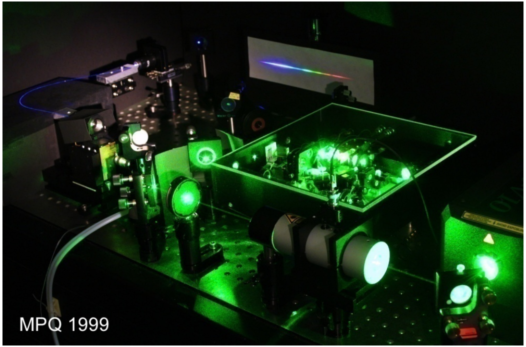
Modules of the 2024 Course

Topics covered	No	Lecture/Date
Introductory presentation; Basic of laser operation I: dispersion theory, atoms	1	11. 09. 2024
Basic of laser operation II: dispersion theory, atoms	2	18. 09. 2024
Laser systems I: 3 and 4 level lasers, gas lasers, solid state lasers, applications	3	25. 09. 2024
Laser systems II: semi-conductor lasers, external cavity lasers, applications	4	02. 10. 2024
Noise characteristics of lasers: linewidth, coherence, phase and amplitude noise, OSA (1)	5	09. 10. 2024
Noise characteristics of lasers: linewidth, coherence, phase and amplitude noise, OSA (2)	6	16. 10. 2024
Optical detection	7	30. 10. 2024
Optical fibers: light propagation in fibers, specialty fibers and dispersion (GVD)	8	06. 11. 2024
Ultrafast lasers I.: Passive mode locking and ultrafast lasers	9	13. 11. 2024
Ultrafast lasers II: mode locking, optical frequency combs / frequency metrology	10	20. 11. 2024
Ultrafast lasers III: pulse characterization, applications	11	27. 11. 2024
Nonlinear frequency conversion I: theory, frequency doubling, applications	12	04. 12. 2024
Nonlinear frequency conversion II: optical parametric amplification (OPA)	13	11. 12. 2024
Laboratory visits (lasers demo)	14	20. 12. 2024

Week 11 content

Content of Week 11

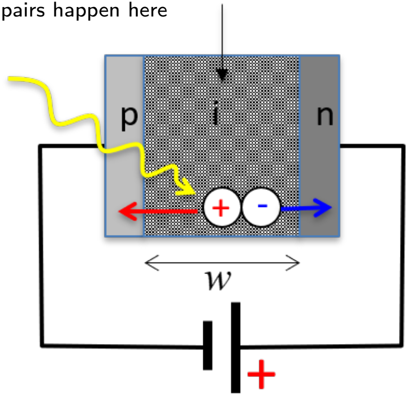
- Limits of direct detection (max. speed).
- Pulse duration using autocorrelation technique
- Full pulse characterization (amplitude, phase, duration) by FROG



Direct detection

What determines the rise time of photodiodes?

Absorption and generation of electron-hole pairs happen here



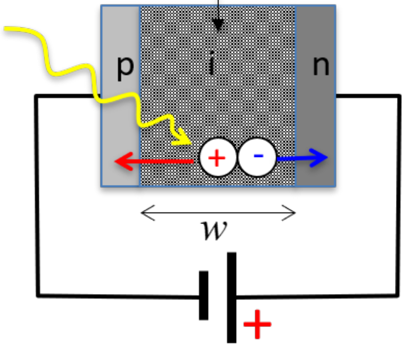
P-i-n photodiode

Max. transient time for the electron (hole) before collection in the circuit:

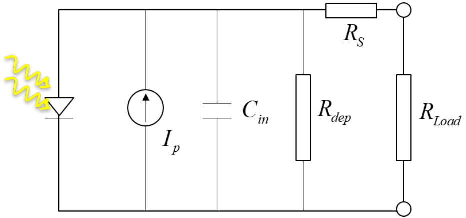
$$w = 40\ \mu\text{m} \quad \tau_{\text{tr}} = 400\ \text{ps}$$
$$w = 4\ \mu\text{m} \quad \tau_{\text{tr}} = 40\ \text{ps}$$

What determines the rise time of photodiodes?

Absorption and generation of electron-hole pairs happen here



P-i-n photodiode



Photodiode $\sim$ two charged plates with depletion region in-between capacitor

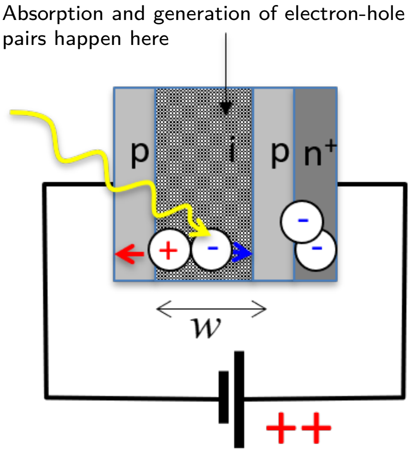
R_{dep}: resistance of the depletion region (very high 1–10's MW)

R_{S}: series resistance of the p and n contacts (very low)

Typical values:

Direct detection

What determines the rise time of photodiodes?



- Reduced width of intrinsic layer = faster transient time to collection circuit
- Lower absorption probability partially compensated with avalanche gain

UPD Series • Available Models

Model	Rise Time (ps)	Band-width (GHz)	Spectral Range (nm)	Quantum Efficiency @ Peak	Sensitive Area (Dia. µm / mm²)	Noise Equiv. Power (W/√Hz)	Dark Current (nA)	Material	Optical Input / Window Type [1]	RF Output Connec-tor Type
UPD-15-IR2-FC	< 15	> 25	800 - 1700	75%	Fiber, 9 µm	1.0×10^{-15}	0.1	InGaAs	Fiber w. FC/APC [5]	SMA
UPD-30-VSG-P	< 30	> 10	320 - 900	40%	200x200 / 0.04	3.0×10^{-15}	0.1	GaAs	Polished, glass	SMA
UPD-35-IR2-P	< 35	> 10	800 -1700	80%	55 / 0.0024	1.0×10^{-15}	0.3	InGaAs	Polished, glass	SMA
UPD-35-IR2-D	< 35	> 10	800 -1700	80%	55 / 0.0024	1.0×10^{-15}	0.3	InGaAs	Diffuse, quartz	SMA
UPD-35-IR2-FR	< 35	> 10	800 -1700	80%	55 / 0.0024	1.0×10^{-15}	0.3	InGaAs	FC/PC receptacle [5]	SMA
UPD-35-IR2-FC	< 35	> 10	800 -1700	80%	Fiber, 9 µm	1.0×10^{-15}	0.3	InGaAs	Fiber w. FC/APC [5]	SMA
UPD-35-UVIR-P	< 35	> 10	**350 -1700**	80%	55 / 0.0024	1.0×10^{-15}	0.3	InGaAs [4]	Polished, MgF_2	SMA
UPD-35-UVIR-D	< 35	> 10	**350 -1700**	80%	55 / 0.0024	1.0×10^{-15}	0.3	InGaAs [4]	Diffuse, quartz	SMA
UPD-300-SP	< 300	> 1.0	320 - 1100	90%	600 / 0.283	3.0×10^{-15}	0.01	Si	Polished, glass	BNC
UPD-300-SD	< 300	> 1.0	320 - 1100	90%	600 / 0.283	3.0×10^{-15}	0.01	Si	Diffuse, quartz	BNC
UPD-300-UP	< 300	> 1.0	**170 - 1100**	90%	600 / 0.283	3.0×10^{-15}	0.01	Si [4]	Polished, MgF_2	BNC
UPD-300-UD	< 300	> 1.0	**170 - 1100**	90%	600 / 0.283	3.0×10^{-15}	0.01	Si [4]	Diffuse, quartz	BNC

Direct detection

How can a fast temporal profile of a few fs be measured?

Direct electronic means are too slow

↓

All optical means

Table 7.2. Examples of data from photodiodes (Antel)

Model (Si PIN)	Rise time [ps]	FWHM pulse response [ps]	−3dB Bandwith [GHz]	Responsivity [V/W]	Responsivity (in terms of density) [V/(W/mm^2)]
AR-S1	< 100	< 180	> 3.5	16	4
AR-S2	< 35	< 65	> 10	5	0.018
AR-S3	< 25	< 45	> 14	3.75	0.014

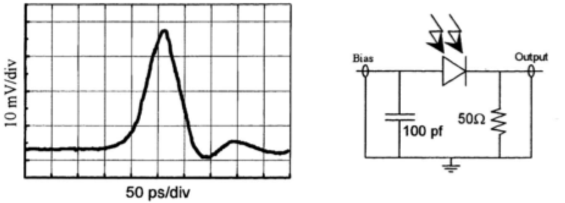
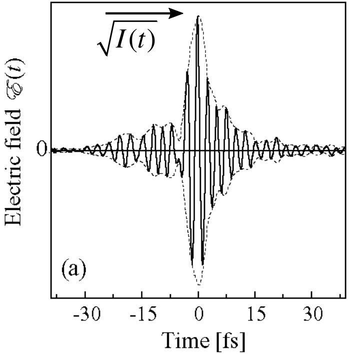


Fig. 7.3. Typical electrical signal given by a photodiode (Antel AR-S2) in response to a femtosecond laser pulse, and the corresponding electric circuit

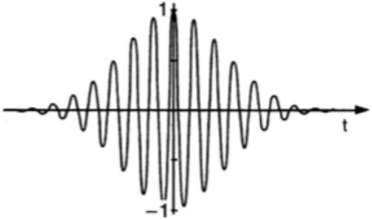
Pulses

Neglecting the spatial dependence for now, the pulse electric field is given by:

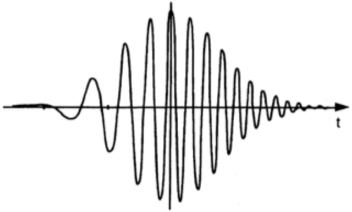
$$E(t) = \mathrm{Re}\left\{ \underbrace{E_0(t)}_{\text{amplitude}=\sqrt{\text{intensity}}} \exp\left\{ i \left[\overbrace{\omega_0}^{\text{Carrier frequency}} t - \underbrace{\phi(t)}_{\text{Phase}} \right] \right\} \right\}$$



Chirped pulses



Transform limited pulse (no chirp)



The pulse is said to be “chirped”

Pulse propagation through media

$$E_0(\omega) = e^{\frac{-(\omega-\omega_0)^2}{4\Gamma}} \quad E(\omega,x) = E_0(\omega)e^{\pm ik(\omega)x}, \quad k(\omega) = \frac{n\omega}{c}$$

$$k(\omega) = k(\omega_0) + k'(\omega-\omega_0) + \frac{1}{2}k''(\omega-\omega_0)^2 + \dots$$

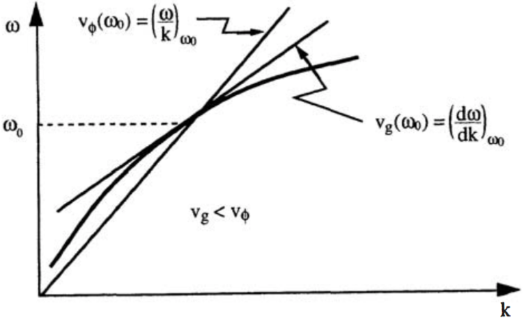
$$E(\omega,x) = \exp\left[-ik(\omega_0)x - ikx(\omega-\omega_0) - \left(\frac{1}{4\Gamma} + \frac{i}{2}k''\right)(\omega-\omega_0)^2\right]$$

$$\varepsilon(t,x) = -\frac{1}{\pi^2}\int_{-\infty}^{+\infty} E(\omega,x)e^{i\omega t}d\omega$$

$$\varepsilon(t,x) = \sqrt{\frac{\Gamma(x)}{\pi}}\exp\left[i\omega_0\left(t - \frac{x}{\nu_\phi(\omega_0)}\right)\right] \times \exp\left[-\Gamma(x)\left(t - \frac{x}{\nu_g(\omega_0)}\right)^2\right]$$

Pulse propagation through media

$$\nu_\phi\left(\omega_0\right)=\left(\frac{\omega}{k}\right)_{\omega_0}, \quad \nu_g\left(\omega_0\right)=\left(\frac{d\omega}{dk}\right)_{\omega_0}, \quad \frac{1}{\Gamma(x)}=\frac{1}{\Gamma}+2ik''x$$



$$\nu_\phi=\frac{c}{n(\omega)}$$

$$\nu_g=\frac{d\omega}{dk}=\frac{1}{dk/d\omega},$$

$$\frac{dk}{d\omega}=\frac{1}{c}\left(n(\omega)+\omega\frac{dn(\omega)}{d\omega}\right)$$

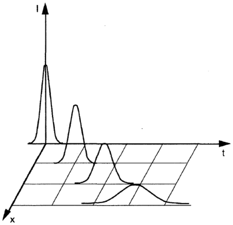
$$\nu_g\approx\nu_\phi\left(1-\frac{\omega}{n(\omega)}\frac{dn(\omega)}{d\omega}\right)$$

Pulse propagation through media

$$\varepsilon(t,x) = \sqrt{\frac{\Gamma(x)}{\pi}} \exp\left[i\omega_0\left(t - \frac{x}{\nu_\phi(\omega_0)}\right)\right] \times \exp\left[-\Gamma(x)\left(t - \frac{x}{\nu_g(\omega_0)}\right)^2\right]$$

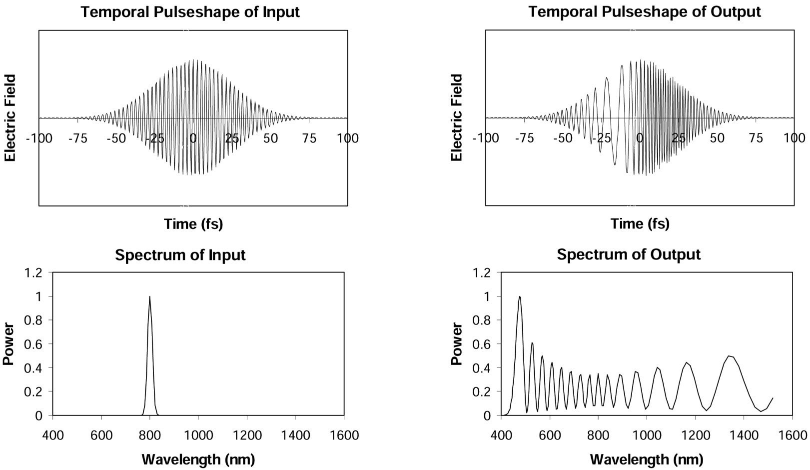
$$\frac{1}{\Gamma(x)} = \frac{1}{\Gamma} + 2ik''x$$

$$k'' = \left(\frac{d^2k}{d\omega^2}\right)_{\omega_0} = \frac{d}{d\omega}\left(\frac{1}{v_g(\omega)}\right) \quad \Gamma(x) = \frac{\Gamma}{1+\xi^2x^2} - i\frac{\xi x}{1+\xi^2x^2}, \quad \xi = 2\Gamma k''$$



Example

Pulse distortion by traveling through glass

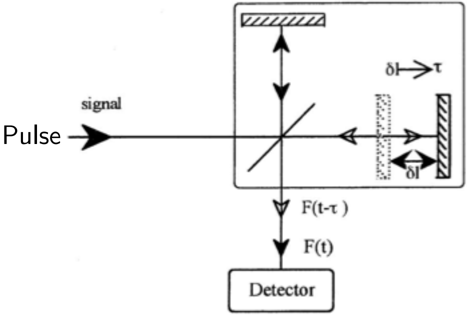


Input pulse: 50 fs, 800 nm, 10 kW peak power
Fibre: 15 cm, 2.5 µm mode diameter

Autocorrelation

All optical Pulse characterization techniques are based on time to space transformations. 1 ps 300 mm in air

7.4.2 & 7.5 (Ruliere, "femtosecond laser pulses")
How to measure characteristics of laser pulses



Second order autocorrelation

$$I_2(\tau) = \int_{-\infty}^{+\infty} \left| \left\{ E(t) e^{i[\omega t + \Phi(t)]} + E(t-\tau) e^{i[\omega(t-\tau) + \Phi(t-\tau)]} \right\}^2 \right|^2 dt$$

$$\begin{aligned} I_2(\tau) = \int_{-\infty}^{+\infty} & |2E^4 + 4E^2(t)E^2(t-\tau) \\ & + 4E(t)E(t-\tau)\left[E^2(t) + E^2(t-\tau)\right]\cos[\omega\tau + \Phi(t) - \Phi(t-\tau)] \\ & + 2E^2(t)E^2(t-\tau)\cos[2(\omega\tau + \Phi(t) - \Phi(t-\tau))]|dt \end{aligned}$$

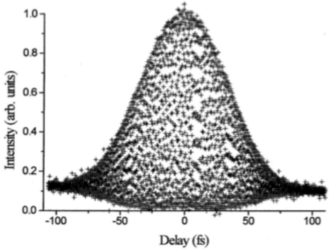


Fig. 7.10. Second-order autocorrelation of some pulse as in Fig. 7.9

$$I_2(\tau = 0) = 2^4 \int E^4(\mathrm{t})\mathrm{dt}$$

$$I_2(\tau \to \infty) = 2 \int E^4(\mathrm{t})\mathrm{dt}$$

Second order autocorrelation

fringes
Are spatially averaged
(same effect as fast sweep)

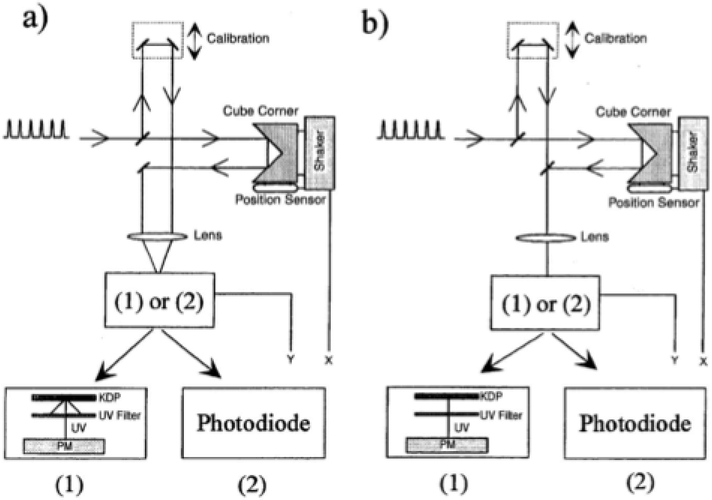
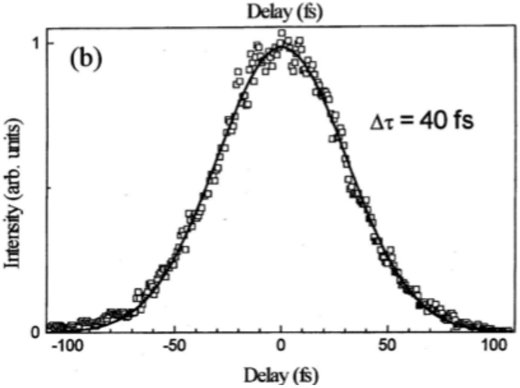


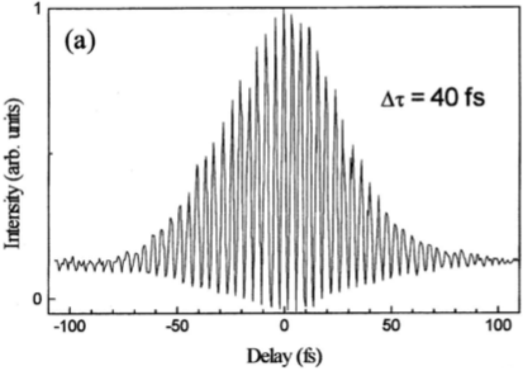
Fig. 7.13. Basic principle of second-order autocorrelator: noncolinear (**a**) and colinear (**b**) arrangements

Second order autocorrelation

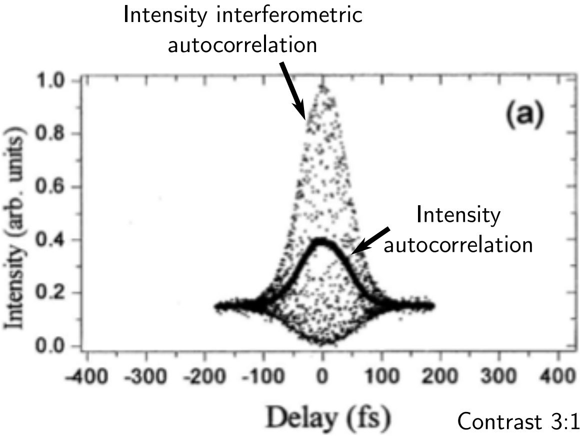
Second order intensity autocorrelation



Second order interferometric autocorrelation



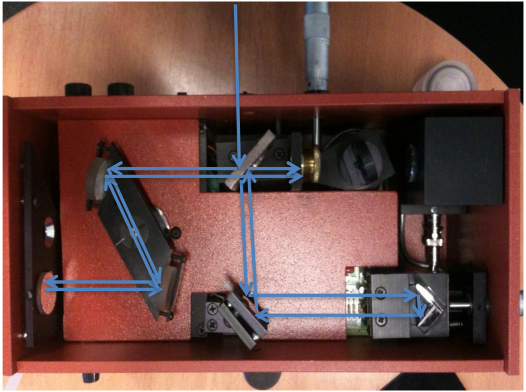
Interferometric vs. Intensity 2nd Order Autocorr.



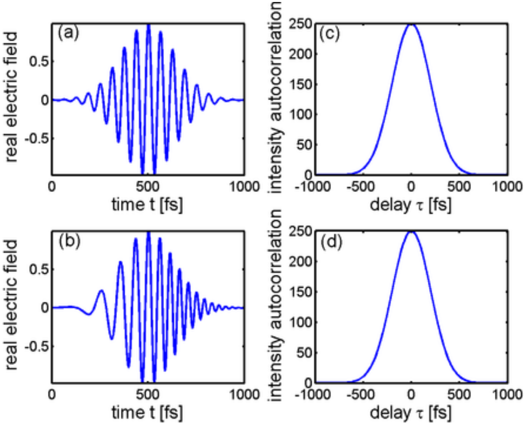
Contrast 3:1

$$G_2(\tau) = \frac{\int_{-\infty}^{+\infty} I(t)I(\mathrm{t}-\tau)dt}{\int_{-\infty}^{+\infty} I^2(t)dt}$$

Second-order Autocorrelation

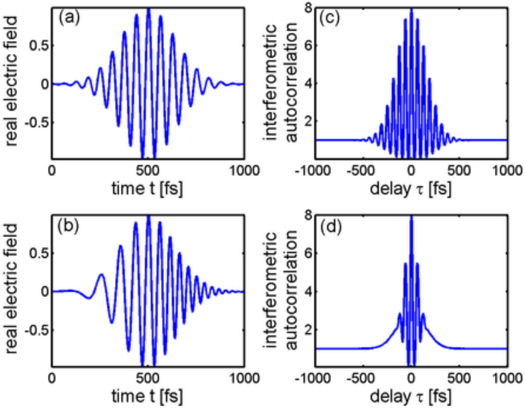


Second-order Autocorrelation



Two ultrashort pulses (a) and (b) with their respective intensity autocorrelation (c) and (d). Because the intensity autocorrelation ignores the temporal phase of pulse (b) that is due to the instantaneous frequency sweep (chirp), both pulses yield **the same intensity autocorrelation**. The zero in this figure has been shifted to omit this background.

Second-order Autocorrelation

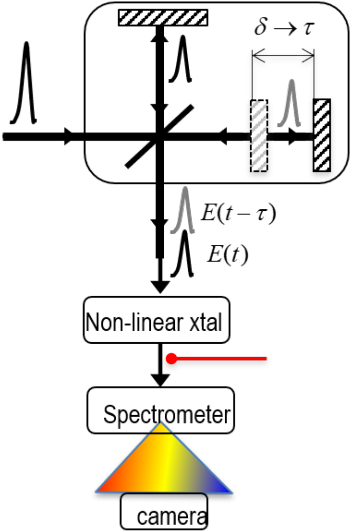


Two ultrashort pulses (a) and (b) with their respective interferometric 2nd order autocorrelation (c) and (d). Because of the phase present in pulse (b) due to an instantaneous frequency sweep (chirp), the fringes of the autocorrelation trace (d) wash out in the wings. Note the ratio 8:1 (peak to the wings), characteristic of interferometric autocorrelation traces.

Second-order Autocorrelation

$I(t)$	Δt	$I(\omega)$	$\Delta\omega$	$\Delta\omega\cdot\Delta t$	$G_2(\tau)$	$\Delta\tau$	$\Delta\tau/\Delta t$
e^{-t^2}	1.665	$e^{-\omega^2}$	1.665	2.772	$e^{-\frac{\tau^2}{2}}$	2.355	1.414
$\mathrm{sech}^2(t)$	1.763	$\mathrm{sech}^2\left(\frac{\pi}{2}\omega\right)$	1.122	1.978	$\frac{3[\tau\cosh(\tau)-\sinh(\tau)]}{\sinh^3(\tau)}$	2.720	1.543
$\frac{1}{e^{t/(t-A)}+e^{-t/(t-A)}}$ A=1/4	1.715	$\frac{1+1/\sqrt{2}}{\cosh\left(\frac{15\pi}{16}\omega\right)+1/\sqrt{2}}$	1.123	1.925	$\frac{1}{\cosh^3\left(\frac{8}{15}\tau\right)}$	2.648	1.544
A=1/2	1.565	$\mathrm{sech}\left(\frac{3\pi}{4}\omega\right)$	1.118	1.749	$\frac{3\sinh(\frac{8}{3}\tau)-8\tau}{4\sinh^3\left(\frac{4}{3}\tau\right)}$	2.424	1.549
A=3/4	1.278	$\frac{1-1/\sqrt{2}}{\cosh\left(\frac{7\pi}{16}\omega\right)-1/\sqrt{2}}$	1.088	1.391	$\frac{2\cosh(\frac{16}{7}\tau)+3}{5\cosh^3\left(\frac{8}{7}\tau\right)}$	2.007	1.570

Frequency Resolved Optical Gating



Frequency Resolved Optical Gating (FROG) is a method that fully resolves the Amplitude and Phase of the pulse

Procedure for FROG:

$$I_{FROG}(\omega,\tau) = \left| \sum_{-\infty}^{+\infty} E_{sig}(t,\tau) e^{i\omega t} dt \right|^2$$

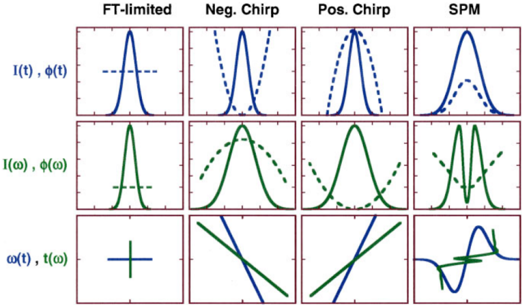
Determine $E_{sig}^{SHG}(t,\tau)$ from $I_{FROG}(\omega,\tau)$

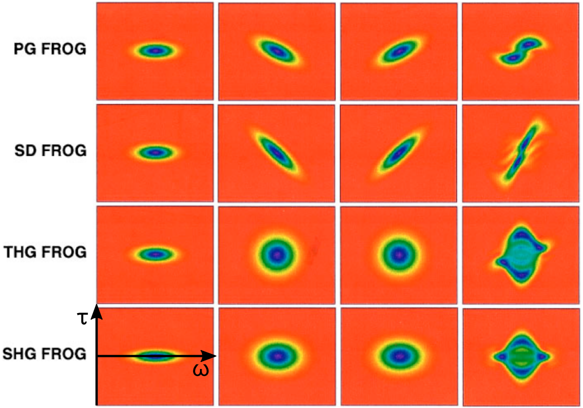
$$E(t) = E_{sig}^{SHG}(t,\tau=0)$$

Different ways for generating $E_{\text{sig}}^{\text{SHG}}(t,\tau)$

$$E_{\text{sig}}(t,\tau) \propto \begin{cases} E(t)|E(t-\tau)|^2 & \text{for PG FROG} \\ E(t)^2 E^*(t-\tau) & \text{for SD FROG} \\ E(t)E(t-\tau) & \text{for SHG FROG} \\ E(t)^2 E(t-\tau) & \text{for THG FROG} \end{cases}$$

FROG traces


FT-limited
Neg. Chirp
Pos. Chirp
SPM
I(t) , ϕ(t)
I(ω) , ϕ(ω)
ω(t) , t(ω)


PG FROG
SD FROG
THG FROG
SHG FROG
τ
ω

Phase retrieval: FROG

The determination of $E_{\text{sig}}(t,\tau)$ from the intensity $I_{\text{FROG}}(\omega,\tau)$ is a phase retrieval problem, i.e finding the amplitude and phase of $E_{\text{sig}}(t,\tau)$, for each t and τ such that the magnitude Square of its Fourier transform is the measured quantity $I_{\text{FROG}}(\omega,\tau)$.

This is done iteratively with an algorithm:

$$\sqrt{I_{FROG}\left(\omega_i,\tau_j\right)} = \operatorname{Re}\left(E_{sig}^{(k)}\left(\omega_i,\tau_j\right)\right)$$

$$E_{sig}^{(k)}\left(t_i,\tau_j\right) = \sum E_{sig}^{(k)}\left(\omega,\tau_j\right) \times e^{i\omega t_i} d\omega$$

Then minimize $Z = \sum_{i,j=1}^{N} \left| E_{sig}^{(k)}\left(t_i,t_j\right) - E^{k+1}\left(t_i\right) E^{k+1}\left(t_i - t_j\right) \right|^2$

Find $E^{k+1}\left(t_i\right)$

Compute $E_{sig}^{(k+1)}\left(t_i,t_j\right) = E^{k+1}\left(t_i\right) E^{k+1}\left(t_i - t_i\right)$