Laser: Theory and Modern Application
ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE (EPFL)

Exercise No.13: Optical parametric amplification. Raman and Brillouin lasing

1. Stimulated Raman Scattering

We describe the motion of a molecule using the equation of a harmonic oscillator with a source field term on the right-hand side:

$$\frac{d^2\tilde{q}}{dt^2} + 2\gamma\frac{d\tilde{q}}{dt} + \omega_v^2\tilde{q} = \frac{\tilde{F}(t)}{m}$$

Dipole Moment and Energy

The dipole moment of a molecule and its energy in an electric field are given as:

$$p = \alpha E, \quad W = \frac{1}{2}\langle p(z,t)E(z,t)\rangle$$

Using $\tilde{p}(z,t) = \epsilon_0\alpha\tilde{E}(z,t)$, we write:

$$\tilde{W} = \frac{1}{2}\epsilon_0\alpha\langle\tilde{E}^2(z,t)\rangle$$

Taking the gradient of energy, the force acting on the vibrational mode is:

$$\tilde{F} = \frac{dW}{dq} = \frac{\epsilon_0}{2}\left(\frac{d\alpha}{dq}\right)_0\langle\tilde{E}^2(z,t)\rangle$$

Optical Fields and Molecular Vibrations

The total optical field is expressed as:

$$\tilde{E}(z,t) = A_L e^{i(k_L z - \omega_L t)} + A_S e^{i(k_S z - \omega_S t)} + \text{c.c.}$$

Using the above, the force becomes:

$$\tilde{F}(z,t) = \epsilon_0\left(\frac{\partial\alpha}{\partial q}\right)_0\left[A_L A_S^* e^{i(Kz-\Omega t)} + \text{c.c.}\right]$$

where $K = k_L - k_S$ and $\Omega = \omega_L - \omega_S$.
The amplitude of molecular oscillations is:

$$q(\Omega) = \frac{\left(\frac{\epsilon_0}{m}\frac{\partial\alpha}{\partial q}\right)_0 A_L A_S^*}{\omega_v^2 - \Omega^2 - 2i\Omega\gamma}$$

Phase Matching

The phase-matching condition is automatically satisfied because optical phonons exhibit a nearly constant dispersion relation, ensuring wave vector equality if frequency equality is met.

Nonlinear Polarization and Raman Susceptibility

The total polarization is:

$$\tilde{P}(z,t) = \epsilon_0 N \alpha(z,t) \tilde{E}(z,t)$$

where $\alpha(z,t) = \alpha_0 + \left(\frac{\partial \alpha}{\partial q}\right)_0 \tilde{q}(z,t)$.

The nonlinear term is:

$$\tilde{P}_{NL}(z,t) = \epsilon_0 N \left(\frac{\partial \alpha}{\partial q}\right)_0 \left[q(\Omega) e^{i(Kz - \Omega t)} + \text{c.c.}\right] \cdot \tilde{E}(z,t)$$

For the Stokes component, we have:

$$\tilde{P}_{NL,S}(z,t) = \epsilon_0 N \left(\frac{\partial \alpha}{\partial q}\right)_0^2 \frac{A_L^2 A_S}{\omega_v^2 - \Omega^2 + 2i\Omega\gamma} e^{ik_S z}$$

Raman susceptibility is defined as:

$$\chi_R(\omega_S) = \frac{\epsilon_0 N}{m} \left(\frac{\partial \alpha}{\partial q}\right)_0^2$$

2. Stimulated Brillouin Scattering (SBS)

The phase-matching condition for SBS is:

$$\omega_2 = \omega_1 - \Omega_B$$

The dispersion relation for acoustic phonons is:

$$\Omega_B = |q_B| v, \quad q_B = k_1 - k_2$$

In optical fibers, acoustic waves propagate either forward or backward. For backward scattering, $q_B = 2k_1$ and:

$$\Omega_B = \frac{v}{c/n}(\omega_1 + \omega_2)$$

Electrostrictive Force

The electromagnetic energy is:

$$U = -\frac{1}{2}\epsilon_0 \alpha E^2$$

The electrostrictive force is the gradient of this energy:

$$F = -\nabla U = \frac{1}{2}\epsilon_0 \alpha \nabla (E^2)$$

Acoustic Wave Equation

The density variations ρ obey the wave equation:

$$\frac{\delta^2 \rho}{\delta t^2} - \Gamma \nabla^2 \frac{\delta \rho}{\delta t} - v^2 \nabla^2 \rho = \nabla f$$

Solving this under the slowly varying amplitude approximation yields:

$$\rho(z,t) = \frac{\epsilon_0 \gamma_e q^2 A_1 A_2^*}{\Omega_B^2 - \Omega^2 - i\Omega\Gamma_B}$$

The SBS gain coefficient is derived from the coupled wave equations for the optical and acoustic fields:

$$g = g_0 \frac{\Gamma_B/2}{(\Omega_B - \Omega)^2 + (\Gamma_B/2)^2}$$