ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Non-linear frequency conversion I

Lasers: theory and applications, lecture 12.2

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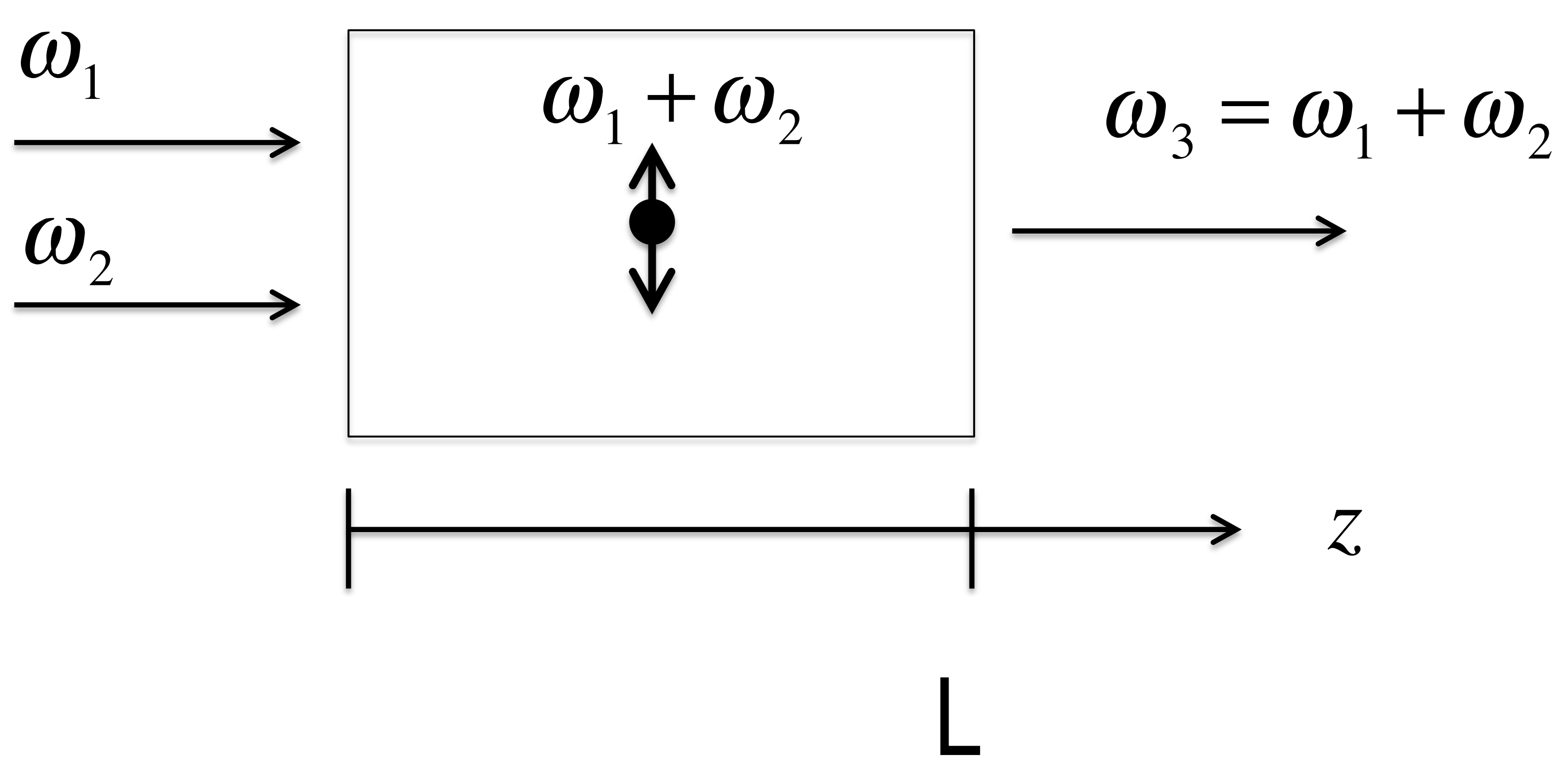
Non-linear frequency conversion I

- Sum frequency generation
- Phase matching

Sum frequency



$$E_3(z,t)=E_3(z)e^{-i\omega_3 t}=A_3(z)e^{ik_3 z}e^{-i\omega_3 t}$$

$$P^{NL}(z,t)=4\varepsilon_o d_{eff}E_1(z)E_2(z)\cdot e^{-i\omega_3 t}$$
$$=4\varepsilon_o d_{eff}A_1(z)A_2(z)\cdot e^{i(k_1+k_2)z}e^{-i\omega_3 t}$$

$$\frac{\partial^2}{\partial z^2}E_3(z,t)-\frac{n^2}{c^2}\frac{\partial^2 E_3(z,t)}{\partial t^2}=-\frac{1}{\varepsilon_o c^2}\cdot\frac{\partial^2 \vec{P}^{NL}(z,t)}{\partial t^2}$$

$$\left[\frac{d^2A_3}{dz^2}+2ik_3\frac{dA_3}{dz}\underbrace{-k_3^2A_3+\frac{n^2(\omega_3)\omega_3^2A_3}{c^2}}_{0}\right]e^{i(k_3z-\omega_3t)}=\frac{-4d_{\text{eff}}\omega_3^2}{c^2}A_1A_2e^{i[(k_1+k_2)z-\omega_3t]}$$

$$\frac{d^2A_3}{dz^2}+2ik_3\frac{dA_3}{dz}=\frac{-4d_{\text{eff}}\omega_3^2}{c^2}A_1A_2e^{i(k_1+k_2-k_3)z}.$$

$$\left|\frac{d^2A_3}{dz^2}\right|<<\left|k_3\frac{dA_3}{dz}\right|$$

Slowly varying enveloppe approximation

$$\boxed{\frac{dA_3}{dz}=\frac{2id_{\text{eff}}\omega_3^2}{k_3c^2}A_1A_2e^{i\Delta kz},}$$

Coupled wave equation

$$\Delta k=k_1+k_2-k_3,$$

Wave-vector mismatch

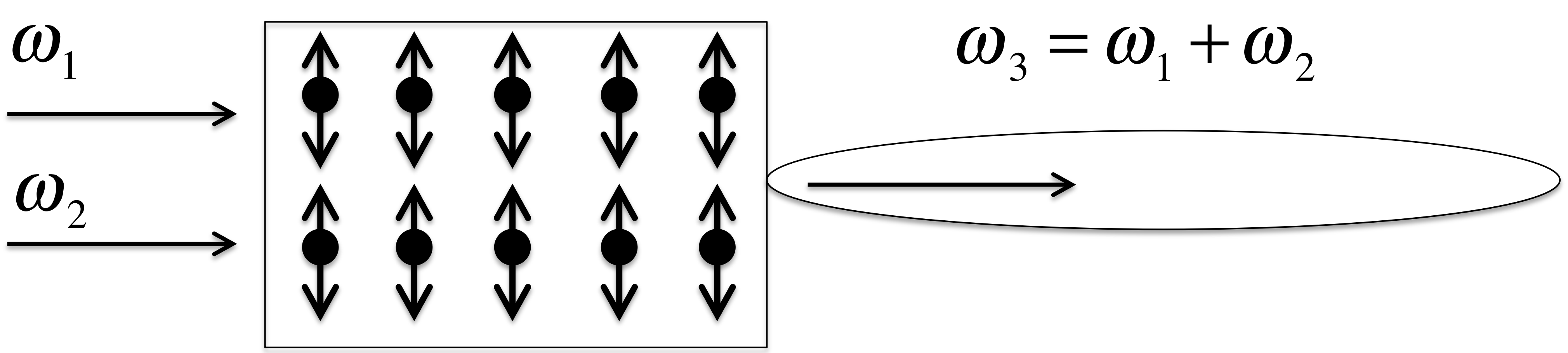
Phase matching

$$\Delta k = 0 \qquad \left(k_1 + k_2 = k_3\right)$$

Perfect phasematching

- coherent addition

$$Power \propto N^2$$



N dipoles

Phase matching

$$\frac{dA_3}{dz} = \frac{2i d_{\text{eff}} \omega_3^2}{k_3 c^2} A_1 A_2 e^{i\Delta k z},$$

Assume A1, A2 constant

$$I_3 = 2 n_3 \varepsilon_o c \left| A_3 \right|^2 = \frac{8 d^2_{eff} \omega^2_3 I_1 I_2}{n_1 n_2 n_3 \varepsilon_o c^2} L^2 \sin c^2 \left(\frac{\Delta k L}{2} \right)$$

$$\Delta k \neq 0 \qquad \left(k_1 + k_2 \neq k_3 \right)$$

Phase mismatch