PART I

General overview on robot manipulators

Serial and parallel robots

SUMMARY

This chapter presents the basic concepts, definitions and classification of serial and parallel robots. Tools to calculate the mobility of parallel robots are described here.[1]

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[1] This chapter has been adapted from the EPFL text book chapter of Professor Reymond Clavel

Contents:

1.1 Definitions, history, fields of application, economic weight, characteristics.

1.1.1 Definitions and history.

If the concept of automaton dates back several centuries, the word robot only appeared very recently. It is derived from the Czech word *"robota"* meaning work or even chore. It was introduced in 1920 by the Czech writer Karel Capek in his anticipation piece "RUR" ("Rossum's Universal Robots") to designate automatic machines called upon to replace men in the most arduous tasks.

Today, the word robot is used to designate the simplest of manipulators, and even household appliances, as well as the most sophisticated machine. It is for this reason that we introduced the notion of industrial robot.

The *robotic systems industry* has grown significantly; applications in the fields of medicine, space, training, agriculture, civil engineering, and even military (non-exhaustive list) have led to a significant extension of this concept of robotic system. Quantitatively, it is undeniable that currently industrial robots play the most important role. Most of the concepts and methods of industrial robotics can be transposed to other sectors. Autonomous robots are now in a phase of marked growth, especially in the areas of household (lawn mower, vacuum cleaner), security, space, maintenance (for example exploration of pipes).

By *industrial robot* (IR) is meant a reprogrammable and versatile automatic machine, designed to move materials, parts, tools, or specialized devices through variable and programmable movements to accomplish various tasks. This definition is similar to the standard proposed by AFNOR (Standard registered NF E 61400 in August 1983) and accepted in the meantime at ISO international level. This definition is still too vague to allow a machine to be systematically classified as being, or not being, an industrial robot.

For instance, common pneumatic *load arms* are not considered as industrial robots because they are not programmable. Similarly, automatic manufacturing and assembly systems *designed for a particular application* must be excluded from the family of industrial robots. The word (rigid automation) refer this kind of automation, as a frozen mechanization of the process, opposite to (flexible automation).

The first industrial robot appeared on the market in the US in 1962. Initially used only in the automotive industry, they gradually conquered other economic sectors such as metallurgy, microtechnology, agri-food, chemistry-pharmacy, and even construction.

Discredited in the 1980s by technological excesses and a frequent inadequacy of needs, industrial robots are making a comeback. Having become cheaper and more efficient, they offer profitable solutions to companies looking for competitiveness. In 2008, we can estimate that the number of industrial robots installed in the world largely exceeded 1 million. About 350,000 IR are installed in Japan. The last two years, we have observed an annual growth of almost 20% depending on the region. The density of robots per 10,000 workers varies between 50 and 150 for industrialized countries (Europe and USA) while it reaches 330 in Japan and around 150 in Korea.

1.1.2 Fields of application, economic weight.

The fields of application are very variable and they evolve over the years. We can cite:

Well-established fields

automotive industry: spot welding, arc welding, installation of wheels, windows, glue joints, etc.
mechanical industries: welding, handling, polishing, deburring, feeding presses (injection, cutting, stamping), laser cutting, water jet cutting, machining by chip removal, etc.
electrical appliance industry: assembly, wiring, gluing, testing, etc.
electronics industry: mechanical assembly, (CMS) SMD component assembly, tests, ...
food and consumer products industry: packaging of chocolates, cookies, cutting of frozen products, handling of storage and destocking, grouping of products in boxes or pallets, etc.
agriculture: processing cheese, milking cows
clothing, footwear: cutting of fabrics, leather or plastic by laser or water jet, deposit of adhesive beads, inspection, packaging, etc.

Under development

the chemical and microbiological laboratory, textile industry, dyes
Micromanipulation (resolution <0.5 micron); currently in the process of progress;
mail order, sorting of packages

In research phase and testing

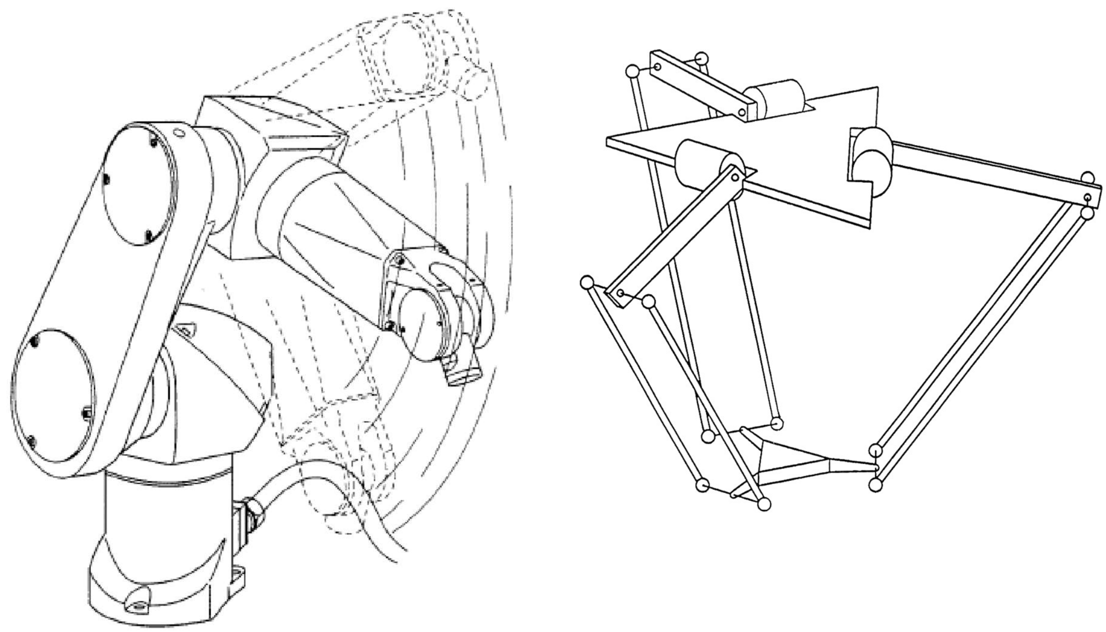
agricultural: sheep shearing, fruit and vegetable sorting, fruit picking, etc.
construction in the workshop or on the site: leveling of screeds, welding, painting, etc.
waste management: sorting and garbage collection
services: help for the disabled and the sick, distribution of mail, food or medicines, cleaning, tanking of vehicles, surveillance, demining, medicine, surgery (will soon be in the development phase)
space: assisted remote manipulation

Study phase

personal robot (by analogy with the PC)
cleaning robot (cleaning, tidying up, cooking, etc.)
space: For this field, the applications are so specific that the transition to the development phase will always be limited.

The search for an increasingly flexible production has pushed the development of robots. Dangerous, unhealthy or monotonous workstations quickly led to the introduction of robots in industry from the beginning of robotics. Some examples of applications: spot welding, arc welding, spray painting, forging, foundry, palletizing of relatively high loads (metal blocks, bundles of newspapers, cartons of milk cartons, etc.). In microtechnology, certain very fine and precise tasks were robotized quite early on, because it is very painful for an operator or more frequently for an operator to concentrate on operations requiring the use of a microscope or binocular. *Microrobotics* (see 5.1) for high precision robots handles this type of application; for example: bonding of chips, assembly of quartz, assembly of integrated optics or optical fibers, assembly of micro-gearboxes and micro-motors.

The majority of robots available on the market are of the serial type (see 1.2 and fig. 1 a). Serial robots comprise a single kinematic chain between the fixed base and the mobile end effector, while parallel robots comprise one or several kinematic chains, closed between the base and the mobile plate (see section 1.3 and fig. 1 b).
Most of the established companies (ABB, Fanuc, Kuka, Stäubli, Yaskawa, Omron , etc.) and several SMEs (for example Unitechnologies, Demaurex, Asyril,…), all offer serial robots as well as parallel robots.



a) Poly-articulated serial robot (Stäubli RX) b) Illustration of the kinematics of the DELTA parallel robot with 3 degrees of freedom

Fig. 1 Examples of robots. **a)** a serial robot and **b)** a parallel robot

Electrical and electronic industries account for nearly 35-40% of worldwide deployed robots, same as for the automotive, motorcycle and bicycle industries (Shimano for example). For reasons of hygiene, cost, working at low temperature and large variations in demand during the year, the food industry is increasingly equipped with robotic installations.

Pick and place applications, for packaging and assembly, are ones of the most robotic-demanding industries, implementing a lot of SCARA type robots (see section 1.2.2) and Deltalike robots (section 1.3); the rest is almost completely covered by 6DOF anthropomorphic robots and Cartesian robots; as well as some robots with cylindrical and spherical coordinates.

1.1.3 Characteristics to be specified to select a robot.

The main characteristic of an industrial robot is probably its flexibility to be deployed for different types of applications. However its specifications, performance parameters that allow to compare the selected robot according to the products offered on the market, are very important to define . The main parameters to be considered when evaluating a robot are defined below:

- ***Degrees of freedom,*** necessary for the application considered. In some cases, a robot is chosen with greater degrees of freedom than required for the following reasons: higher workspace, ease of installation, availability on the market,…..

- ***The maximal load*** that can to be transported plus that of the grasper or tool used.

- ***The working volume*** *(workspace)* (fig. 2), which results from the geometry of the robot and the physical constraints on the joints, represents the volume reached by the robot's wrist; the shape of this volume plays an important role regarding the targeted applications.

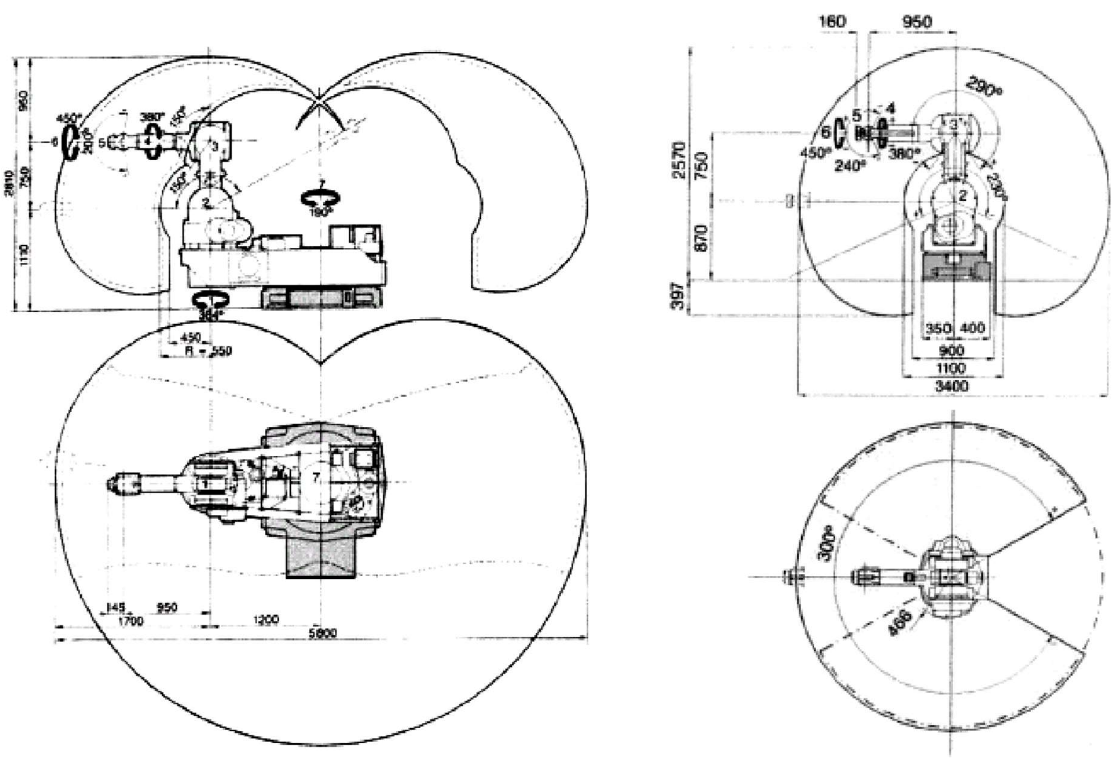


Fig. 2 Working volume of Comau 3 7.16 P and 3 6.25 R angular robots; they represent the space reachable by the point P center of the wrist during the movement of the axes of the wearer.

- ***Maximum acceleration*** is sometimes a useful indication, especially on short displacements where maximum speed is not reached.

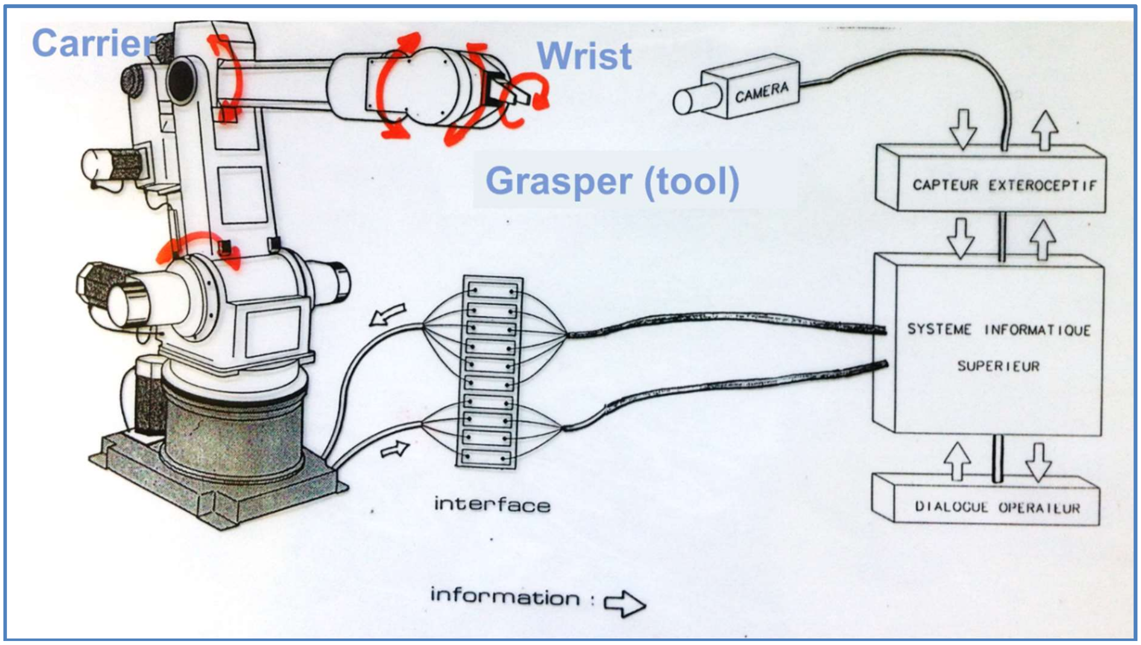
- ***The cadence*** is measured as the number of outward and return movements per unit of time between two end points. This value can be supplemented by more precise information concerning certain speeds and accelerations. Often, the maximum speed is given for the arm endpoint; however, the speed of each degree of freedom is of the same importance.

- ***The precision***, generally specified for the robot's end effector. However, the *repeatability* (relative precision) is often more important than the absolute precision. When good absolute precision is required, it is necessary to calibrate the robot to operate an error correction using a precise measurement approach, similar as we do when calibrating measurement devices and machines.

- ***The environment*** in which the robot operates is of paramount importance. It is essential to specify certain particular conditions of use such as the presence of abrasive dust, humidity, vibrations, etc. If the robot can be disturbed by the environment in which it works, it should not be forgotten that the reverse can occur and that the robot can pollute the workspace intended for operations requiring great cleanliness. On the other hand, the nature of the product handled can exclude certain technological choices (for instance, use of brushed motors in an explosive environment).

An industrial robot is required to grab and move objects (parts or tools). It therefore assumes a mechanical structure. The mastery of an object in Cartesian space implies its location and orientation. To position a point of the object in space, you need 3 degrees of freedom. The point corresponding to the object being fixed, if one wishes to orient the object in any way, it is necessary to have in addition three new degrees of freedom.

For serial type robots and for some hybrids, the first three degrees of freedom define what is called the ***"carrier"***, while the next three constitute the *wrist.* At least 6 axes are therefore required to completey define the position of an object in space. Note that if the human arm has 7 (3 at the shoulder, 2 at the elbow and 2 at the wrist), this is a redundancy insofar as the elbow allows us to bypass the obstacles.


Carrier
Wrist
Grasper (tool)
CAMERA
CAPTEUR EXTEROCEPTIF
SYSTEME INFORMATIQUE SUPERIEUR
DIALOGUE OPERATEUR
interface
information :

1.2 Robots: geometries and applications

1.2.1 Introduction.

Without going too much into detail, we can classify fixed robots according to 2 main groups: Serial and parallel robots.

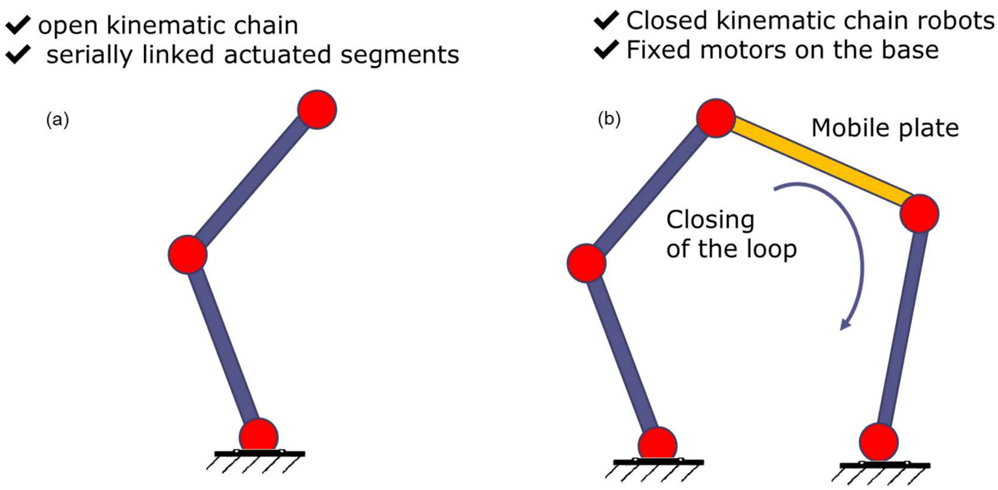


Fig. 1 Illustration of serial and parallel robots.

- **serial robots** are characterized by the fact that a single kinematic chain connects the fixed base to the mobile terminal organ (1.1 / fig. 1 a)
- **parallel robots** are characterized by the fact that the kinematic chain(s), ensuring the connection between the fixed base and the mobile plate are all closed to the frame(section 1.1 / fig. 1 b). Some robots combine serial and parallel kinematics; in this case we talk about hybrid kinematics (fig. 1c opposite). Considering the characteristics of these kinematics, for simplification we will classify the hybrid robots with the parallel robots which are studied in section 1.3.

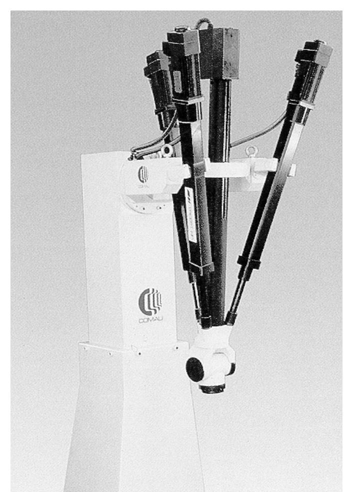


Fig. 1c- The Tricept hybrid robot industrialized by the companies NEOS, COMAU and Siemens.

Notice:
Apart from the Pollard robot (see section 1.3), most of the robots designed before the 1980s were of the serial type; the main reasons are as follows:
- similarity to the human arm, overhead cranes or certain construction machines,
- ease of calculation/implementation of coordinate transformations that allow the position of the robot axes to be determined according to the tool position or the reverse.

1.2.2 Serial geometries.

Remember that for serial robots, the first three degrees of freedom define what is called the *"carrier",* while the following three constitute the *wrist.*

Regarding the carrier, the axes (called main axes) can be in translation or in rotation. If the order is not considered, there are 4 main configurations: TTT, RTT, RRT, RRR.

The TTT configuration (3 translations) is well suited to our Cartesian representation of space (fig. 2). It corresponds to the designer's world. The positions of the axes relative to each other do not vary during a movement (one speaks of constant differential geometry in space). The calculation of trajectories and the control (no variation of inertia) are thereby simplified. This variant is particularly interesting for precision assembly or for large manipulations; its mechanics can be quite delicate (fig. 3).

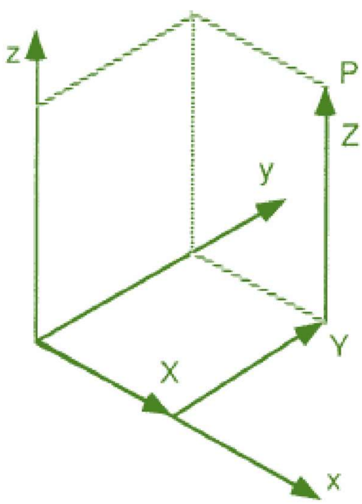


Fig.2 Cartesian system (TTT)

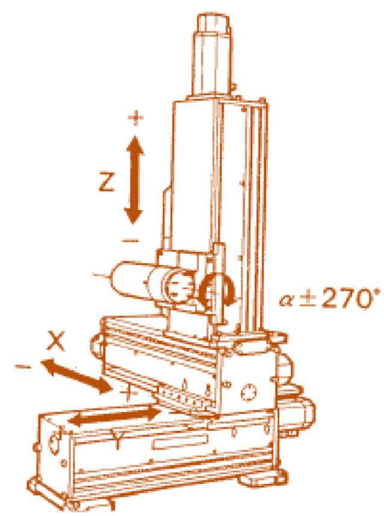


Fig.3 Cartesian robot

The RTT configuration (with cylindrical coordinates) is the combination of a rotation and two translations (fig.4). The construction complications (prismatic guides) and the variable resolution according to the extension radius increasingly reduce its use. It leads to a variant, SCARA, which has been successful in assembly (fig. 8). The SCARA robot replaces it advantageously for medium precision applications and a Cartesian robot for high precision.

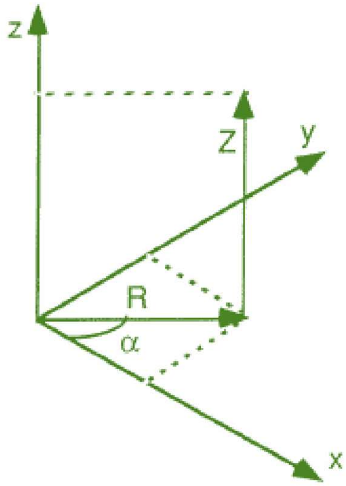


Fig.4 Cylindrical system (RTT)

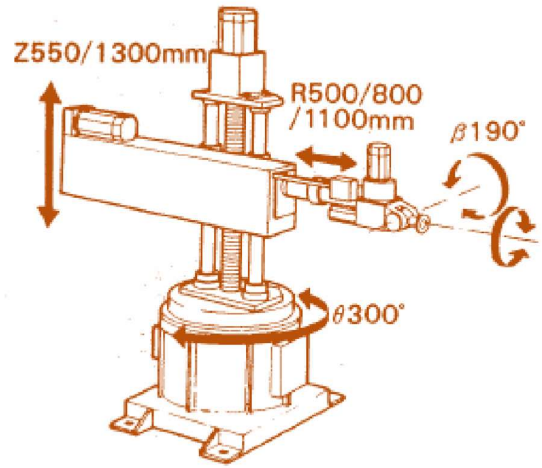


Fig.5 Cylindrical robot

In the RRT configuration (fig.6) there is a wide variety of combinations. The oldest of these is the spherical coordinate robot (fig. 7).

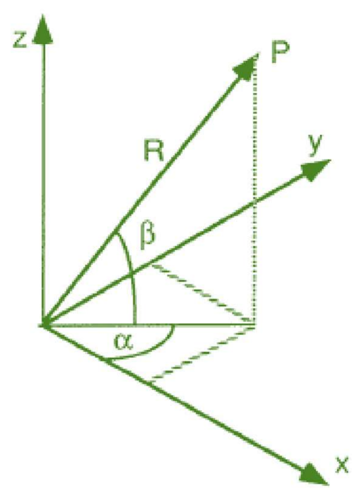


Fig.6 Spherical system (RRT)

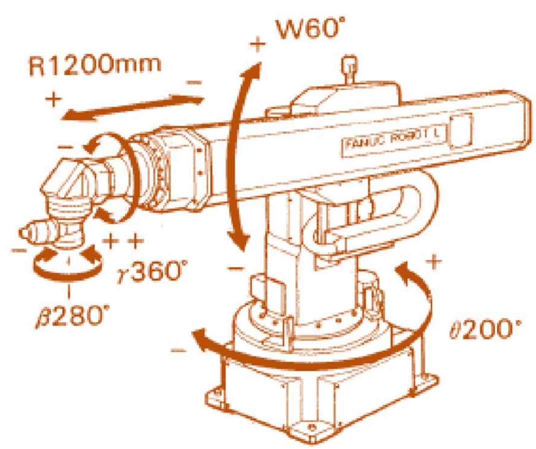
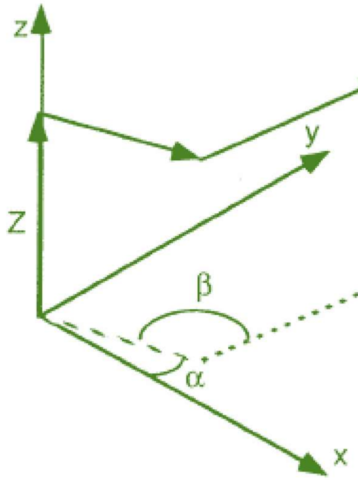


Fig.7 Robot with spherical configuration

Other industrial robots available on the market have translation on a vertical axis (fig.8 and fig.9). This RRT configuration, called *SCARA (Selective Compliance Assembly Robot Arm),* was developed in Japan in the 1980s. It is the most common configuration in the field of assembly, whereas the "turret" type (fig.7) sees its use greatly reduced.



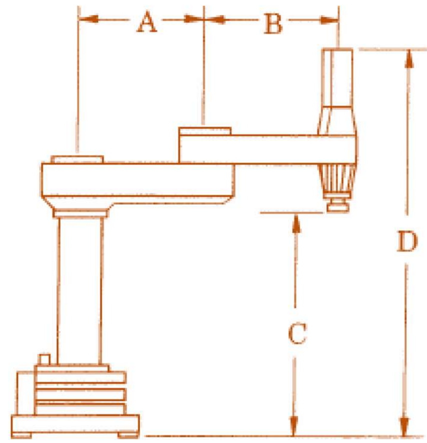


Fig. 8 SCARA configuration (RRT)

The angular coordinate RRR system also called polyarticulated robot (fig. 10 and 11) is certainly the simplest from a mechanical point of view (formed only by rotary axes), but it requires sophisticated control, for example to travel in a straight line. Great attention must also be paid to the design of the rotary axes which must be very rigid to ensure a high natural frequency, necessary for good repeatability. Some manufacturers (Stäubli for example) have made sure that all the wiring goes inside the robot, through the joints which, as a result, become more complicated.

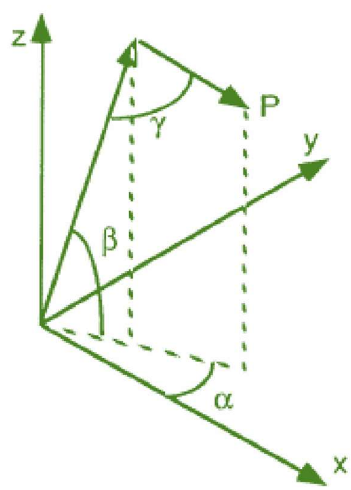


Fig.10 angular configuration (RRR)

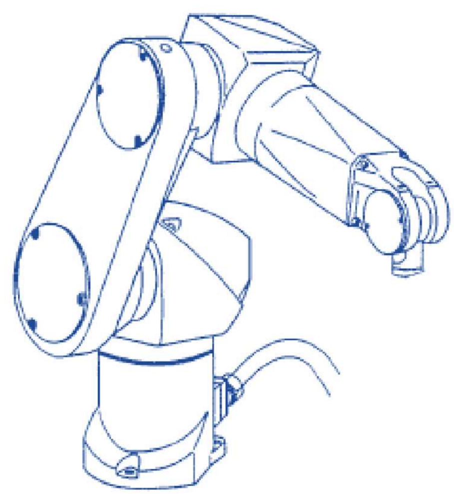


Fig. 11 Polyarticulated angular robot

The three axes of the carrier are not necessarily essential, if you are working according to a plane, for example. But the economy in axes is most often done at the wrist. By considering certain symmetries (a cylindrical body only requires 5 coordinates for the description of its position and a sphere only requires 3) or by limiting ourselves to a family of simple operations, we can reduce the number of axes of the industrial robot to 5, 4 or even 3.

1.2.3 Applications.

In the early days of robotics, a dream was to design the universal robot. The range of the various applications mentioned under 1.1.2 shows that this dream is a utopia and that this path would inevitably lead to far too expensive, complicated and unreliable solutions.

Originally focused primarily on the automobile (in this industrial branch, operations are now designed from the outset for robots), the highest percentage of recent installations is now found in the field of electrical and electronic construction (assembly). Then, there are newly won sectors such as mechanics, the processing of plastics and metals (polishing, deburring, etc.). Industrial robots have made their appearance in new fields ranging from food processing to clothing, including micro-manipulation; we can say without any doubt that no sector will be "spared" by robotization.

Most of the applications mentioned under 1.1.2 in well-established fields are carried out with serial robots. Parallel and hybrid robots, which are often more complicated to control, will have outlets in sectors requiring very high speeds (in particular feed) or very high precision and stiffness (high precision assembly and probably machining).

<u>Some examples of serial robot applications:</u>

- polishing and deburring of foundry parts, fittings, handles (door, bicycle brake, etc.); watch cases and strap parts, cutlery, etc.; these operations are generally carried out by angular robots with 5 or 6 degrees of freedom;

- assembly of microsystems such as accelerometers for air bags, micro-motors, chip - printed circuit or grid connection, assembly of watches, etc.; these operations are usually performed by Cartesian robots with 4 degrees of freedom (x, y, z, θz); in general, Cartesian robots are well suited for precision work (constant resolution throughout the entire work volume and no variation in inertia);

- assembly of video recorders, CD players, telephone devices, bicycle hubs, printers, ... carried out by SCARA robots; for the use of such robots with 4 degrees of freedom to be possible, it is essential to design the product to be assembled accordingly and to define a single vertical insertion direction;

- spot welding of automobile bodies; these applications are practically always carried out by angular robots with 6 degrees of freedom; sometimes, to save space in the chains, the first 2 axes are parallel and horizontal and the 3rd is perpendicular to them; they must be able to handle a fairly high load, given the mass of the welding clamp (from 50 to 150 kg); this robot work guarantees better quality through greater regularity of the operation; moreover, this type of activity is extremely difficult for the operators. At somewhat different scales, these operations are found in household appliances;

- palletizing newspaper packages for shipping; these applications are generally carried out by SCARA robots which in this case have the particularity of having their translation axis positioned in the first, or even in the second position; this arrangement makes it possible to obtain movements of large vertical amplitude without excessively penalizing inertia; this kind of operation does not require great precision;

- turret-type RRT robots have been used in the automobile industry and also for handling in food processing; the first applications (Robots Unimation) used hydraulic energy; given that in general 4 degrees of freedom (x, y, z, □z) are sufficient for these handling operations, the choice of “turret” robots is not judicious; in the particular case, it is the load that can be handled which led to the choice of this hydraulic robot;

- an ABB robot of the so-called pendular RRT type was presented in 1984 for assembly applications; its complexity and competition from SCARA have seriously limited its distribution.

When dealing with robotic systems, one must be aware that the robot is a particular element and that the "accessories" that are classified in the periphery play an important role both in terms of the functions required and on the cost plan (the total cost of an installation may vary, depending on the case, between 2 and 10 times the price of the robot); don't forget the costs of design, programming, tune-up, assembly, wiring, training and warranty. For the troubleshooting of installations, remote diagnostics will make it possible to limit maintenance and repair costs, while virtual reality and artificial intelligence should lead to greater ease of programming.

The choice of robot is linked to the choice of the periphery and vice versa; a robot with an additional axis could, in certain cases, seriously simplify the periphery or the choice of a robot capable of a greater volume of work could reduce the importance of the transfer systems.

As with the design of any system or product, it is essential to have a comprehensive approach. In addition, consultation between product designers and designers of automation systems from the start of the project allows for interesting simplifications and substantial advantages in terms of cost and reliability.

1.3 Parallel and hybrid robots: mobility, types, applications

1.3.1 Characteristics

Most industrial robots built to date are of the serial type, that is to say that their mobile structure is an open chain formed of a succession of segments linked together by bonds with a degree of freedom; each articulation is controlled by an actuator located at the location of the articulation or on one of the preceding segments; in the latter case, a mechanism ensures the transmission between the actuator and the joint in question.

These robots have the advantage of having a large volume of work and of being relatively simple in terms of calculations related to their control. Their main faults are summarized below:

- high inertia due to the masses distributed over the entire kinematic chain (actuators, transmission components);
- lack of rigidity by placing elastic elements in series;
- fatigue and wear of the power connections ensuring the supply of the actuators (cables, flexible pipes);
- fatigue and wear of the connections ensuring the flow of information between the sensors and the control; in terms of safety, this point is essential since a transmission error can have disastrous consequences on the robot's movements.

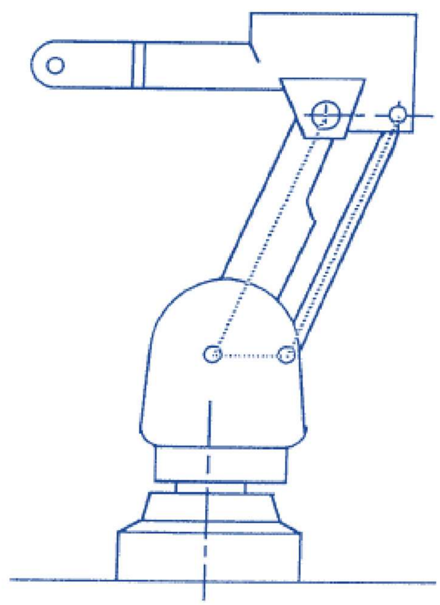


Fig. 1
Example of a robot with a flat polygonal loop: ABB IRB 2'400.

To reduce the moving masses, some robots on the market include one or more closed loops forming a plane articulated polygon; FIG. 1 represents a typical case. All the articulations of a loop are not independent, it is possible to choose in the loop the one that you want to motorize; this makes it possible to bring the motors back to the base, and therefore to reduce the moving masses.

Certain authors [FICHTER 86] [DOMBRE 88] consider that a parallel robot is characterized by the fact that the different mobile bodies form several kinematic chains which all directly connect the base to the terminal organ; some authors speak in this case of fully parallel robots. The robot type corresponding to this definition is the platform STEWART [STEWART 65] in the variant shown in Figure 2.

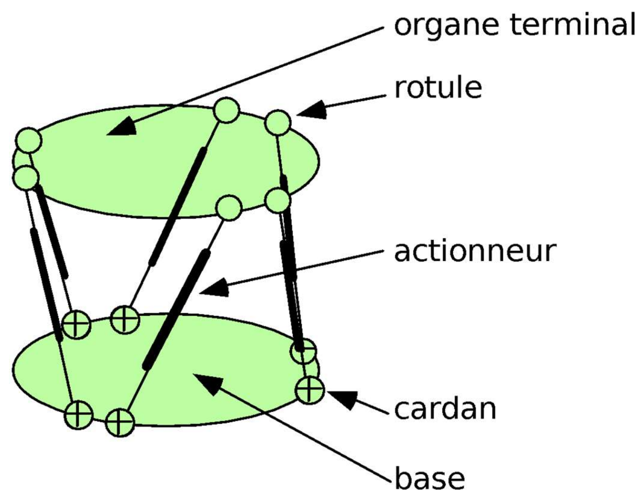
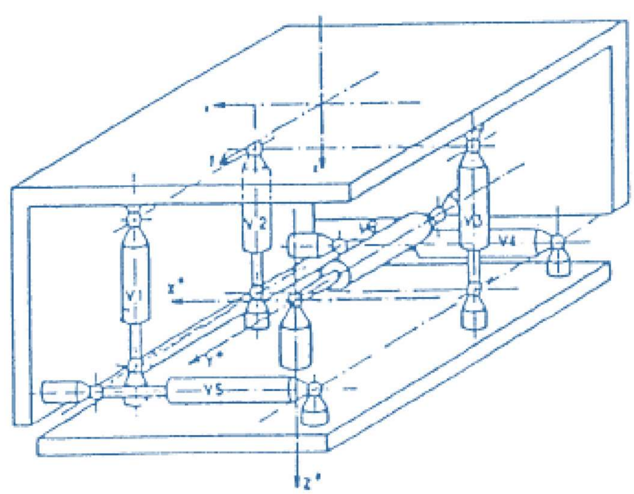


Fig.2
platform **Stewart-Gough**

This mobile platform with 6 degrees of freedom has been developed to perform the function of a flight simulator; a mechanism of the same type has been used previously for the tire test [GOUGH 62]; several experimental robots have been built on the basis of this concept; in particular the robot "GadFly" GEC [DWOLATZKY 88] and the wrist "SPACE 1" (system poignet avec contrôle des efforts; wrist system with effort control) [REBOULET 88]. Artigue offers a mobile structure on an identical principle, but with a different spatial arrangement [ARTIGUE 84] (see Fig.3). The serialization of several STEWART platforms makes it possible to create a redundant robot in the shape of a horn [KWASNY 87].

Fig. 3
Parallel manipulator with 6 degrees of freedom used as an active wrist for mounting a suspension triangle; stroke of each hydraulic actuator: ± 5 mm.



1.3.2 Definition

We will refer to a **parallel robot** as a robot comprising at least 2 bodies connected by more than one kinematic chain, excluding robots which differ from robots with a series structure only in that they comprise one or more loops each forming a plane polygon.

A kinematic chain is a set of segments arranged alternately with links.

1.3.3 Mobility of a kinematic structure

Definitions

Degree of freedom (DOF):
the number of degrees of freedom of a system is the number of independent variables necessary for the total description of the kinematic structure.

Movable kinematic structure:
a kinematic structure is said to be movable when the number of DOF is greater than or equal to 1; let MO be the mobility of the structure considered; MO is equal to the number of DOF of the kinematic structure.

Mobility of a kinematic structure.
Consider a kinematic structure formed of n solid parts; the number of degrees of freedom of this set before any assembly with a view to creating the kinematic structure considered is obviously MO = 6n.

Each connection between 2 parts decreases the mobility of the assembly by a value corresponding to the number of generalized forces (NFG=nombre de forces généralisées) present in the connection considered; with k connections MO takes the following value:

$$MO = 6n - \sum_{i=1}^{k} NFG_i$$

The number of generalized forces (NFGi) intervening in a connection is the complement to 6 of the number of degrees of freedom (MOi), that is to say: NFGi = 6 - MOi hence:

$$MO = 6n - 6k + \sum_{i=1}^{k} MO_i$$

$$MO = 6(n - k) + \sum_{i=1}^{k} MO_i$$

Grübler's formula

Let us consider that one of the parts is fixed (its 6 DOFs are blocked) and take it as a reference (frame for a robot); MO thus takes the following value:

$$MO = 6(n - k - 1) + \sum_{i=1}^{k} MO_i$$

Creating a loop in a given structure consists in adding a link without adding a part.

Adding a part necessarily leads to adding an additional connection.

These 2 observations lead to the following relation for a system of n parts, k connections and bo loops (fr. boucle):

n = k - bo + 1

With this expression, the mobility MO takes the following form:

$$MO = \sum_{i=1}^{k} MO_i - 6bc$$

The mobility of a kinematic structure of which one of the elements is fixed on a fixed basis is therefore equal to the sum of the mobilities of all the links minus 6 times the number of loops.

This expression can be used to systematically search for new robot geometries; it is also very interesting for determining the mobility of a given kinematic system; hyperguidances (over-constraints) or excessive mobility can easily be highlighted by this formulation.

1.3.4 Parallel robots, by groups and by examples

Patents and literature have enabled us to identify a certain number of kinematic structures which have led or can lead to concepts of parallel robots conforming to the definition given above. We have put them together in 4 groups.

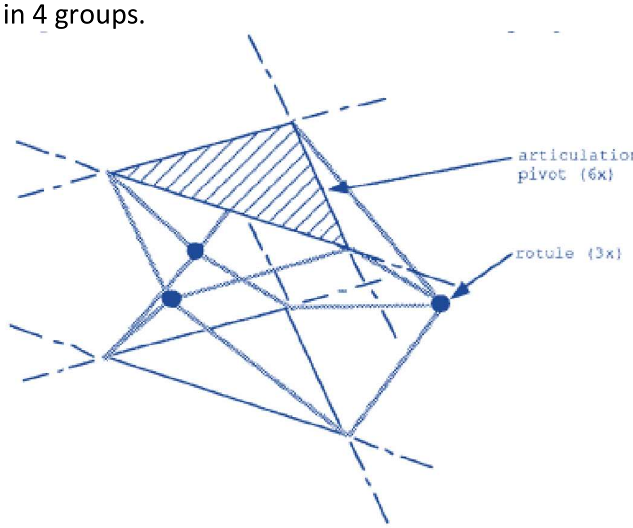


Fig.4
Principle of the poly-articulated mechanism invented by Lambert [LAMBERT 83].

The **first group** concerns robot modules comprising 3 degrees of freedom; these modules [LAMBERT 83] (see Fig.4) [KRUEGER 85] [KIMURA 83] will often be placed in series with each other to form proboscis-shaped robots. The basic principle is the same for these 3 inventions; it is a module comprising 2 plates connected by 3 (in certain cases 4) kinematic chains each provided with an actuator; The combination of the actuators' movement ensures the displacement of one plate in relation to the other according to a translation and 2 rotations. Based on a similar principle, Lande and David [LANDE 78] have proposed a joint allowing 3 rotations; this mechanism can be used as a robot wrist.

The second group concerns robots with 5 or 6 degrees of freedom comprising a carrier formed by 2 or 3 parallel kinematic chains [POLLARD 38] (see Fig. 5) [SHUM 81] [ASEA 85] (see Fig. 6) [DWOLATZKY 88]. These structures work according to the same principle: 3 arms work together towards the same point; the movement of these arms causes the displacement of this point; the orientations of the wrist are ensured either by one of the arms (Pollard, Asea, Shum), or by a support passing through a fixed point (Dwolatzki)

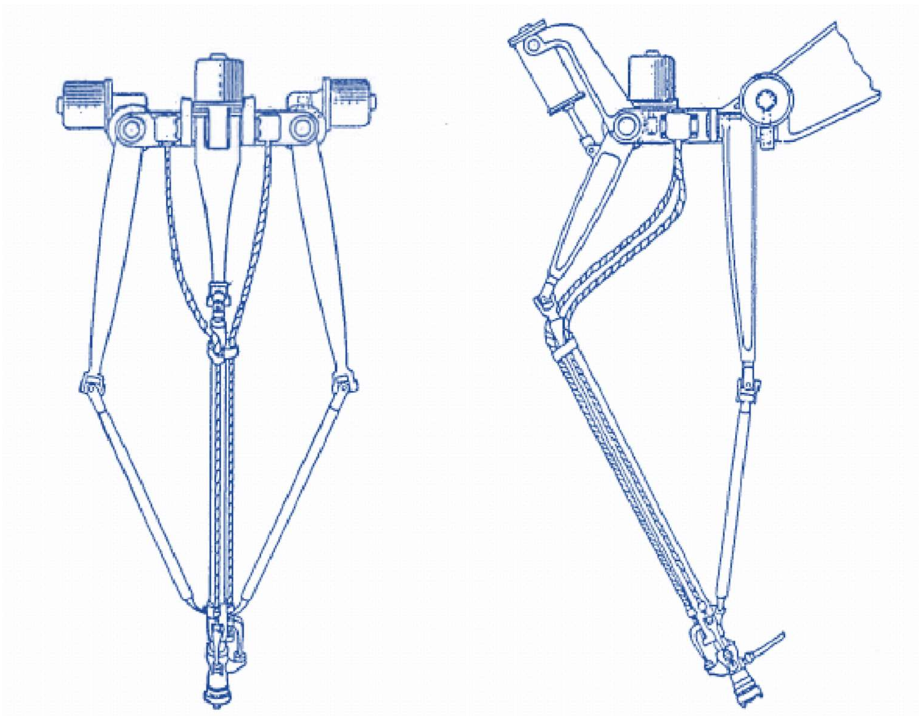


Fig.5 The "POLLARD" robot designed to paint cars [POLLARD 38]; in 1938, the electronic and computer means did not allow such a robot to be controlled.

We can also include in this group the miniature robot named "Colibri" [BURCKHARDT 88] [LIU 90] developed at the Institute of Microtechnology of the EPFL; a variant of this robot with 6 degrees of freedom is shown in figure 7.

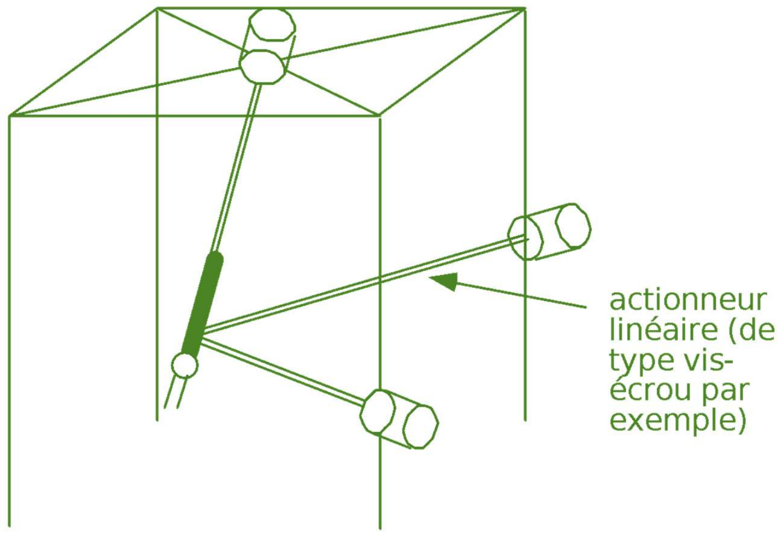


Fig.6 Development stage of the Asea IRB 1'000 pendular robot [ASEA 85]; the actuators are linked to the frame by gimbals and the junction of the 2 lower modules with the wrist support is ensured by 2 ball joints.

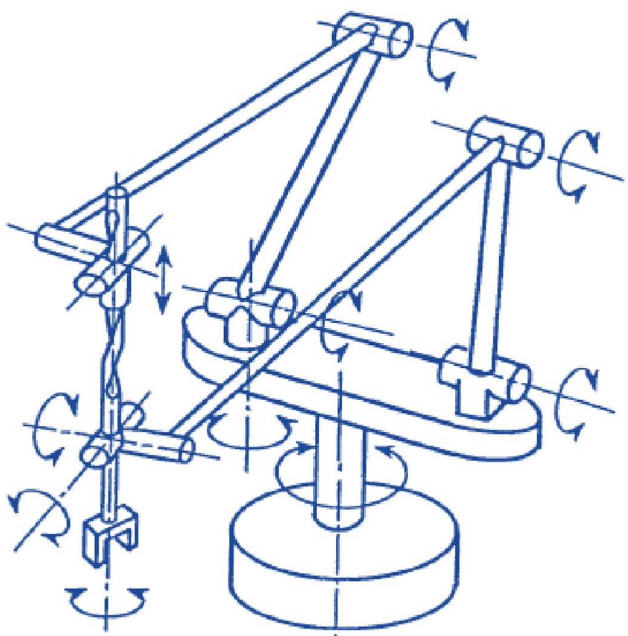


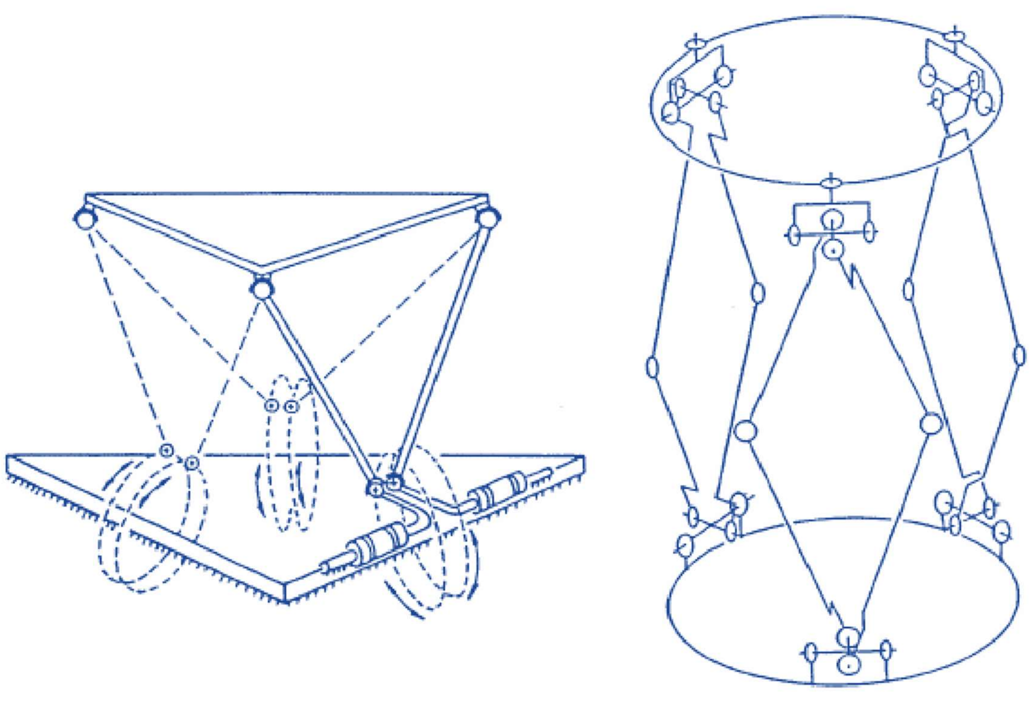
Fig.7 Parallel robot "Colibri" with 6 degrees of freedom, that implements a pivoting platform on which are mounted 2 articulated arms with 2 and 3 degrees of freedom respectively; the loop is closed by the screw system linking the ends of the 2 arms.

The **third group** concerns structures with 6 degrees of freedom implemented using kinematic chains directly connecting the terminal organ to the base [STEWART 65] [HUNT 83] [INOUE 83] [BEHI 88].

The structure mentioned by Hunt (fig.8a) is similar to a Stewart platform in which the 6 kinematic chains fitted with linear actuators are replaced by kinematic chains each comprising a rotating actuator fixed to the frame; the fixed position of the actuators is an interesting advantage in terms of reliability; a compilation of this structure with that of the DELTA robot led Pierrot [PIERROT 91] to propose a "HEXA" robot (fig.9).

The positioning mechanism proposed by Inoué (fig.8b) is related to Hunt's structure; the difference, which is important from a construction point of view, lies in the arrangement of the motors.

The Behi manipulator (Fig.10a), formed of 3 kinematic chains each comprising 2 linear actuators, is very close to the first principle highlighted by Stewart during the development of his flight simulator [STEWART 65] (Fig.10b).



a) b)

Fig.8 a) Mobile structure with 6 degrees of freedom mentioned by Hunt [HUNT 83].
b) Positioning mechanism proposed by Inoué [INOUE 83].

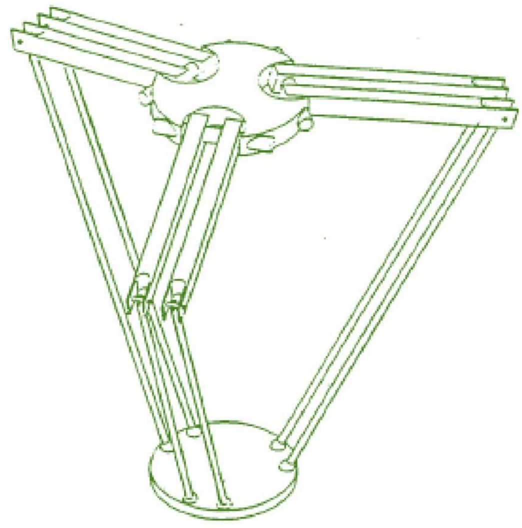


Fig.9 Robot HEXA proposed by Pierrot LIRMM (Laboratory for Computer Science, robotics and microelectronics Montpellier

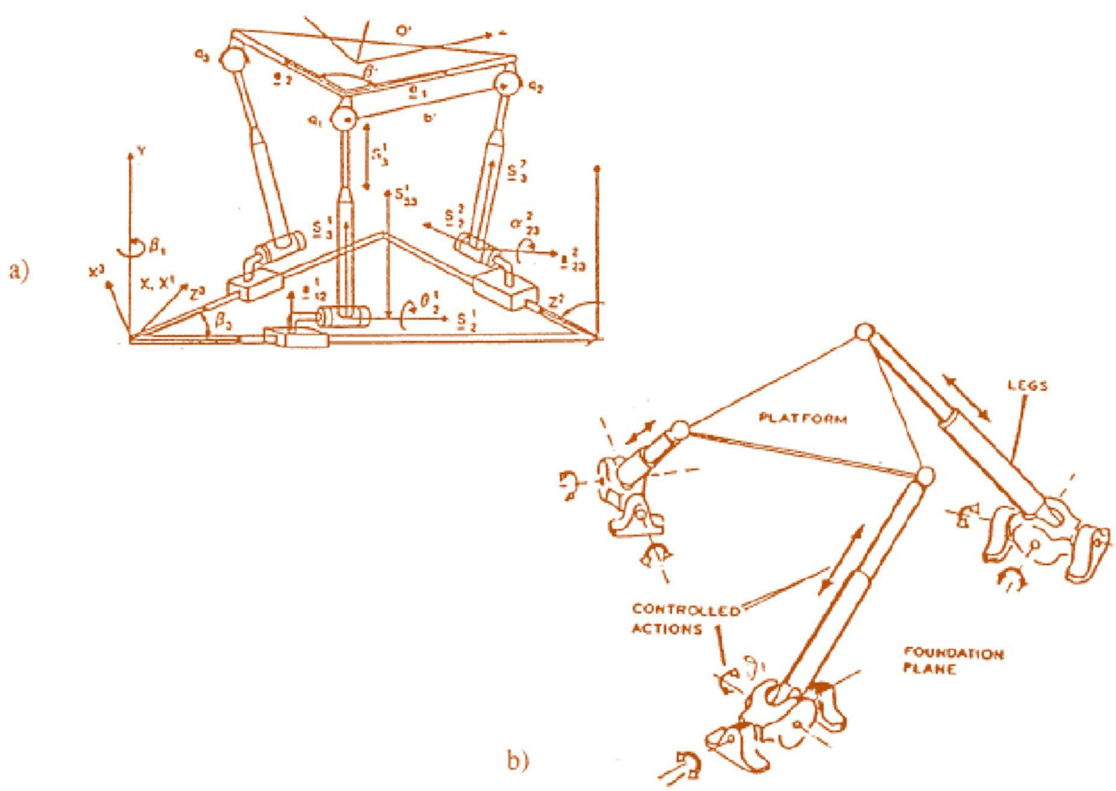


Fig. 10 a) manipulator Behi [BEHI 88]
b) Flight simulator Stewart [STEWART 65]

The **fourth group** includes parallel structures making it possible to move the carrying element of the wrist while keeping its 3 orientations constant.

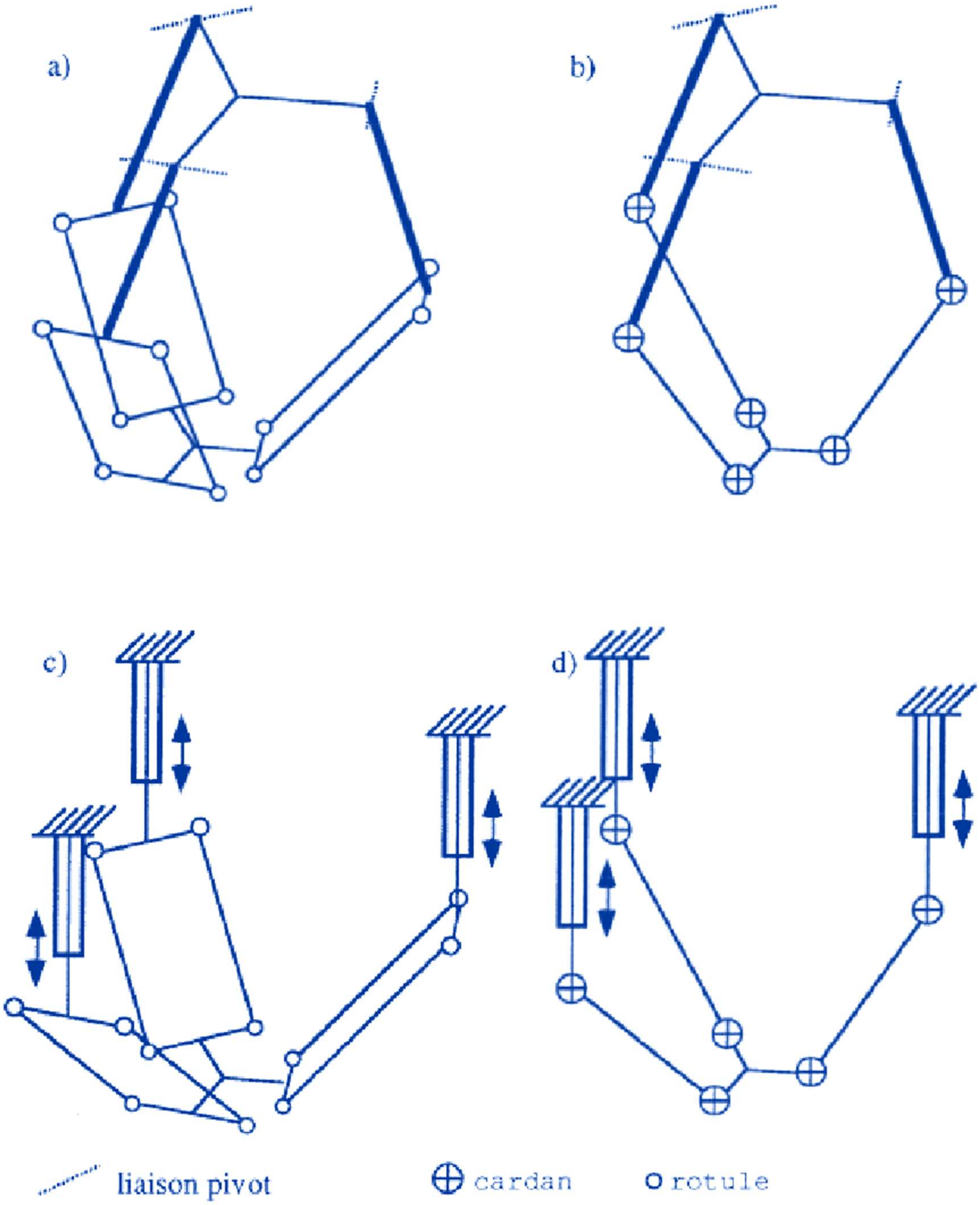
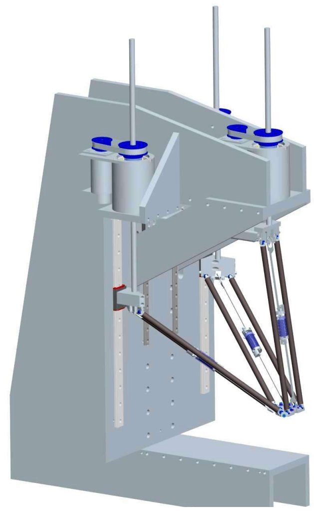


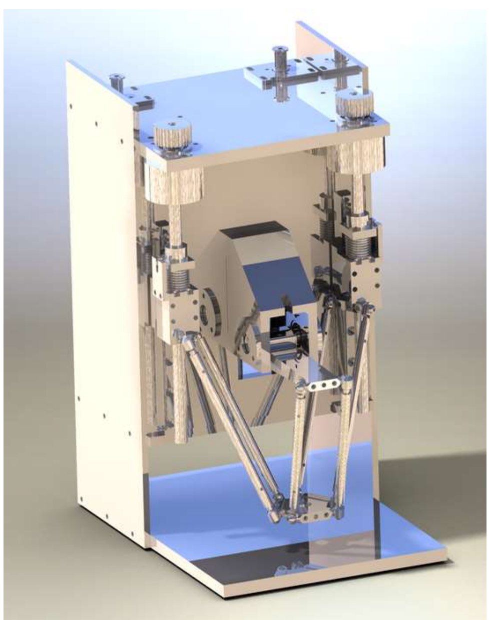
Fig.11 The 4 basic structures of the DELTA robot; cases a) and b) have rotating actuators (pivot linkage), cases c) and d) have linear actuators (prismatic linkage); in cases a) and c), the links between the nacelle and the arms are ensured by 3 pairs of parallel bars and 12 ball joints; for cases b) and d), these links are ensured by 3 bars and 6 gimbals.

In c) and d), the rectilinear guides can be arranged differently, for example on the edges of a pyramid; this gives the DELTA Keops. A combination with a rotating arm and 2 rectilinear guides gives the DELTA Ibis robot.

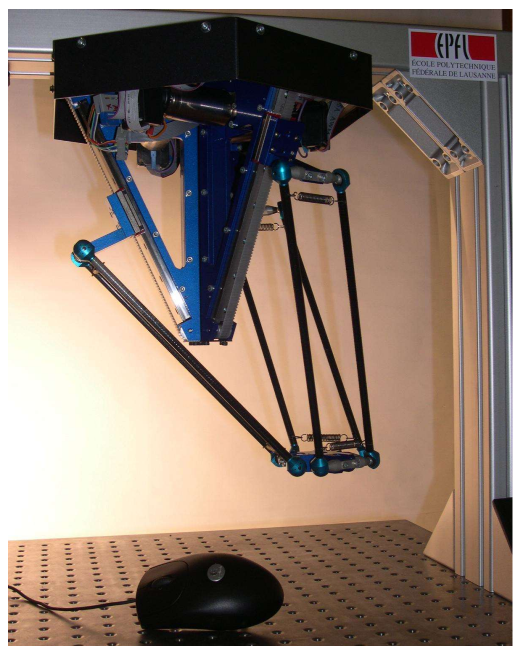
a) Vertical linear delta with 3 columns



b) Vertical linear delta with offset frame



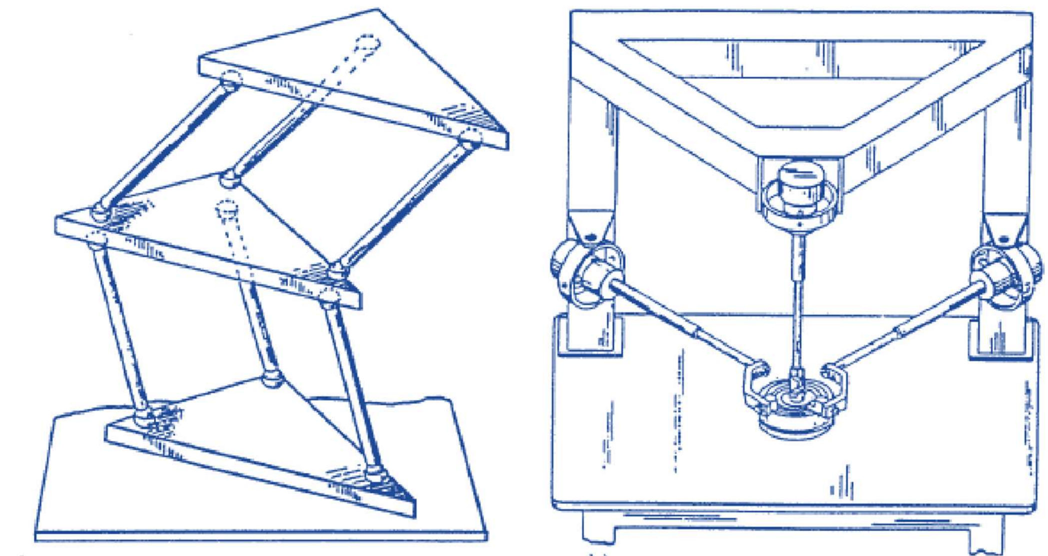
c) Delta Ibis



d) Delta Keops

Fig. 11 ' Four versions of Delta leading to specific characteristics for pin insertion (a), for precision work in interaction with a rotary table (b), for watchmaking assemblies and/or integration in a micro-factory (c) and for handling on a large surface (d). In this last case, an additional parameter (the angle between the straight guide and the vertical) can be judiciously used to favor the vertical force or the working surface.

The solution presented in figure 12a of Flory [FLORY 67] is a sort of parallelogram of space if the platforms cannot rotate around a vertical axis; figure 12b shows an original solution of Simunovic [SIMUNOVIC 84].



a) b)

Fig. 12 a) structure **Flory mobile**; to ensure that the orientation of the mobile platforms is maintained, it is necessary for each stage to include a bar fitted with a gimbal at each end; the position of the central platform represents an additional degree of freedom.

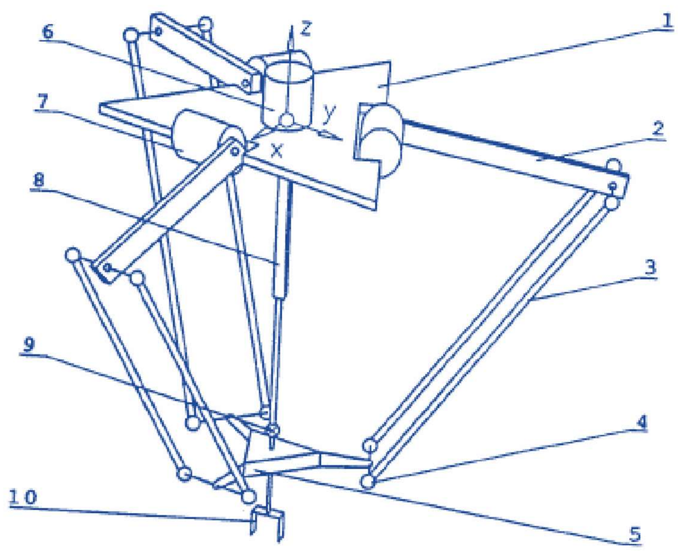
b) manipulator ***Simunovic***.

1.3.5 Description of the DELTA concept and definition of its constituents.

Figure 13 will serve as a support for the description of the DELTA4 robot and its operation.

This robot with 4 degrees of freedom is essentially a **parallelogram of space** [CLAVEL 85]; it mainly consists of a "**fixed base**" (1) integral with a support frame of the setup (not illustrated) and a mobile plate (5); the name given to this last part is "**nacelle**". The link between the fixed base (1) and the nacelle (5) is ensured by 3 main kinematic chains; each of them is composed of an "**arm**" (2) mounted in a pivot articulation on the fixed base and of 2 "**parallel bars**" (3) each provided with an articulation (4) at each end; the assembly formed by 2 parallel bars and the 2 joint elements to the arm and to the nacelle will be called "**parallelogram**". Each arm (2) is driven by an "**actuator**" (7) which is often a motor-gearbox-sensor assembly. The "**gripper**" (10) will be pivotally mounted on the nacelle and driven by a motor (6) via the "**telescopic shaft**" (8) thanks to a cardan-type articulation (9) at each of its ends; this solution may, depending on the case, be replaced by other variants such as:

- a small motor-reducer mounted on the nacelle,
- a rotating flexible cable driven by a motor fixed on the frame; a reducer mounted on the nacelle (worm gear, for example) transmits the rotation of the cable to the gripper,



1. Base (fixed to the frame)
2. Arm
3. Forearm
4. Ball joint (spherical joint)
5. Platform (mobile plate)
6. Motor for the 4th degree of freedom
7. Arm motor
8. Telescopic linkage
9. Cardan (universal joint)
10. Gripper

Fig. 13 Illustration of the DELTA4 robot.

1.3.6 Mobility of the DELTA robot with 6 parallel bars.

A simple examination of FIG. 13 shows that this mobile structure without the 4th axis is composed of 5 loops; the account of the mobility of the joints gives the following result:

- 3 joints at MO = 1 (arm motor)
- 12 ball joints (MO = 3)

The mobility of the whole is therefore: 12x3 + 3x1 - 5x6 = 9 *whereas we expected* ***3.***

Using the formula of Grübler.

n = 11
k = 15
MO = 6 (11 - 15 - 1) + 12x3 + 3x1 = -30 + 39 = 9.

This result is due to the fact that with a ball joint at each end, **each parallel bar can rotate around its longitudinal axis**, which gives 6 additional mobilities. This undesired movement is constructively limited by the effect of the springs for maintaining the parallel bars in contact with the spherical joints. A gimbal *at one end* of each of the six bars would lead to the expected mobility of 3.

1.3.7 Singularities of the DELTA robot

For serial manipulators, the singular positions result from the loss of degrees of freedom while in a parallel manipulator, they result of the gain of one or more degrees of freedom [FICHTER 86]. Most of the time, passing through a singularity causes over constraint, to then inducing the loss of control of one or more degrees of freedom.

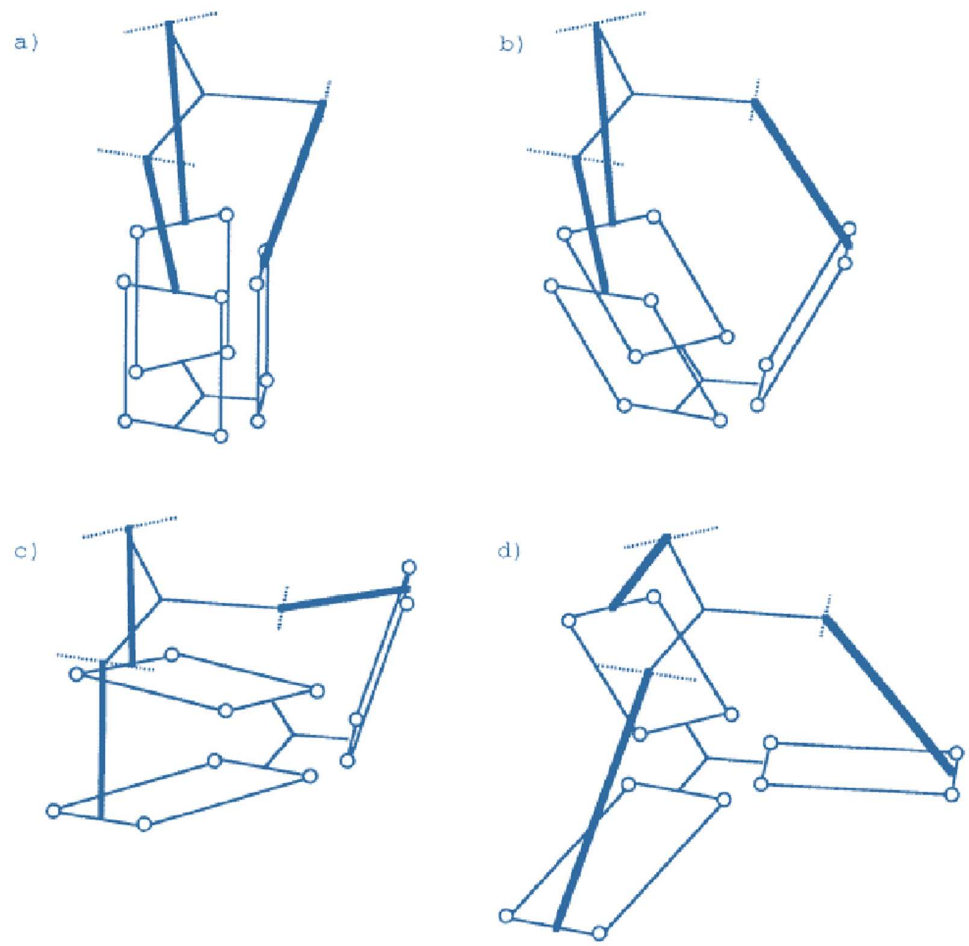


Fig. 14 Representation of the 4 types of singularities of the DELTA robot.

The singularities of the Delta, as implemented using the 6 parallel bars and spherical joints, were determined from the analysis of the relative position of the 6 bars attached to the "nacelle". We consider that each bar transmits only a force whose line of action is merged with the line linking the 2 spherical joints at its ends.

The position of the nacelle is fully defined and its stability ensured if the following conditions are met

- the lines of action of the forces (the parallel bars in this case) are located in 3 different non-parallel planes;
- no more than 2 lines of action are parallel.

These conditions highlight the 4 types of singularities shown in figure 14

a) when the 6 bars (3 pairs) are all parallel to the same direction (*fig.14.a*), the nacelle moves on a portion of a sphere and rotate around the vertical axis, while it should not rotate.

b) if 4 bars (2 pairs) are parallel (fig.14.b), the nacellle keeps one degree of freedom, i.e.: movement on an arc of a circle;

c) if 4 bars (2 pairs) are in the same plane or in 2 parallel planes (fig.14.c), the nacelle moves in one degree of freedom, i.e.: rotates around the axis linking the two joints located between the nacelle and the two other bars.

d) when the 3 parallelograms are located in 3 parallel or merged planes (fig.14.d), the nacelle has 3 degrees of freedom, i.e.: 2 tilts (rotations) around the axes belonging to the plane of the nacelle (horizontal axes), while it should not rotate, and a translation of small amplitude perpendicular to the same plane

Appendix

1.4 A use case study of the parallel structure of the DELTA robot.

The goal of this paragraph is to highlight, using the example of the angular variant of the robot Delta, the study of a parallel robot.

As already mentioned in the previous section, the key element of the DELTA robot are the 3 **parallelograms of the space** formed by 3 main kinematic chains linking the fixed base (frame) to the nacelle (mobile plate); the kinematics necessary for the rotation of the gripper will not be discussed here.

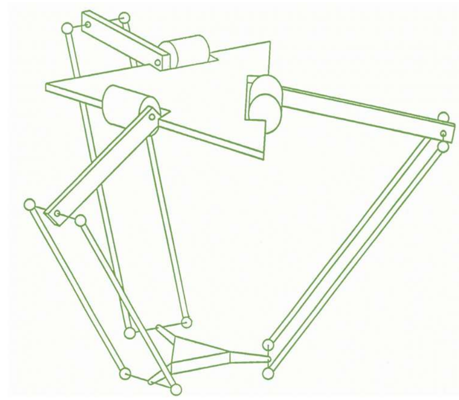


Fig. 15. Illustration of the angular variant of the DELTA robot

In this section, we present the different **geometrical parameters** of this parallel structure. We will then point out its **limit displacements**; that help to derivate the workspace of this robot. The two **geometrical models, direct** and **inverse,** which relate the mobile plate coordinates with the angles of the joint arms.

Simplifying assumptions.

During all the following geometric and kinematic studies, the joints will be considered ideal (without play, without elasticity and without friction) and the segments will be assumed geometrically exact and perfectly rigid. This last hypothesis was confirmed a posteriori by determining the natural frequencies of the various prototypes realized at EPFL (measured values: 120 to 250 Hz).

Parametric lengths.

We consider the ternary symmetry configuration formed of 3 identical kinematic chains arranged according to a period of 120 ° (see Fig. 15). The parametric lengths and characteristic angles of this structure are shown in figure 16.

RA: distance between the center of the fixed base and the axis of rotation of the arm;

RB: distance between the center of the nacelle and the side of the parallelogram integral with the nacelle;

R : difference between the lengths RA and RB; this distance defines the position of point Ai; the latter is obtained by an amplitude translation RB of the kinematic chain i which brings the point Bi to the center of the nacelle, the point Oi to Ai and the point Ci to Ci '; the axis of rotation of the thus translated arm is named ai.

LA: arm length;

LB: length of a parallel bar;

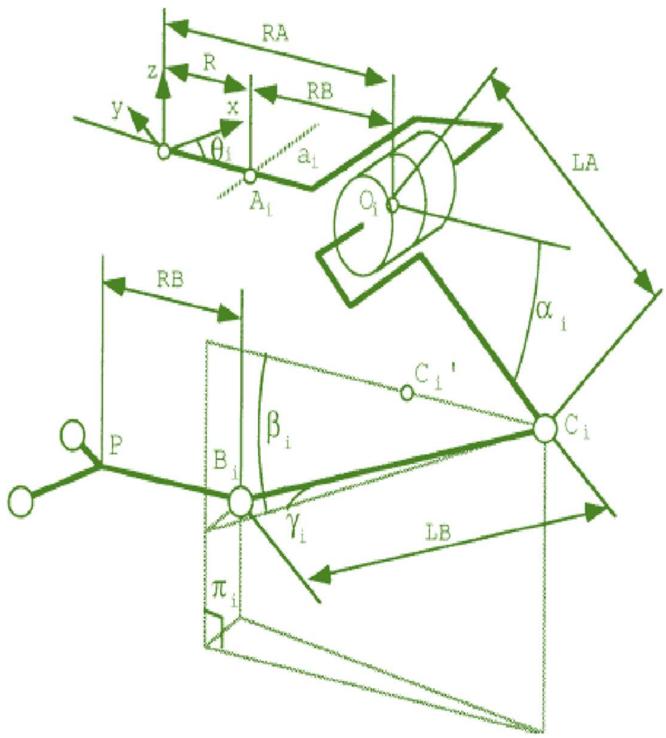


Fig.16. Parametric lengths and characteristic angles of the DELTA robot; to simplify, each parallelogram is represented by a single segment.

Characteristic angles.

αi: angle between the i[th] arm and the plane of the fixed base; in the following, we consider that the fixed base is horizontal; by convention, the angle αi is positive when the arm is located on the side of the nacelle;

βi: angle between the plane of the i[th] parallelogram and the horizontal plane, measured in a vertical plane βi containing the arm i;

γi: angle between the vertical plane πi and one of the bars of the parallelogram i;

θi: angle between the plane πi and the plane Oxz.

Dimensionless parametric lengths.

In order to generalize the study of the relationships between the parametric lengths of the robot and its different characteristics, we choose to relate all the dimensions to the reference length LA.

r = R / LA | **b = LB / LA**

Global dimension of the DELTA robot.

To qualify the global dimension DG of the DELTA robot, we have chosen to consider as a reference the sum of the lengths LA and LB expressed in millimeters; this value can be formulated as follows:

DG = LA (1 + b)

For example, the prototype whose LA = 200 mm and LB = 380 mm, DG=580 and the Delta is reference as DELTA 580.

Simplification of the geometry

When describing the operation of the robot, we have seen that the 3 orientations of the nacelle are constant; kinematically, we can therefore consider the equivalent robot whose kinematic chain is shown in Figure 17; it is obtained by translating the parallel arm-bar assemblies as mentioned in the definition of R.

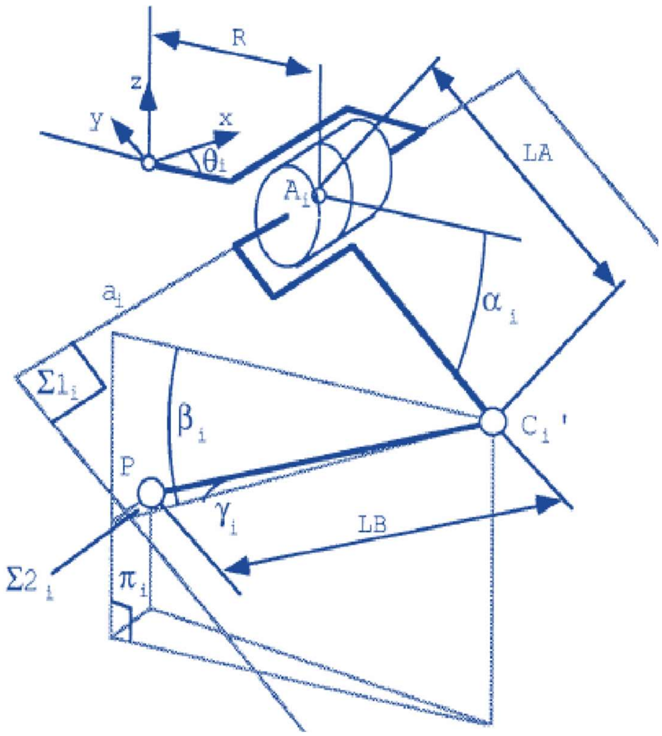


Fig. 17. Equivalent kinematic chain considering that the *nacelle is reduced to a point*.

<u>Workspace</u>

Geometric limits. By the fact that the dimensions of the components are finite and that they have restricted relative movements, the movement of the nacelle is limited to a portion of the space. It is still necessary to impose restrictions on certain movements to avoid ambiguities when calculating the coordinate transformations and the previously mentioned singularities.

Limit 1: $-\gamma_{max} \le \gamma_i \le \gamma_{max}$

<u>Source:</u> the construction constraints of the joints at the 2 ends of the parallel bars limit this angle to γ_{max} = 40 °;

limit 2: 30 ° ≤ $\alpha i + \beta i$ ≤ 180 °

<u>For the following 2 reasons:</u>

- avoid interference between the arm and the parallel bars as well as between the parallel bars and the arm motors when $\alpha i + \beta i$ is small;
- avoid coordinate transformation ambiguities which occur when the angle ($\alpha i + \beta i$) becomes greater than 180 °; this gives 2 sets of joint instructions for a single point in the operational space.

Workspace

The working volume is the area that point P, the center of the mobile plate, can reach. Mounting a tool will cause the working point to shift downward and translate the working volume accordingly.

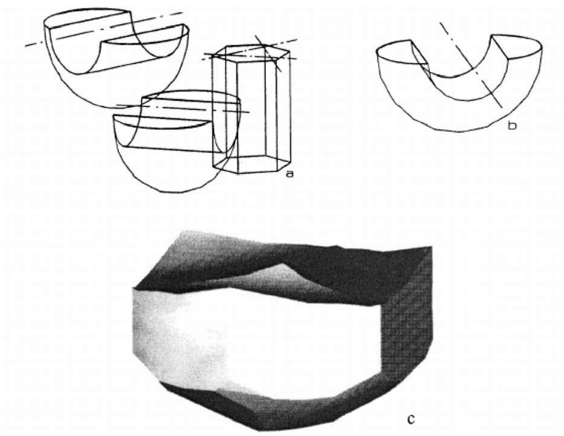


Fig. 18. The working volume of the DELTA robot (in c) is given approximately by the intersection of the right hexagonal prism a) with the 3 bodies of revolution b) whose axes are indicated in a).

Diameter D of the cylinder inscribed in the working volume:

This dimension D is directly given by limit 1:

$$D = 2\ LB \sin(\gamma_{max})$$

if we relate this value to LA:

$$d = D / LA = 2\ b \sin(\gamma_{max}) \approx 1,3\ b \text{ for } \gamma_{max} = 40°$$

It is interesting to notice that d depends only on b for a given angle γ_{max}.

Working surface:

the value of d determined above makes it possible to calculate the surface S of the hexagon, projection of the working volume on a horizontal plane; the dimensionless value of this surface will be given by $s = S / LA^2$.

$$s = 3\ \frac{b^2 \sin^2\gamma_{max}}{\cos 30°} = 2\sqrt{3}\ \sin^2\gamma_{max}\ b^2$$

$$s = 1,431 b^2 \quad \text{pour } \gamma_{max} = 40°$$

Height of the working volume:

Let Zh and Zb be the coordinates along z respectively of the top and the bottom of the working volume; the height H of the working volume will be given by: Zh - Zb; zh and zb are the dimensionless values of Zh and Zb.

According to the diagram in figure 19, we obtain directly:

$$zh = -\sqrt{1+b^2-2b\ \cos(\alpha_i+\beta_i)_{min}}$$

$$zh = -\sqrt{1+b^2-b\sqrt{3}} \quad \text{for } (\alpha_i+\beta_i)_{min} = 30°$$

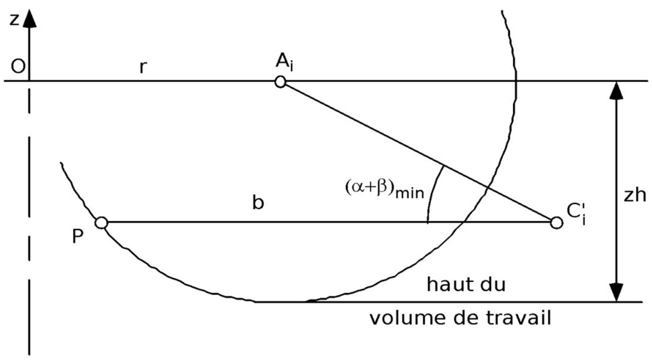


Fig. 19 Determination of the upper limit of the working volume.

To determine the lower limit of the working volume (dimension zb), let us refer to figure 20. The point of intersection of the surface generated by the arc of a circle centered in Ci 'by a rotation of axis ai (Cf . Fig. 17) with α+β= 180 ° with the vertical edge of the hexagonal prism opposite each arm is called T; the intersection of the same surface with the 2 adjacent vertical edges gives the points T '; due to the ternary symmetry, the T' points defined by each of the arms coincide 2 by 2.

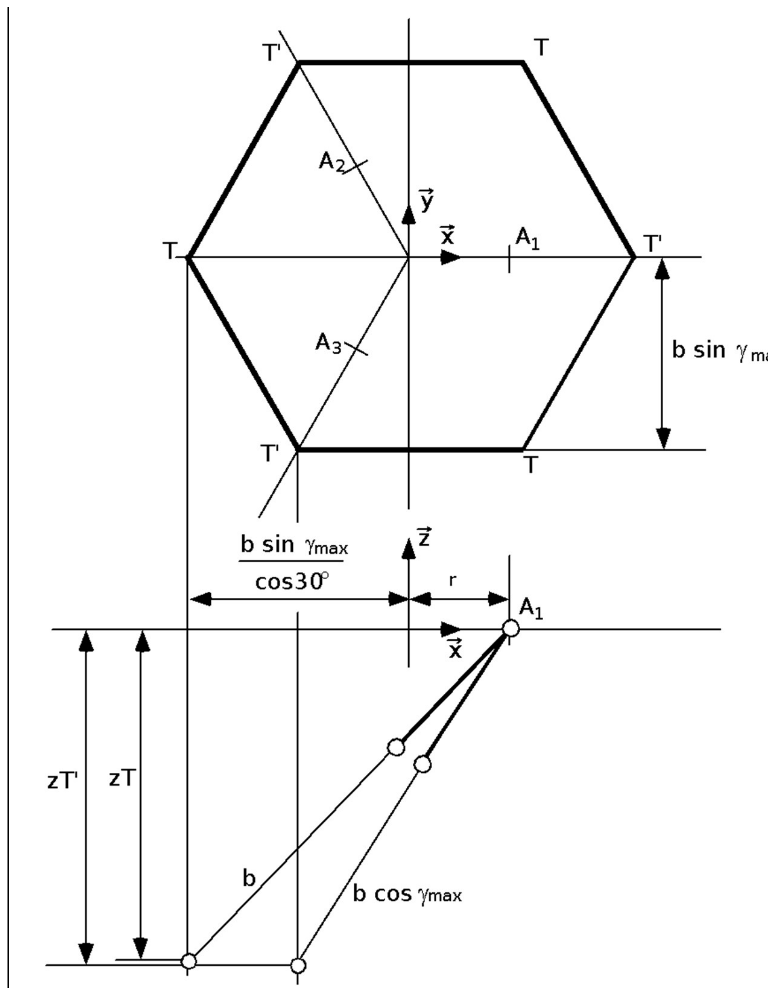


Fig.20- Determination of the lower limit of the working volume zb = max. (zT, zT').

The coordinates of T and T 'along z have the following value:

$$z_T = -\sqrt{(1+b)^2 - \left(r + 2\,\frac{b \sin \gamma_{max}}{\sqrt{3}}\right)^2}$$

$$z_{T'} = -\sqrt{(1+b\cos\gamma_{max})^2 - \left(r + \frac{b \sin \gamma_{max}}{\sqrt{3}}\right)^2}$$

The value zb is the maximum value of zT and zT'.

Note that the increase in the angle γ_{max} leads to an increase in the working surface (see: "working surface"), but decreases the height over which the hexagonal prism is complete; the low height of the work zone gained by an angle γmax markedly higher than 40 ° makes this approach unattractive in terms of use and seriously complicates the construction of the joints.

We can draw from the 2 preceding expressions the condition so that zT = zT ':

$$r = \sqrt{3}\,\frac{1-\cos\gamma_{max}}{\sin\gamma_{max}}$$

For γ_{max} = 40 °, r = 0.63 so that the points T and T' are located at the same height; it is under this condition that the most regular shape of the lower cap of the working volume is obtained.

h will therefore have the following value:

if r ≥ 0.63

$$h = \sqrt{(1+b)^2 - \left(r + 2\,\frac{b \sin \gamma_{max}}{\sqrt{3}}\right)^2} - \sqrt{1+b^2 - b\sqrt{3}}$$

if r ≤ 0.63

$$h = \sqrt{(1+b\cos\gamma_{max})^2 - \left(r + \frac{b \sin \gamma_{max}}{\sqrt{3}}\right)^2} - \sqrt{1+b^2 - b\sqrt{3}}$$

To better understand the complete working volume with the lower cape, it is interesting to know the maximum height h_{max}; this is given by the distance between the top of the working volume (zh) and the lowest point that the mobile plate can reach; this point is located on the Oz axis at the dimensionless value:

$\sqrt{(1+b)^2-r^2}$,

Hence:

$$h_{max} = \sqrt{(1+b)^2-r^2} - \sqrt{1+b^2-b\sqrt{3}}$$

Table 21 and fig. 21 give a synthetic view of zh, h, hmax, d, s and v (equal to h.s); these values are determined with γ_{max} = 40 ° and $(\alpha i + \beta i)_{min}$ = 30 °.

r	b	zh	h	hmax	d	s	v
	1.75	-1.02	1.06	1.68	2.25	4.38	4.67
	2	-1.24	1.01	1.71	2.57	5.72	5.79
0.5	2.25	-1.47	0.95	1.74	2.89	7.24	6.87
	2.5	-1.71	0.88	1.75	3.22	8.94	7.87
	2.75	-1.95	0.81	1.76	3.54	10.82	8.76
	3	-2.19	0.74	1.77	3.86	12.88	9.47
	1.75	-1.02	0.94	1 , 65	2.25	4.38	4.14
	2	-1.24	0.89	1.69	2.57	5.72	5.09
0.63	2.25	-1.47	0.83	1.71	2.89	7.24	5.98
	2.5	-1.71	0.76	1.73	3.22	8.94	6.76
	2.75	-1.95	0.68	1.74	3.54	10.82	7.4
	3	-2.19	0.61	1.76	3.86	12.88	7.84
	1.75	-1.02	0.76	1.61	2.25	4.38	3.34
	2	-1.24	0.71	1.65	2.57	5.72	4.04
0.8	2.25	-1.47	0.64	1.68	2.89	7.24	4.65
	2.5	-1.71	0.57	1.69	3.22	8.94	5.11
	2.75	-1.95	0.5	1.71	3.54	10.82	5.4
	3	-2.19	0.42	1.72	3.86	12.88	5, 46
	1.75	-1.02	0.49	1.54	2.25	4.38	2.17
	2	-1.24	0.44	1.58	2.57	5.72	2.53
1	2.25	-1, 47	0.38	1.62	2.89	7.24	2.77
	2.5	-1.71	0.32	1.64	3.22	8.94	2.82
	2.75	-1.95	0.25	1, 66	3.54	10.82	2.66
	3	-2.19	0.17	1.68	3.86	12.88	2.23

Table 21 Influence of r and b on the dimensions (h, hmax, d = D / LA , $s = S / LA^2$, $v = V / LA^3 = s.h$) and the position (zh) of the workspace.

Examples of realized robots and their corresponding workspace:
The following working volumes do not consider any limitation caused by the 4th axis.

Micro-DELTA 240
RA = 67 mm RB = 17 mm r = 0.625
LA = 80 mm LB = 160 mm b = 2
D = 206 mm H = 71.5 mm Hmax = 136 mm
$S = 3.66\ 10^4\ mm^2$
$V = 2.62\ 10^6\ mm^3$

DELTA 580
RA = 150 mm RB = 25 mm r = 0.625
LA = 200 mm LB = 380 mm b = 1.9
D = 489 mm H = 183 mm Hmax = 336 mm
$S = 20, 7\ 10^4\ mm^2$
$V = 37.9\ 10^6\ mm^3$

DELTA 740	RA = 194 mm	RB = 30 mm	r = 0.631
	LA = 260 mm	LB = 480 mm	b = 1.846
	D = 617 mm	H = 240 mm	Hmax = 435 mm
	$S = 33\ 10^{4}\ mm^{2}$		
	$V = 79.2\ 10^{6}\ mm^{3}$		

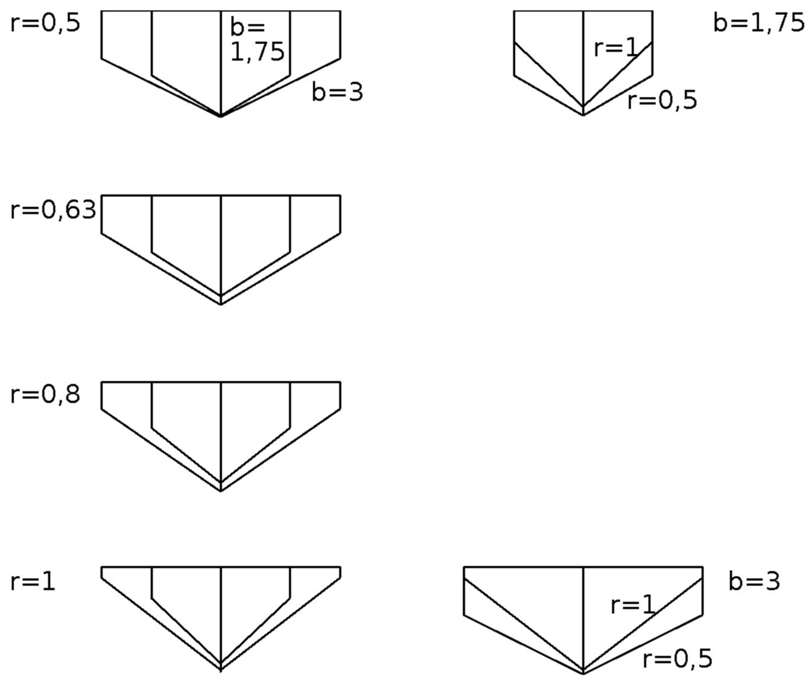


Fig. 21 Simplified representation of the influence of r and b on the dimensions h and d of the working volume.

1.4.1 Geometric models.

1.4.1.1 Introduction.

An important characteristic of the DELTA robot is the fact that, whatever are the movements of its mobile plate and its position, the 3 orientations of its nacelle are blocked. This does not consider the singularity configurations, where one or several of these orientations are no more blocked.

One of the typical applications of the DELTA robot is the packaging of light components; this operation essentially consists of picking up a part at a given position and placing it at another position following a trajectory compliant with the geometrical constraints of the environment (dimensions, position of the conveyors, obstacles,…).

The direct and inverse geometric models are the relations which express the positions of the end effector of the robot (the nacelle in this case) as a function of the active joint coordinates and vice versa. The knowledge of direct and inverse geometric models is therefore absolutely essential to program the DELTA robot in the operational space (pick and place target points as well as the whole trajectory). These models will also be essential for the dynamic modeling.

The fact that the nacelle only performs translational movements allows a simple formulation of geometric models. We consider the simplification of representation described under 1.3.8, fig. 17 which reduces the nacelle to a point.

On this basis, Sternheim [STERNHEIM 87] established a method to determine the direct and inverse models of the DELTA robot; for this operation, it involves the angles βi and γi (Cf. Fig. 17) which define the directions of the parallel bars.

We propose below a method of analytical geometry which consists in expressing the intersections of a sphere with 3 circles.

We refer to figure 17; the point P can be considered as the center of a sphere of radius LB; the points Ai are the centers of circles of radius LA belonging to the planes π_i and the points Ci 'are given by the intersections of the three circles of radius LA with the sphere of *radius LB* centered at P.

Coordinates of the points Ci:

$$[(R + LA \cos\alpha_i) \cos\theta_i, (R + LA \cos\alpha_i) \sin\theta_i, - LA \sin\alpha_i]$$

Equation of the sphere with center P (x, y, z) and radius LB:

$$(X - x)^2 + (Y - y)^2 + (Z - z)^2 = LB^2$$

The points Ci belong to the sphere:

$$[(R + LA \cos\alpha_i)\cos\theta_i - x]^2 + [(R + LA \cos\alpha_i)\sin\theta_i - y]^2 + [- LA \sin\alpha_i - z]^2 = LB^2$$

$$x^2 - 2x(R + LA \cos\alpha_i)\cos\theta_i + y^2 - 2y(R + LA \cos\alpha_i)\sin\theta_i + z^2 + 2z\, LA \sin\alpha_i = LB^2 - LA^2 - R^2 - 2 R\, LA \cos\alpha_i]$$

[eq.1]

These three equations can be solved according to x, y, z for the direct model and according to α_i for the inverse model. In order to preserve the possibilities of dimensional control of the different equations, we will not introduce here the dimensionless factors r and b.

1.4.1.2 Direct geometric model.

By setting:

$$D_i = - LB^2 + LA^2 + R^2 + 2 R\, LA \cos\alpha_i$$

$$E_i = 2(R + LA \cos\alpha_i)\cos\theta_i$$

$$F_i = 2(R + LA \cos\alpha_i)\sin\theta_i = E_i \, tg\theta_i$$

$$G_i = - 2 LA \sin\alpha_i$$

$$H_1 = E_1G_2 - E_1G_3 - E_2G_1 + E_2G_3 + E_3G_1 - E_3G_2$$

$H_2 = - E_1F_2 + E_1F_3 + E_2F_1 - E_2F_3 - E_3F_1 + E_3F_2$

$H_3 = - E_1D_2 + E_1D_3 + E_2D_1 - E_2D_3 - E_3D_1 + E_3D_2$

$H_4 = F_1D_2 - F_1D_3 - F_2D_1 + F_2D_3 + F_3D_1 - F_3D_2$

$H_5 = - F_1G_2 + F_1G_3 + F_2G_1 - F_2G_3 - F_3G_1 + F_3G_2$

Then :

$$x = z\frac{H_5}{H_2} + \frac{H_4}{H_2} \qquad \textbf{[eq.2]}$$

$$y = z\frac{H_1}{H_2} + \frac{H_3}{H_2} \qquad \textbf{[eq.3]}$$

[eq.2] et [eq.3] considered in [eq.1] for i=1, leads to :

$$z = \frac{-M \pm \sqrt{M^2 - 4\,L\,N}}{2\,L} \qquad \textbf{[eq.4]}$$

with:

$$L = \frac{H_5^2 + H_1^2}{H_2^2} + 1$$

$$M = 2\,\frac{H_5\,H_4 + H_1\,H_3}{H_2^2} - \frac{H_5\,E_1 + H_1\,F_1}{H_2} - G_1$$

$$N = \frac{H_4^2 + H_3^2}{H_2^2} - \frac{H_4\,E_1 + H_3\,F_1}{H_2} + D_1$$

1.4.1.3 Inverse geometric model:

Considering, $Q_i = 2\,x\cos\theta_i + 2\,y\sin\theta_i$

$$S = \frac{1}{LA}\left(-x^2 - y^2 - z^2 + LB^2 - LA^2 - R^2\right)$$

Solving the equations [eq.1] gives the following expression:

$$tg\frac{\alpha_i}{2} = \frac{-2z \pm \sqrt{4z^2 + 4R^2 - S^2 + Q_i^2\left(1 - \frac{R^2}{LA^2}\right) + Q_i\left(-2\frac{R\,S}{LA} - 4\,R\right)}}{-2\,R - S - Q_i\left(\frac{R}{LA} - 1\right)}$$

Observations about the results obtained for the direct and inverse geometric models:

- function of the geometry and the origin chosen for the angle αi, the absolute value of the latter is always less than 180°, therefore tg (αi / 2) cannot be infinite; however, mathematical singularities of type 0/0, generating numerical noise can appear; in order to rule out this problem, Codourey [CODOUREY 91] used with identical premises, a different way to solve the inverse geometric model; however, this resolution is slightly more computationally demanding;

- the two values for each αi arise because each arc intersects the sphere at two points; the useful value is the smaller of the two. The other value corresponds to a position where the angle $\alpha i + \beta i$ is greater than 180 °; we will therefore take the sign "-" in the expression of tg($\alpha i/2$);

- the two values obtained for each of the coordinates x, y, z, to establish the direct transformation, originate from the fact that through the 3 points Ci 'can pass two spheres of a given radius; their respective centers are placed symmetrically with respect to the plane defined by the three points considered; the minimum value of z will be the one to be taken into consideration; one will choose the sign "-", in the expression [3.4.4];

- to make the numerical calculations faster, it is advantageous to choose one of the angles θi equal to 0; we will therefore take $\theta 1 = 0$, $\theta_2 = 120$ ° and $\theta_3 = 240$ °; with these values, Q1 =2x, F1 = 0, H2, H4 and H5 have 2 terms less.

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