this be correct?			

## Exercise. 2 (21pts)

Consider the structure represented by the opposite figure. Two kinematic chains, composed by an arm and a forearm each, are controlled using two brushless DC motors combined with gear transmissions.

Two possibilities are made for the arms and forearms:

Aluminium arms,  $m_a = 2$  g,  $J_a$ \* = 40 gcm<sup>2</sup> Carbon fiber arms,  $m_a = 1.2$  g,  $J_a$  \* =25 gcm<sup>2</sup>

Aluminium forearms,  $m_{fa} = 2 g$ Carbon fiber forearms,  $m_{fa} = 1.2 g$ 

The length of each arm is  $l_a$  = 80 mm \*) the moment of inertia is considered around the axis of rotation of  $\theta(\theta_1 \text{ or } \theta_2)$ 

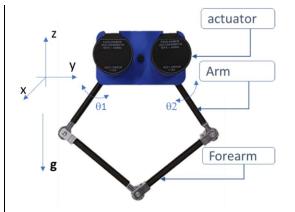


Figure 1- Pantograph-like structure. **g** is the earth's gravitation.  $\theta_1$  or  $\theta_2$  are equal to zero in the horizontal

These two rotational axes are designed as follows:

Motor Flat 42, Series 4221 ... BXT R

- Moment of inertia Jm = 70 gcm²
- Nominal torque = **140 mNm**
- Nominal velocity = 2500 rpm

Reduction gear,

- Gear transmission ratio n= 3,1
- Moment of inertia of the reducer is neglected
- Efficiency = 95%

In general, the Matrix of inertia associated with the inverse dynamic model of this robot is submitted to coupling. To simplify this model, we neglect the moment of inertia of the forearms and we assume that their mass **m\_fa** is all concentrated as punctual in the elbow.

**2.1** For each axis, the motor inertia as reported to the arm shaft is:

- (A) 672,7 10<sup>-7</sup> kgm<sup>2</sup>
- (B) 7,3 10<sup>-7</sup> kgm<sup>2</sup>
- (C) 217 10<sup>-7</sup> kgm<sup>2</sup>
- (D)22.6 10<sup>-7</sup> kgm<sup>2</sup>

**2.2** Using Aluminium Arms, excluding the motor inertia, the total inertia as reported to the arm shaft is:

- (A) 168 10<sup>-7</sup> kgm<sup>2</sup>
- (B) 40 10<sup>-7</sup> kgm<sup>2</sup>
- (C) 128 10<sup>-7</sup> kgm<sup>2</sup>
- (D) 32 10<sup>-7</sup> kgm<sup>2</sup>

**2.3** Using carbon fibre arms, excluding the motor inertia, the total inertia as reported to the arm shaft is:

- (A)102 10<sup>-7</sup>kgm<sup>2</sup>
- (B) 25 10<sup>-7</sup> kgm<sup>2</sup>
- (C) 76.8 10<sup>-7</sup> kgm<sup>2</sup>
- (D) 23 10<sup>-7</sup> kgm<sup>2</sup>

**2.4** for the **aluminium configuration**, the optimal reduction gear ratio is

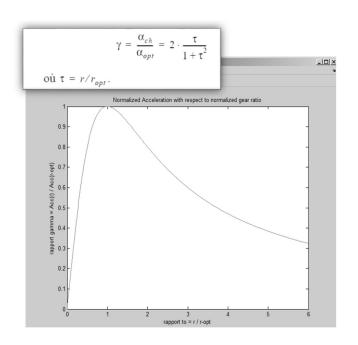
(A) 1.55

- (B) 0,6
- (C) 1,35
- (D) 0,7

**2.5** for the **carbon fibre configuration**, the optimal reduction gear ratio is

- (A) 1.2
- (B) 0,6
- (C) 1,35
- (D) 0,7

Here is a plot of the relative output acceleration over the optimal acceleration  $\gamma(\tau)$  in function of the relative reduction gear ratio tau (r optimal corresponds to tau = 1, optimal acceleration corresponds to  $\gamma$ =1)



**2.6** In the aluminium configuration, the combination (motor + gear ratio = 3) will allow the following acceleration capability (according to the optimal acceleration)

- (A) ~ 80%
- (B) ~ 40%
- (C) ~ 50%
- (D) ~ 60%

**2.7** In the carbon fibre configuration, the combination (motor + gear ratio = 3) will allow the following acceleration capability (according to the optimal acceleration)

- (A) ~ 65%
- (B)  $\sim 60\%$
- (C) ~ 80%
- (D) ~ 100%

#### Exercise. 3 (3, 3, 3, 2, 2 = 13 pts)

This exercise continues from the previous exercise 2.

**3.1** We need to have a resolution of a minimum of **12 microns** at the side of the elbow. The minimal number of quadrature periods (the encoder pitch) is

- (A) 500 qp
- (B) 1000
- (C) 4000 microns
- (D) 10000 qp

qp refers to quadrature periods

**3.2** The controller sampling frequency is 2 kHz and the output velocity is computed from derivation over 2 sampling periods. The resolution of the velocity at the elbow is then:

- (A) 10,13 mm/sec
- (B) 20,26 mm/sec
- (C) 5,05 mm/sec
- (D) 2.52 mm/sec

**3.3** The available nominal torque at the side of the arm shaft is:

- (A) 0,412 Nm
- (B) 0,434 Nm
- (C) 0,140 Nm
- (D) 0,133 Nm

**3.4.** To improve the velocity resolution, it is better to:

(A) Change the actuator

- (B) Reduce the sampling frequency
- (C) Increase the sampling frequency
- (D) None of the above responses

	would like to make hoosing among diff		ess sensitive to the output required torque. To that, is better to :			
(A) Ch	oose a motor with	a lower constant	of (B) Choose a motor with a higher constant of			
velocity (C) Choose a motor with a lower factor of regulation		h a lower factor	velocity of (D) Choose a motor with a higher factor of regulation			
	e 4 (1,1,2,2,3,2, 2,		ous exercise 2 and 3			
	is exercise continu	es ironi the previo	us exercise 2 and 5			
<b>4.1</b> Wh	at is the number of	degrees of freedo	m of the described pantograph structure? (1 pnt)			
(A) 1	(B) 2	(C) 3	(D) 4			
<b>4.2</b> Wh	4.2 What is the number of mobility of the described pantograph structure? (1pnt)					
(A) 2	(B) 0	(C) -1	(D) -2			
<b>4.3</b> Cho	oose the correct ans	swer: (2 pnts)				
(A) This structure is normally constrained (C) This structure is over-constrained			(B) This structure has internal mobilities (D) None of the above responses apply			
<b>4.4</b> Cor	ncerning the Jacobia	an matrix, which or	ne of the following statements is correct? (2 pnts)			
	Jacobian matrix de Jacobian matrix is		the robot position bot position			
(C) The	Jacobian matrix cone of the above res	orresponds to Mati	•			
The fol			e next part of the exercise:			
•	<ul> <li>The efficiency of the gear transmission is assumed to be ideal, as 100%.</li> </ul>					
•	• The centre of mass of the arms is in the mid of the arms $(l_a/2)$ .					

- We neglect the moment of inertia of the forearms and we assume that their mass **m\_fa** is all concentrated as punctual in the elbow.
- $\theta_1$  or  $\theta_2$  are equal to zero in the horizontal

**4.5** The inverse dynamic model of the axis 1 expressed according to the output of the reducer, (rotational axis  $\theta_I$ ) is given by : (3pnts)

(A) 
$$\Gamma_1 = \{J_a + n^2 J_m + M_{fa} l_a^2\} \ddot{\theta}_1 + M_a g \frac{l_a}{2} \sin(\theta_1) + M_{fa} g l_a \sin(\theta_1)$$

**(B)** 
$$\Gamma_1 = \{J_a + n^2 J_m + M_{fa} l_a^2\} \ddot{\theta}_1 + M_a g \frac{l_a}{2} \cos(\theta_1) + M_{fa} g l_a \cos(\theta_1)$$

(C) 
$$\Gamma_1 = \{J_a + n^2 J_m + M_{fa} l_a^2\} \ddot{\theta}_{1d} + M_a g \frac{l_a}{2} \sin(\theta_{1d}) + M_{fa} g l_a \sin(\theta_{1d})$$

(D) 
$$\Gamma_1 = \{J_a + n^2 J_m + M_{fa} l_a^2\} \ddot{\theta}_{1d} + M_a g \frac{l_a}{2} \cos(\theta_{1d}) + M_{fa} g l_a \cos(\theta_1 d)$$

**4.6** The inverse dynamic model a priori of the axis 1 expressed according to the output of the reducer, (rotational axis  $\theta_l$ ) is given by : (3 pnts)

(A) 
$$\Gamma_{1\_ap} = \{J_a + n^2 J_m + M_{fa} l_a^2\} \ddot{\theta}_1 + M_a g \frac{l_a}{2} \sin(\theta_1) + M_{fa} g l_a \sin(\theta_1)$$

**(B)** 
$$\Gamma_{1\_ap} = \{J_a + n^2 J_m + M_{fa} l_a^2\} \ddot{\theta}_1 + M_a g \frac{l_a}{2} \cos(\theta_1) + M_{fa} g l_a \cos(\theta_1) \}$$

(C) 
$$\Gamma_{1\_ap} = \{J_a + n^2 J_m + M_{fa} l_a^2\} \ddot{\theta}_{1d} + M_a g \frac{l_a}{2} \sin(\theta_{1d}) + M_{fa} g l_a \sin(\theta_{1d})$$

(D) 
$$\Gamma_{1\_ap} = \{J_a + n^2 J_m + M_{fa} l_a^2\} \ddot{\theta}_{1d} + M_a g \frac{l_a}{2} \cos(\theta_{1d}) + M_{fa} g l_a \cos(\theta_1 d)$$

- 4.7 The torque a priori is mainly required to: (2 pnts)
  - (A) know the internal forces/torques at each joint.
  - (B) improve the steady state positioning accuracy of a robot
  - (C) improve the dynamic positioning accuracy of a robot
  - (D) project the joint forces generalized to the forces/torques at the tool level.
- **4.8** Please chose which statement is not correct. (2pnts)
  - (A) The method of Newton-Euler of dynamic modeling allows to know the internal forces.
  - (B) The Systematic approach studied in the lecture and obtained from the method of Lagrange is only valid for serial robots.
  - (C) The method of Newton Euler studied in the lecture is valid for serial and paralle robots.
  - (D) The method of Newton Euler studied in the lecture is valid only for a category of serial robots.

- **4.9** Regarding the development of the dynamic model of a robot, which of the following expressions is not correct (2 pnts)
  - (A) The Lagrange method gives the same result as the Newton-Euler method.
  - (B) For Cartesian robots, the Newton approach is sufficient.
  - (C) The Newton-Euler method gives a more accurate model.
  - (D) The Lagrangian approach is more systematic than the Newton-Euler approach
- **4.10** Please chose which statement is not correct. (2 pnts)
  - (A) The torque a priori is not sensitive to sensor noise
  - (B) The torque a priori helps to size sensors for closed loop control
  - (C) The torque a priori helps to improve control performance
  - (D) The torque a priori helps to size the actuators of the robot.

## **Exercice 5 Position control (16 pts)**

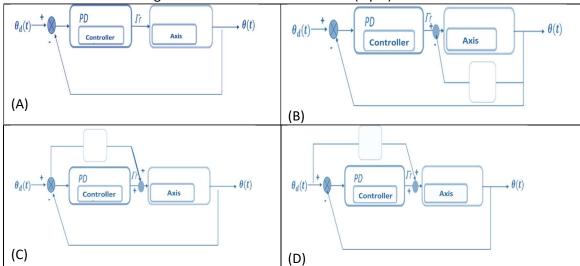
**Important**: All questions in this exercise assume that the DC motors of the exercise 2 are driven in current, without transmission viscosity and without friction.

Please answer as **True** or **False** on the answer sheet for the following statements. 1 pnt /question

- 5.1 In the absence of dry and viscous friction, the proportional controller is sufficient to control the position of the axis in question.
- 5.2 In the absence of dry and viscous friction, the integral action is never required to control the position of the axis in question.
- **5.3** The derivative action of a PID allows the oscillations to be damped around the desired position.
- The integrating action of a PID allows oscillations to be damped around the desired position.
- **5.5** The derivative action of a PID reduces the steady state error

- 5.6 The minimal controller to control the position of the robot of exercise (2) is (2 pnts)
- (A) P
- (B) PI
- (C) PD
- (D) PID

**5.7** Which of the following control schemes is not correct? (3 pts)



- **5.8**. To compensate for the gravity effect, we can (2 pnts)
  - (A) Oversize the torque a priori
  - (B) Increase the proportional control gain of the PD position controller.
  - (C) Reduce the derivative control gain of the PD position controller.
  - (D) Implement an integrator action in the position controller.
- **5.9**. To compensate for the friction effect, we can (2 pnts)
  - (A) Oversize the torque a priori
  - (B) Increase the proportional control gain of the PD position controller.
  - (C) Reduce the derivative control gain of the PD position controller.
  - (D) Implement an integrator action in the position controller.
- **5.10**. Which statement is not correct? (2 pnts)
  - (A) The exact compensation contribution is sensitive to sensor noise
  - (B) The a priori compensation contribution is sensitive to sensor noise
  - (C) The PD controller is sensitive to sensor noise
  - (D) The exact compensation control approach is more general than the a priori control approach

# Exercise 6 (3\*7 + 3\*2 = 27pts)

- **6.1.** The angle and axis of rotation given by the quaternion  $\{\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0, 0\}$  are the following: (3pnts)
  - **A)** 45° around  $\left[\frac{\sqrt{2}}{2}, 0, 0\right]^T$  **C)** 60° around the Y axis

B) 90° around the X axis

- A) 90° around the Y axis
- **6.2.** What is the quaternion corresponding to a rotation of 120° around the axis  $[-1, 0, 1]^T$ ? (3pnts)
  - **A)**  $\left\{\frac{1}{2}, -\frac{\sqrt{6}}{4}, 0, \frac{\sqrt{6}}{4}\right\}$

B)  $\{-\frac{1}{2}, -\frac{\sqrt{6}}{4}, 0, \frac{\sqrt{6}}{4}\}$ D)  $\{\frac{\sqrt{3}}{2}, -\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}\}$ 

C)  $\{-\frac{1}{2}, 0, -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\}$ 

- 6.3. What is the quaternion corresponding to a rotation of 90° around the Y axis? (3 pnts)
  - **A)**  $\{\frac{1}{2}, 0, \frac{\sqrt{3}}{2}, 0\}$

**c)** {0, 0, 1, 0}

- **B)**  $\{0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\}$  **D)**  $\{\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}, 0\}$
- 6.4. What is the quaternion corresponding to the rotation in 6.3 followed by the rotation in 6.2?
  - A)  $\{\frac{\sqrt{2}}{4}, -\frac{\sqrt{12}}{4}, \frac{\sqrt{2}}{4}, 0\}$ C)  $\{\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}, \frac{1}{2}\}$

- B)  $\{\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{4}, 0, \frac{\sqrt{2}}{2}\}$ D)  $\{\frac{\sqrt{2}}{2}, -\frac{\sqrt{12}}{4}, \frac{\sqrt{2}}{4}, \frac{1}{2}\}$
- **6.5.** What is the matrix of direction cosines for a rotation of 120° around the axis  $[-1, 0, 1]^T$ ? (3
  - A)  $\begin{bmatrix} \frac{\sqrt{6}}{2} & -\frac{\sqrt{6}}{4} & \frac{3}{4} \\ \frac{\sqrt{6}}{2} & \frac{1}{2} & \frac{\sqrt{6}}{2} \\ \frac{3}{2} & -\frac{\sqrt{6}}{2} & \frac{\sqrt{6}}{2} \end{bmatrix}$

 $\mathbf{B}) \begin{bmatrix} \frac{1}{4} & -\frac{\sqrt{6}}{4} & -\frac{3}{4} \\ \frac{\sqrt{6}}{4} & -\frac{1}{2} & \frac{\sqrt{6}}{4} \\ -\frac{3}{4} & -\frac{\sqrt{6}}{4} & \frac{1}{4} \end{bmatrix}$ 

c)  $\begin{bmatrix} -1 & -\frac{\sqrt{6}}{4} & -\frac{3}{4} \\ \frac{\sqrt{6}}{2} & 0 & \frac{\sqrt{6}}{2} \\ -\frac{3}{4} & -\frac{\sqrt{6}}{4} & 1 \end{bmatrix}$ 

- $\mathbf{D}) \begin{bmatrix} -\frac{3}{4} & -\frac{\sqrt{6}}{4} & -\frac{3}{4} \\ \frac{\sqrt{6}}{2} & 1 & \frac{\sqrt{6}}{2} \\ -\frac{3}{2} & -\frac{\sqrt{6}}{2} & -\frac{3}{2} \end{bmatrix}$
- 6.6. What is the matrix of direction cosines for a rotation of 90° around the Y axis? (3 pnts)

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A)  $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 

 $\mathbf{B}) \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ 

$$\mathbf{A)} \begin{bmatrix} \frac{3}{2} & -\frac{\sqrt{6}}{2} & \frac{1}{4} \\ -\frac{\sqrt{6}}{4} & \frac{1}{4} & \frac{\sqrt{6}}{4} \\ \frac{1}{4} & \frac{\sqrt{6}}{4} & \frac{3}{4} \end{bmatrix}$$

$$\mathbf{B}) \begin{bmatrix} 0 & -\frac{\sqrt{6}}{4} & \frac{1}{2} \\ -\frac{\sqrt{6}}{4} & 1 & \frac{\sqrt{6}}{4} \\ \frac{1}{2} & \frac{\sqrt{6}}{4} & 0 \end{bmatrix}$$

$$\textbf{C)} \begin{bmatrix} \frac{3}{4} & -\frac{\sqrt{6}}{4} & \frac{1}{4} \\ -\frac{\sqrt{6}}{2} & \frac{1}{2} & \frac{\sqrt{6}}{2} \\ \frac{1}{4} & \frac{\sqrt{6}}{4} & \frac{3}{4} \end{bmatrix}$$

$$\mathbf{D)} \begin{bmatrix} \frac{3}{4} & -\frac{\sqrt{6}}{4} & \frac{1}{4} \\ -\frac{\sqrt{6}}{4} & -\frac{1}{2} & \frac{\sqrt{6}}{4} \\ -\frac{1}{4} & -\frac{\sqrt{6}}{4} & -\frac{3}{4} \end{bmatrix}$$

**6.8.** The homogeneous matrix that represents a rotation around an axis which does not pass from the origin, centered at p is: (2 pnts)

$$\mathbf{A)} \ \ H \ = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{B)} \quad H = \begin{bmatrix} R & p - Rp \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{C)} \ \ H \ = \begin{bmatrix} R & R - pR \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{D)} \quad H = \begin{bmatrix} R & Rp \\ 0 & 1 \end{bmatrix}$$

6.9. Which statement is not correct? (2 pnts)

- (A) The Jacobian Matrix derivates from the geometric model
- (B) The Jacobian matrix is a kinematic matrix
- (C) If the number of DOFs equals the number of actuators, the Geometric model is represented by a square Matrix.
- (D) The method of Newton Raphson requires the Jacobian Matrix to invert a geometric model
- **6.10.** Which answer is not correct? The Jacobian Matrix may have the following use: (2 pnts)
  - (A) It projects the velocities between joint and tool spaces
  - (B) It deduces the torque resolution function of the sensor resolution
  - (C) It projects the resolutions between joint and tool spaces
  - (D) It is used to mplement tool space controllers.

## Exercise 7 (16 pts, 2 pts each)

Let us consider the robot Delta with only ball bearings as implemented by the company Force Dimension.





This robot belongs to the family of Delta robots. All its joints are pivots!

**7.1** This kinematics is

(A) Serial (B) Parallel (C) Hybrid (D) Cartesian

**7.2** The number of joints of this structure is:

- (A) 3 (B) 5 (C) 15 (D) 21
- **7.3** The number mobility of this structure is
- (A) 3 (B) 9 (C) -3 (D) -9
- **7.4** Choose the correct answer:
  - (A) This structure is normally constrained (B) This structure has internal mobilities
  - (C) This structure is over-constrained (D) None of the above responses apply
- **7.5 Select the correct answer.** This implementation is dedicated to haptic applications, requiring less friction, because of:
  - ess metion, because or:
  - (A) the answer of question 7.4 (B) the implementation of pivots as ball bearings and the answer to 7.4
  - (C) the implementation of pivots as ball (D) None of the above responses apply bearings

Force Dimension provides in its catalogue another structure with the all the DOFs of the wrist actuated, in addition to the grasp. This structure is called the Sigma 7.



- **7.6** This kinematics (Sigma 7) is
- (A) Serial (B) Parallel (C) Hybrid (D) Cartesian
- **7.7** The number mobility of this structure is
- (A) 7 (B) 11 (C) 1 (D) -5
- **7.8** Choose the correct answer:
  - (A) This structure is normally constrained
- (B) This structure has internal mobilities
- (C) This structure is over-constrained
- (D) None of the above responses apply