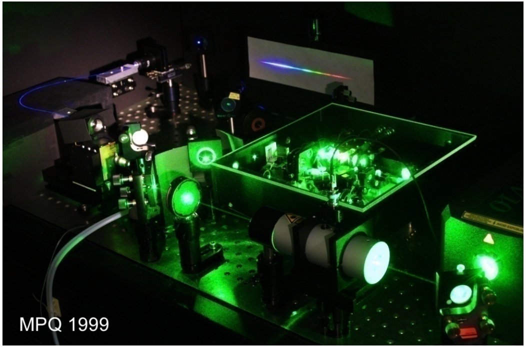
Modules of the 2024 Course

Topics covered	No	Lecture/Date
Introductory presentation; Basic of laser operation I: dispersion theory, atoms	1	11. 09. 2024
Basic of laser operation II: dispersion theory, atoms	2	18. 09. 2024
Laser systems I: 3 and 4 level lasers, gas lasers, solid state lasers, applications	3	25. 09. 2024
Laser systems II: semi-conductor lasers, external cavity lasers, applications	4	02. 10. 2024
Noise characteristics of lasers: linewidth, coherence, phase and amplitude noise, OSA (1)	5	09. 10. 2024
Noise characteristics of lasers: linewidth, coherence, phase and amplitude noise, OSA (2)	6	16. 10. 2024
Optical detection	7	30. 10. 2024
Optical fibers: light propagation in fibers, specialty fibers and dispersion (GVD)	8	06. 11. 2024
Ultrafast lasers I.: Passive mode locking and ultrafast lasers	9	13. 11. 2024
Ultrafast lasers II: mode locking, optical frequency combs / frequency metrology	10	20. 11. 2024
Ultrafast lasers III: pulse characterization, applications	11	27. 11. 2024
Nonlinear frequency conversion I: theory, frequency doubling, applications	12	04. 12. 2024
Nonlinear frequency conversion II: optical parametric amplification (OPA)	13	11. 12. 2024
Laboratory visits (lasers demo)	14	20. 12. 2024

Week 9 content

Content of Week 9

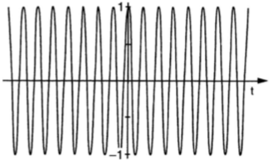
- Basics of pulsed laser sources
- Kerr Lens Mode Locking
- Carrier Envelope Frequency
- Characterizing pulses using autocorrelation technique
- Wiener Khinchine theorem
- Frequency Metrology

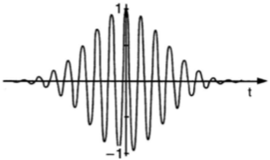


Basics of pulses

Source: 2.2.2 (Ruliere, "femtosecond laser pulses")

Time domain



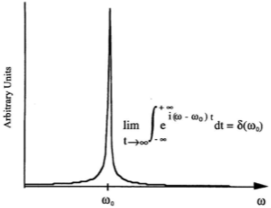


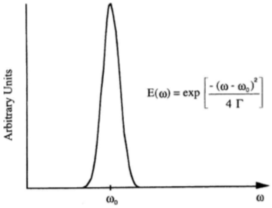
$$E_y = \mathrm{Re}\left(E_0 e^{i\omega_0 t}\right) \qquad E_y = \mathrm{Re}\left(E_0 e^{-\Gamma t^2 + i\omega_0 t}\right)$$

$\downarrow$ **Fourier Transform** $\downarrow$

A finite duration monochromatic wave has a non-zero spectral width

Frequency domain

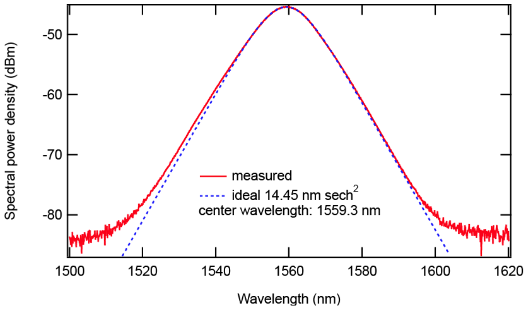




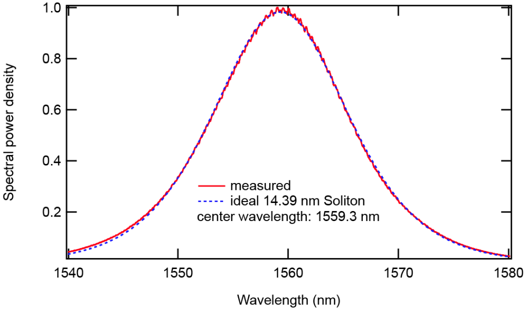
Example

Experimental spectrum of Hyperbolic secant pulse of a 200 fs ultra-fast faser at telecom wavelength

Linear scale



Same in log scale



Elemental relation between time and frequency domain (time-bandwidth product):

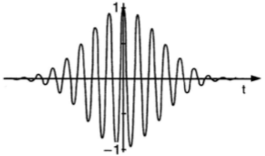
$$\Delta\omega \cdot \tau > \frac{1}{2}$$

Minimum pulse duration of mode-locked lasers

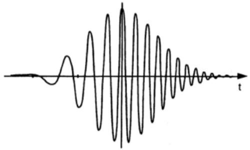
Laser medium	**Gain bandwidth** $\Delta\lambda_{\text{gain}}$	**Minimum puls duration** $\Delta\tau \approx \frac{\lambda^2}{c\cdot\Delta\lambda}$
Argon-ion	$0.7 \cdot 10^{-2}$ nm	150 ps
Ruby	0.2 nm	5 ps
Nd:YAG	10 nm	350 fs
Er-glass	40 nm	150 fs
Dye	100 nm	10 fs
Ti-Sapphire	400 nm	3 fs

Chirped pulses

$$E_y = \mathrm{Re}\left(E_0 \mathrm{e}^{\left(-\Gamma t^2 + i\omega_0 t\right)}\right)$$



$$E_y = \mathrm{Re}\left(E_0 \mathrm{e}^{\left[-\Gamma t^2 + i\left(\omega_0 t - a t^2\right)\right]}\right)$$



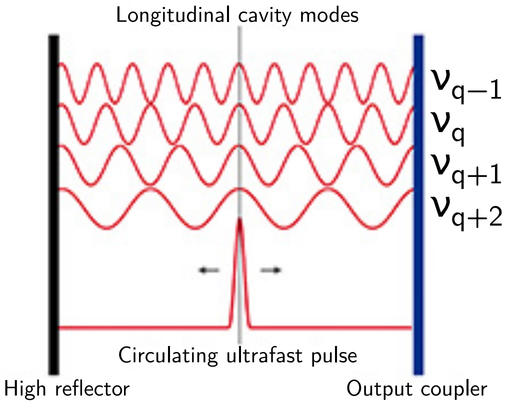
Instantaneous frequency:

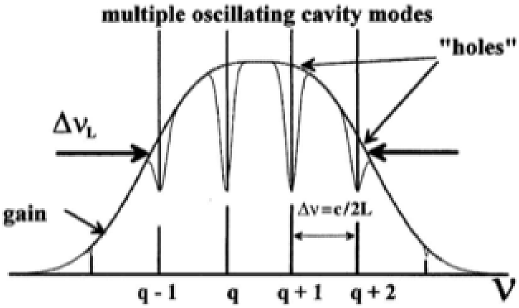
Instantaneous frequency:

The pulse is said to be “chirped”

Mode-locking

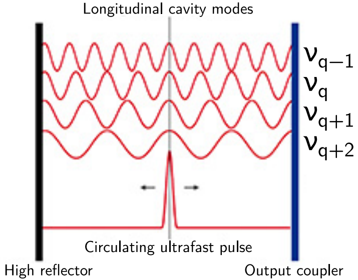
Chapter 3. (Ruliere, “femtosecond laser pulses”)





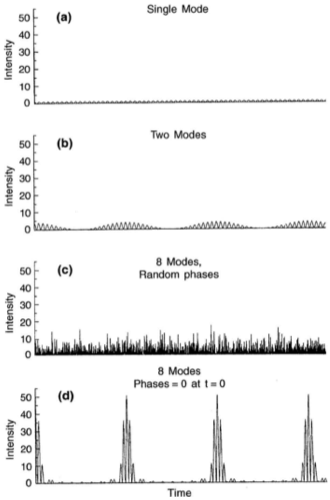
Fresnel representation

Frequency domain interpretation



$$\sum_n x_o \sin\left(2\pi\left(\nu_o + n\Delta\nu_{\text{cavity}}\right)\cdot t + \phi_o\right)$$

Constant phase for all modes, i.e. "mode locking"



Mode-locking

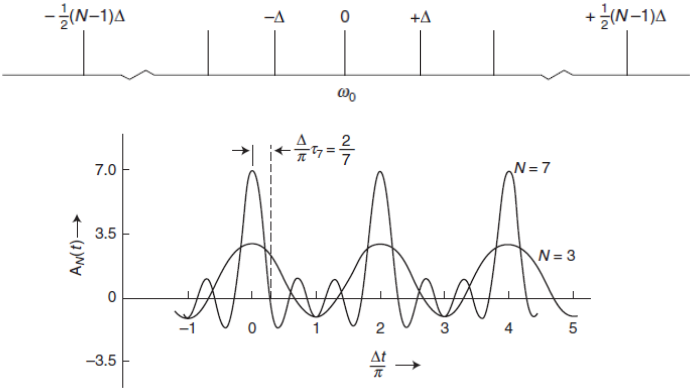
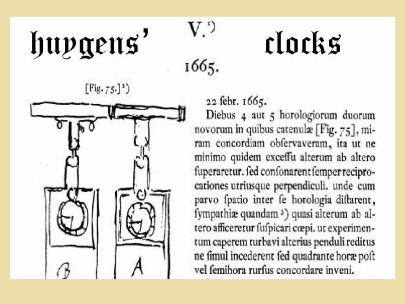


Figure 11.19: The function $A_N(t) = \sin(\frac{1}{2}N\Delta t)/\sin(\frac{1}{2}\Delta t)$ vs. $\Delta t/\pi$

Nonlinear oscillator

Synchronization of two pendulum: Huygens, 17$^{\text{th}}$ century

In 1657, **Christiaan Huygens** revolutionized the measurement of time by creating the first working pendulum clock. In early 1665, Huygens discovered "…an odd kind of sympathy perceived by him in these watches [two pendulum clocks] suspended by the side of each other." The pendulum clocks swung with exactly the same frequency and 180 degrees out of phase; when the pendulums were disturbed, the antiphase state was restored within a half-hour and persisted indefinitely.


huygens' clocks

V.[1]

1665.

[Fig. 75.][2]

22 febr. 1665.

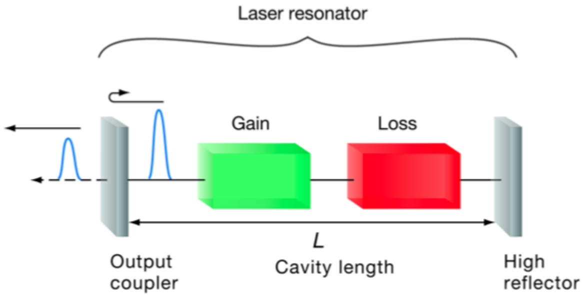
Diebus 4 aut 5 horologiorum duorum novorum in quibus catenulæ [Fig. 75], miram concordiam obſervaveram, ita ut ne minimo quidem exceſſu alterum ab altero ſuperaretur. ſed conſonarent ſemper reciprocationes utriusque perpendiculi. unde cum parvo ſpatio inter ſe horologia diſtarent, ſympathiæ quandam [3]) quasi alterum ab altero afficeretur ſuſpicari cœpi. ut experimentum caperem turbavi alterius penduli reditus ne ſimul incederent ſed quadrante horæ poſt vel ſemihora rurſus concordare inveni.

Active Mode locking: AM modulation

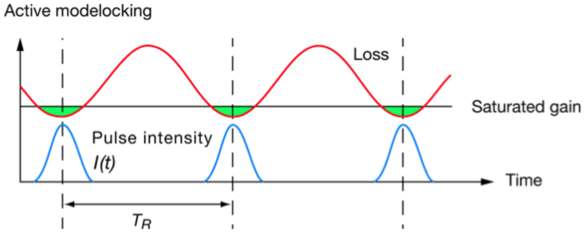
Active mode locking requires external loss modulation

Time domain interpretation

Sinusodial modulation of the loss leads to coupling of different longitudinal modes



From: U. Keller, Nature



Active Mode locking: AM modulation

Frequency domain interpretation

Scalar electric field:

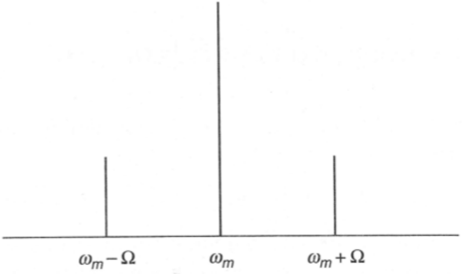
$$E_m(z,t) = \varepsilon_m \cdot \sin k_m z \cdot \sin(\omega t + \phi_n)$$

Amplitude: $\varepsilon_m = \varepsilon_0(1 + \varepsilon \cos \Omega t)$

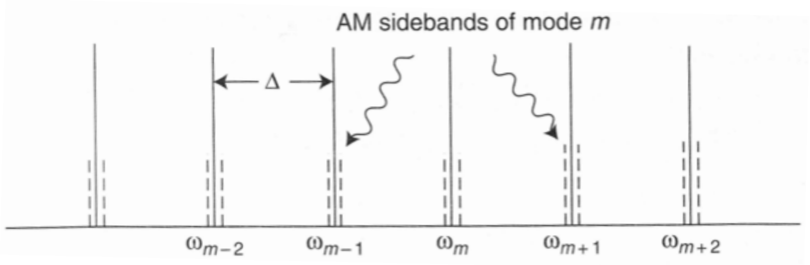
$$E_m(z,t) = \varepsilon_0(1 + \varepsilon \cos \Omega t) \sin(\omega t + \phi_m) \sin k_m \cdot z =$$

$$\begin{cases} = \varepsilon_0 \left[\sin(w_n t + \phi_m) + \frac{\varepsilon}{2} \sin\left((\omega_m + \Omega) t + \phi_m\right) \right] \\ = \varepsilon_0 \left[\sin(w_n t + \phi_m) + \frac{\varepsilon}{2} \sin\left((\omega_m - \Omega) t + \phi_m\right) \right] \end{cases}$$

Active mode-locking: AM modulation



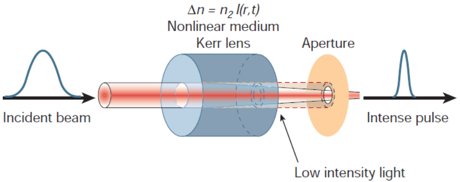
Strong coupling to the nearest neighbor



Passive: Kerr Lens Mode locking

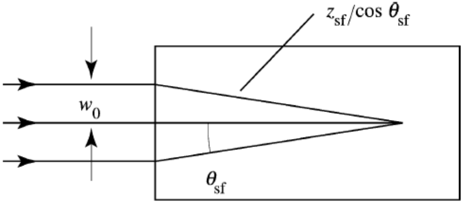
Derivation of critical self focusing: The Kerr nonlinearity

Intensity dependent refractive index



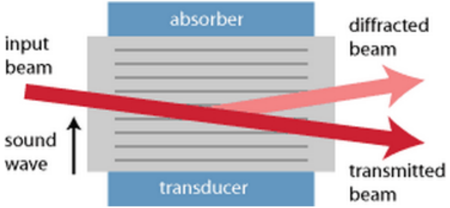
Power for critical self focusing

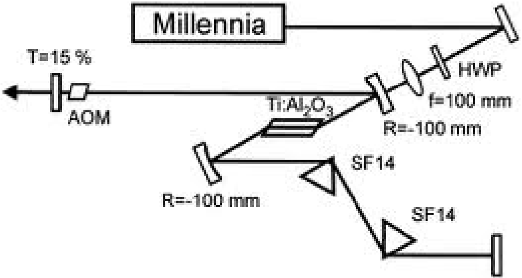
$$P_{cr} = \frac{\pi(0.61)^2\lambda_0^2}{8n_0n_2} \approx \frac{\lambda_0^2}{8n_0n_2} \quad \theta_{dif} = \theta_{sf}$$



Active mode-locking: AM modulation

Acousto-optic modulator





Condition for mode-locking

Active mode-locking: FM modulation

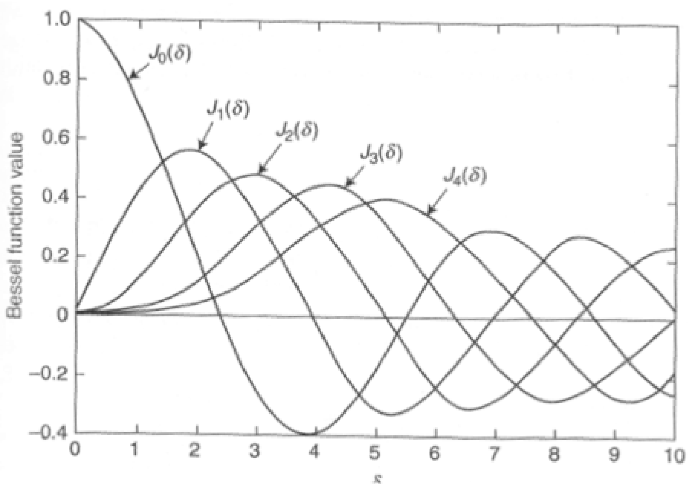
Scalar electric field:

$$E_m(z,t) = \varepsilon_m \cdot \sin k_m z \sin\left(\omega_m t + \phi_0 + \delta \cos \Omega t\right)$$

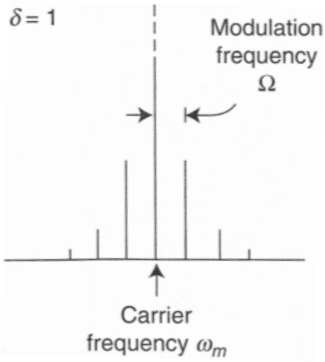
Suppose amplitude is constant but phase varying in time

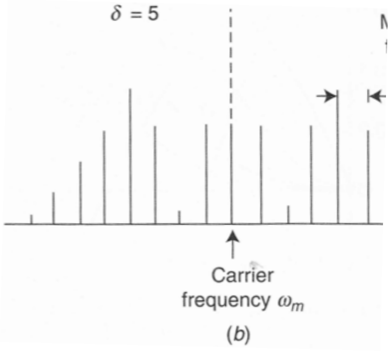
$$\begin{aligned}
\sin\left(\omega_m t + \phi_m + \delta \cos \Omega t\right) &= J_0(\delta) \cdot \sin\left(\omega_m t + \phi_m\right) \\
&+ J_1(\delta)\left[\sin\left((\omega_m + \Omega)t + \phi_m\right) + \sin\left((\omega_m - \Omega)\, t\right)\right] \\
&- J_2(\delta)\left[\sin\left((\omega_m + 2\Omega)\, t + \phi_m\right) + \sin\left((\omega_m - 2\Omega)\, t\right)\right] \\
&- J_3\left(\delta\right)\left[\sin\left((\omega_m + 3\Omega)\, t + \phi_m\right) + \sin\left((\omega_m - 3\Omega)\, t\right)\right] \\
&\vdots
\end{aligned}$$

Active mode-locking: FM modulation



Active mode-locking: FM modulation

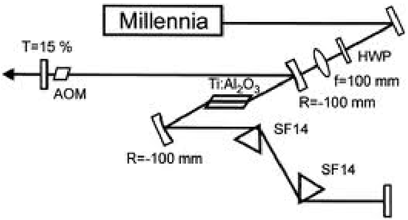




Active mode-locking: FM modulation

Electro-optic modulator

Linear electro-optic effect (Pockels effect)



Passive mode-locking: saturable absorber

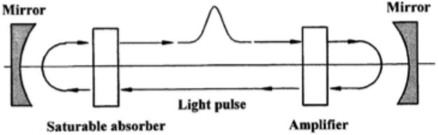
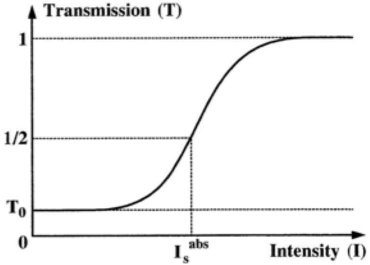
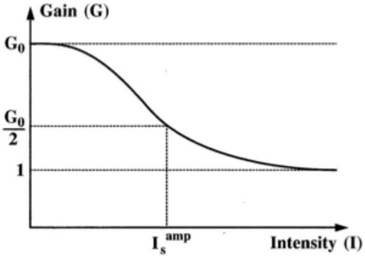


Fig. 3.12. Round-trip pulse in a laser cavity including saturable absorber and amplifying medium





Passive mode-locking: saturable absorber

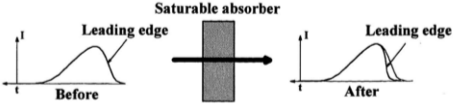


Fig. 3.15. Illustration of pulse shape modification after crossing a saturable absorber

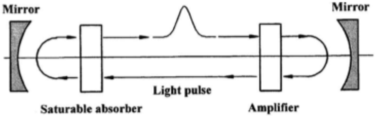


Fig. 3.12. Round-trip pulse in a laser cavity including saturable absorber and amplifying medium

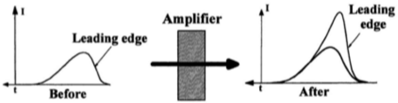
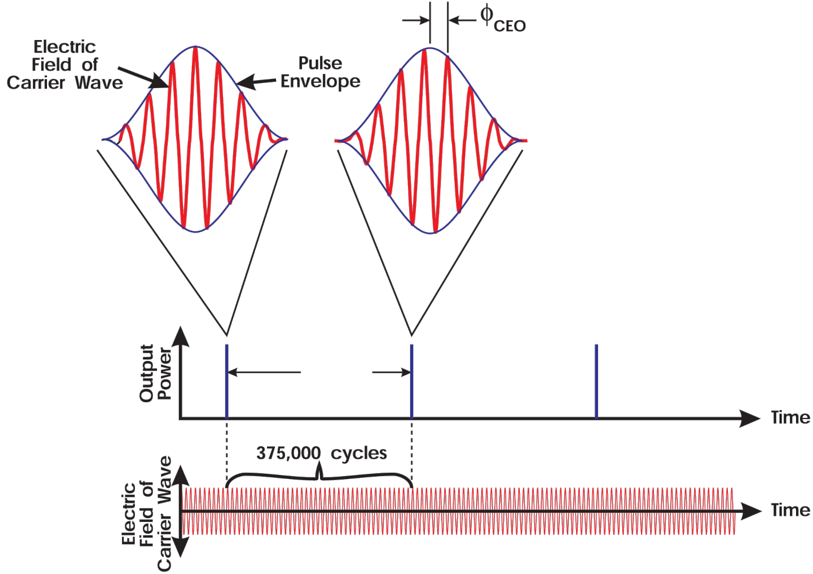


Fig. 3.16. Illustration of pulse shape modification after crossing an amplifying medium

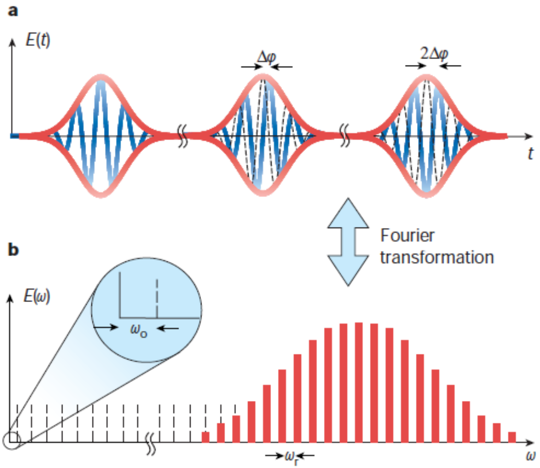
Carrier Envelope Frequency



$$\nu_{\mathrm{CEO}} = \frac{d\Phi}{dt} = \frac{\Delta\Phi}{2\pi T}$$

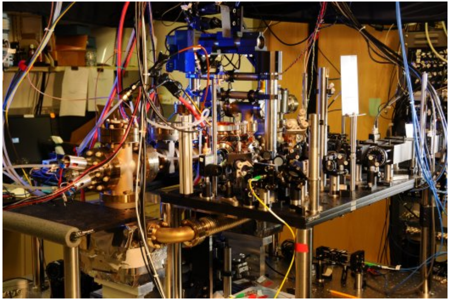
$$\nu_m = m f_{\mathrm{rep}} + \nu_{\mathrm{CEO}}$$

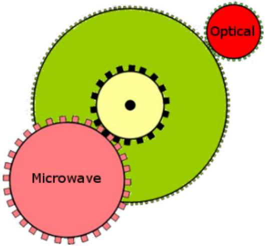
Carrier Envelope Frequency

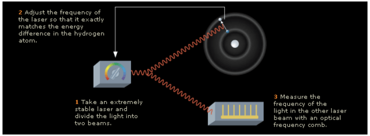


Applications of ultra-short pulses: Optical Clocks

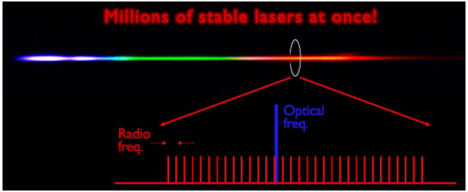
Optical atomic clocks (NIST, Boulder, Colorado)

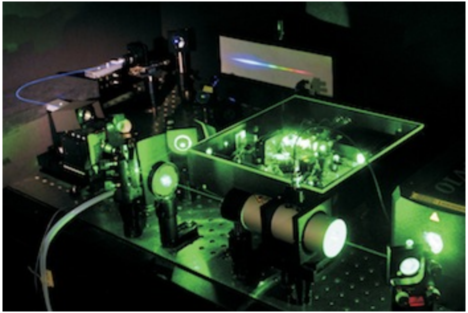


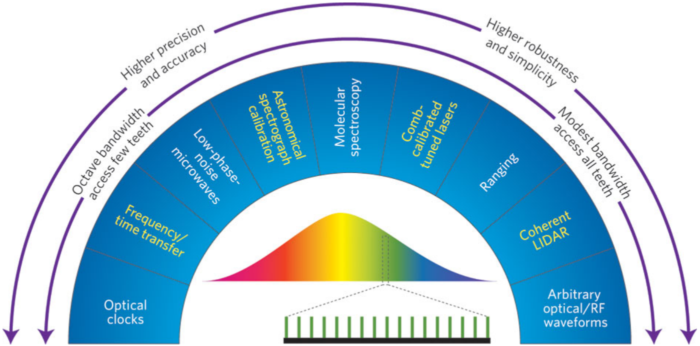




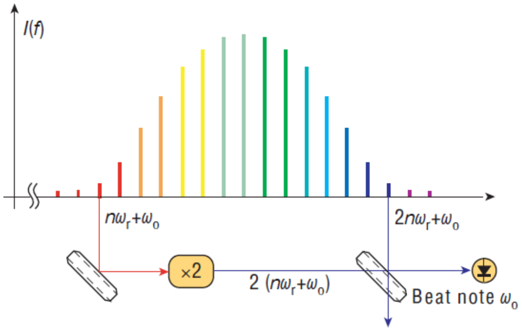
Applications of frequency combs







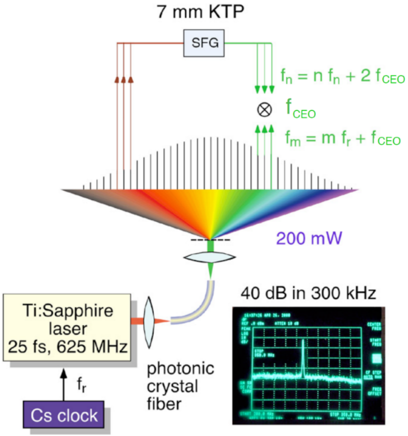
Measuring the carrier envelope frequency

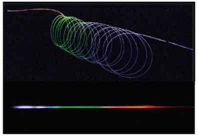


$$f_n = n f_{\text{rep}} + f_0$$

$$2f_n - f_{2n} = 2\left(n f_{\text{rep}} + f_0\right) - \left(2n f_{\text{rep}} + f_0\right) = f_0$$

Measuring the carrier envelope frequency



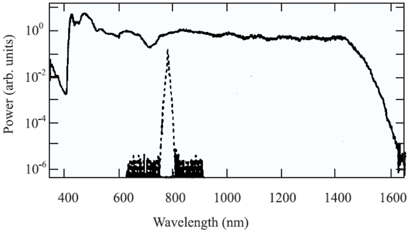


„Rainbow Fiber“

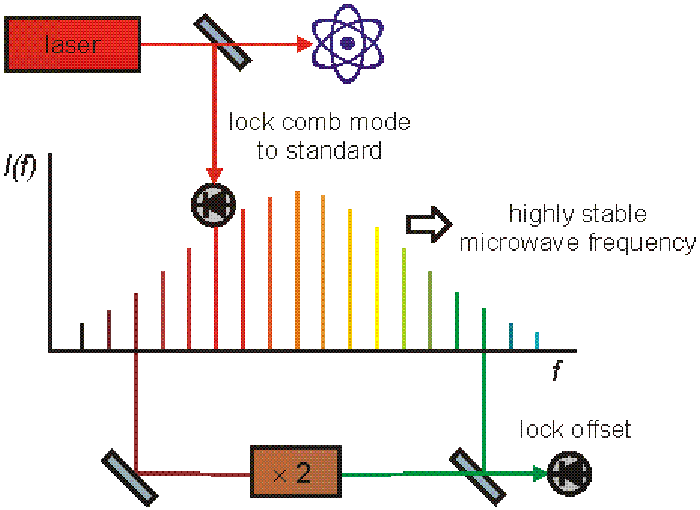
(Lucent Technologies, 1999)

“Photonic Crystal Fiber”

J.C. Knight, W.J. Wadsworth, P. St. Russel
University of Bath, UK



Principle of an atomic clock



Optical atomic clock: state of the art

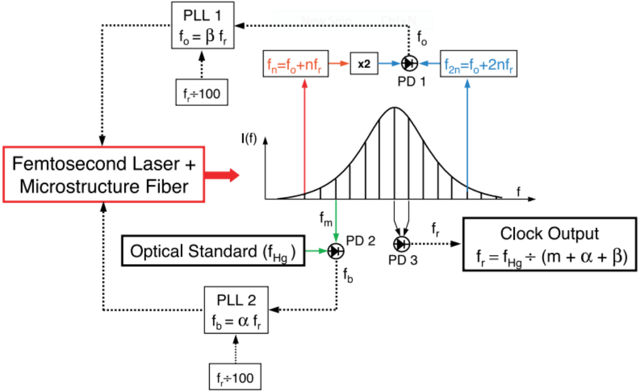
An Optical Clock Based on a Single Trapped $^{199}Hg^+$ Ion

S. A. Diddams,[1*] Th. Udem,[1†] J. C. Bergquist,[1] E. A. Curtis,[1,2] R. E. Drullinger,[1] L. Hollberg,[1] W. M. Itano,[1] W. D. Lee,[1] C. W. Oates,[1] K. R. Vogel,[1] D. J. Wineland[1]

A clockwork based on a mode-locked femtosecond laser provides output pulses at a 1-gigahertz rate that are phase-coherently locked to the optical frequency. By comparison to a laser-cooled calcium optical standard, an upper limit for the fractional frequency instability of $7 \cdot 10^{-15}$ is measured in 1 second of averaging – a value substantially better than that of the world's best microwave atomic clocks.

Fractional frequency instability:

$$\sigma_y(\tau) \approx \left\langle \frac{\Delta\nu_{\rm rms}}{\nu_0} \right\rangle_\tau$$

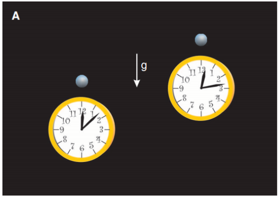


Best clocks to date

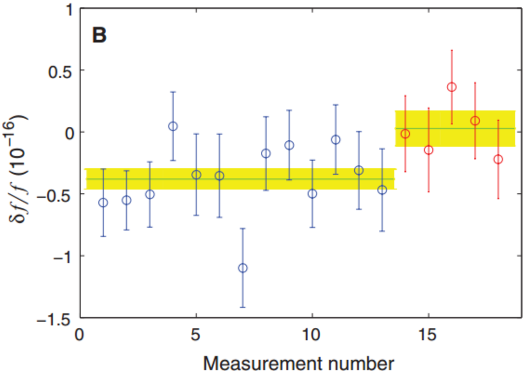
Optical Clocks and Relativity

C. W. Chou,* D. B. Hume, T. Rosenband, D. J. Wineland

Observers in relative motion or at different gravitational potentials measure disparate clock rates. These predictions of relativity have previously been observed with atomic clocks at high velocities and with large changes in elevation. We observed time dilation from relative speeds of less than 10 meters per second by comparing two optical atomic clocks connected by a 75-meter length of optical fiber. We can now also detect time dilation due to a change in height near Earth's surface of less than 1 meter. This technique may be extended to the field of geodesy, with applications in geophysics and hydrology as well as in space-based tests of fundamental physics.

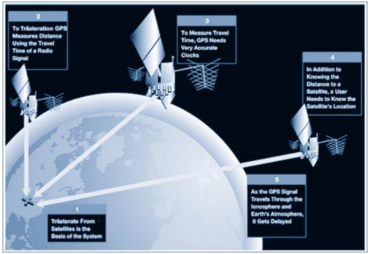


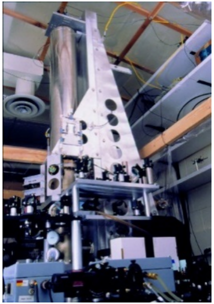
$$\sigma_y(\tau) \approx \left\langle \frac{\Delta v_{\mathrm{rms}}}{v_0} \right\rangle_\tau$$



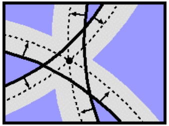
Gravitational time dilation at the scale of daily life. (A) As one of the clocks is raised, its rate increases when compared to the clock rate at deeper gravitational potential

Atomic clocks





1999 – NIST-F1 begins operation with an uncertainty of $1.7 \cdot 10^{-15}$, or accuracy to about one second in 20 million years, making it one of the most accurate clocks ever made (a distinction shared with similar standards in France and Germany).



Three spheres are necessary to find position in two dimensions, four are needed in three dimensions.