Laser: Theory and Modern Application

ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE (EPFL)

Exercise No.11: Second-order autocorrelation techniques

Solution Set (Exercise Sheet 11)

11.1 Spectral FWHM and Chirp Rate

To calculate the spectral FWHM, start from the real part of the electric field:

$$\text{real}(\epsilon(t,x)) = E_0 e^{-\Gamma(x)\left(t-\frac{x}{v_g(\omega_0)}\right)^2}.$$

The initial pulse for $x = 0$ is:

$$\text{real}(\epsilon(t,0)) = E_0 e^{-\Gamma(0)t^2}.$$

When $t = t_{\text{FWHM}}/2$, the electric field is half its initial value:

$$\text{real}(\epsilon(t_{\text{FWHM}}/2,0)) = \frac{E_0}{2} = E_0 e^{-\Gamma(0)\left(\frac{t_{\text{FWHM}}}{2}\right)^2}.$$

From this, we calculate $\Gamma(0)$:

$$\Gamma(0) = -\ln(0.5)\left(\frac{2}{t_{\text{FWHM}}^2}\right) = 6.9315 \times 10^{27}\,\text{Hz}^2.$$

To compute $\Gamma(x)$, we need:

$$\xi = 2\Gamma(0)\frac{d^2k}{d\omega^2}.$$

From the MATLAB code, we find:

$$\left.\frac{d^2k}{d\omega^2}\right|_{\omega_0} = 4.4572 \times 10^{-26}, \quad \xi = 617.9007.$$

Finally:

$$\Gamma(50\,\text{cm}) = \frac{\Gamma(0)}{1+\xi^2(50\,\text{cm})^2} - i\frac{\xi \cdot 50\,\text{cm}}{1+\xi^2(50\,\text{cm})^2} = 7.2543 \times 10^{24} - i3.2334 \times 10^{-2}.$$

A pulse is chirped if its frequency changes over time. The optical frequency is:

$$\omega(t) = \omega_0 + \text{Im}[\Gamma(x)]\frac{x}{v_g(\omega_0)} - 2\text{Im}[\Gamma(x)]t.$$

The instantaneous frequency depends on time, indicating the pulse is chirped. The chirp rate is:

$$\frac{d\omega}{dt} = -2\text{Im}[\Gamma(x)] = 2\frac{\xi x}{1+\xi^2 x^2}.$$

11.2 FROG (Frequency-Resolved Optical Gating)

Define:

$$E_{\text{sig}}(t,\tau) = E(t)E(t-\tau).$$

Then:

$$E_{\text{sig}}(t,\tau) = \int_{-\infty}^{\infty} \hat{E}_{\text{sig}}(t,\Omega)e^{-i\Omega\tau}d\Omega.$$

Insert this into the equation for $I_{\text{FROG}}(\omega,t)$:

$$I_{\text{FROG}}(\omega,t) = \left|\int_{-\infty}^{\infty} E(t)E(t-\tau)e^{-i\omega t}dt\right|^2 = \left|\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{E}_{\text{sig}}(t,\Omega)e^{-i\Omega\tau}e^{-i\omega t}d\Omega dt\right|^2.$$

To recover $\hat{E}_{\text{sig}}(t,\Omega)$, we use:

$$\hat{E}_{\text{sig}}(t,\Omega) = \int_{-\infty}^{\infty} E_{\text{sig}}(t,\tau)e^{i\Omega\tau}d\tau.$$

$$\hat{E}_{\text{sig}}(t,0) = \int_{-\infty}^{\infty} E_{\text{sig}}(t,\tau)d\tau = \int_{-\infty}^{\infty} E(t)E(t-\tau)d\tau = E(t)\int_{-\infty}^{\infty} E(t-\tau)d\tau. = E(t)\cdot\text{constant}$$

In SHG-FROG: - $I_{\text{sig}}(t)$ and $I_{\text{sig}}(-t)$ are identical, making it impossible to determine the pulse's direction of time.

To break this symmetry, a chirp can be added to one arm of the setup, such as inserting a piece of glass.

Why is SHG more sensitive than THG?

SHG is more efficient in conversion and has a higher signal-to-noise ratio compared to THG.

11.3 First-Order Autocorrelation

The first-order autocorrelation function $G(\tau)$ is:

$$G(\tau) = \int_{-\infty}^{\infty} F(t)F(t-\tau)dt.$$

For a Michelson interferometer with a delay $\Delta l = c\tau$, the output intensity is:

$$I_1(\tau) = \int_{-\infty}^{\infty} |E(t)+E(t-\tau)|^2 dt.$$

Expanding:

$$I_1(\tau) = \int_{-\infty}^{\infty} \left(|E(t)|^2 + |E(t-\tau)|^2 + E(t)^*E(t-\tau) + E(t)E^*(t-\tau)\right)dt.$$

This simplifies to:

$$I_1(\tau) \propto 2\int_{-\infty}^{\infty} I(t)dt + 2G(\tau).$$

For a pulse centered at 835 nm with a pulse duration $\Delta t = 28.4$ fs, the electric field is:

$$\vec{E} = A(t)\cos(\omega_0 t), \quad A(t) = \sqrt{I}\,\text{sech}\left(\frac{t}{\Delta t\cdot 1.76}\right).$$

Use the autocorrelation to calculate the power spectrum for a Michelson interferometer with a 30 μm delay. The Jupyter Notebook for autocorrelation calculation can be found on moodle.