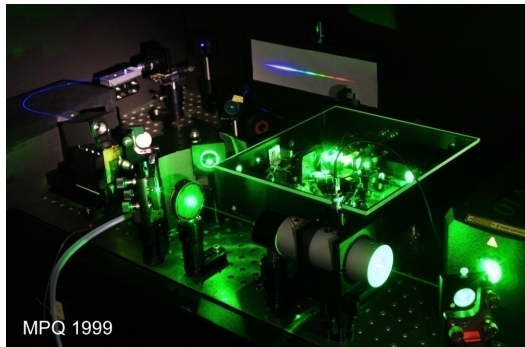


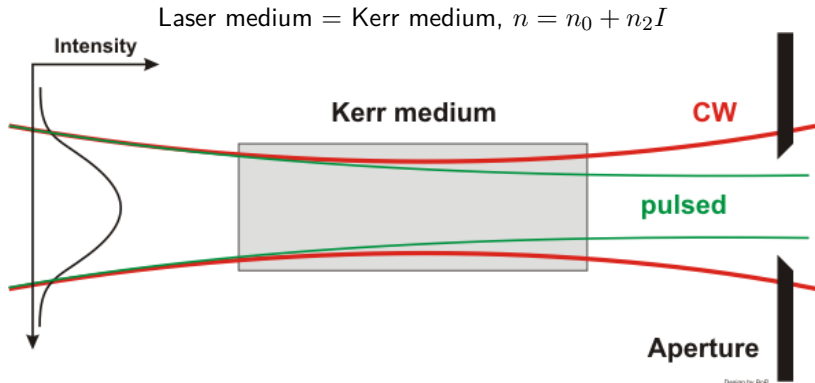
Topics covered	No	Lecture/Date
Introductory presentation; Basic of laser operation I: dispersion theory, atoms	1	11. 09. 2024
Basic of laser operation II: dispersion theory, atoms	2	18. 09. 2024
Laser systems I: 3 and 4 level lasers, gas lasers, solid state lasers, applications	3	25. 09. 2024
Laser systems II: semi-conductor lasers, external cavity lasers, applications	4	02. 10. 2024
Noise characteristics of lasers: linewidth, coherence, phase and amplitude noise, OSA (1)	5	09. 10. 2024
Noise characteristics of lasers: linewidth, coherence, phase and amplitude noise, OSA (2)	6	16. 10. 2024
Optical detection	7	30. 10. 2024
Optical fibers: light propagation in fibers, specialty fibers and dispersion (GVD)	8	06. 11. 2024
Ultrafast lasers I.: Passive mode locking and ultrafast lasers	9	13. 11. 2024
Ultrafast lasers II: mode locking, optical frequency combs / frequency metrology	10	20. 11. 2024
Ultrafast lasers III: pulse characterization, applications	11	27. 11. 2024
Nonlinear frequency conversion I: theory, frequency doubling, applications	12	04. 12. 2024
Nonlinear frequency conversion II: optical parametric amplification (OPA)	13	11. 12. 2024
Laboratory visits (lasers demo)	14	20. 12. 2024

Content of Week 10

- Characterizing pulses using autocorrelation technique
- Mode locking via saturable absorbers
- Frequency Metrology
- Pulse propagation
- Dual-Comb Spectroscopy



Passive mode locking with Kerr lens effect



Mode locking: saturable absorber

Saturable absorber

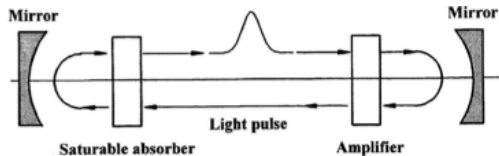
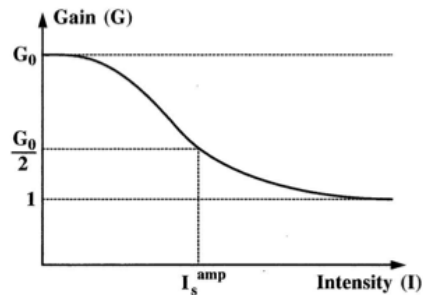
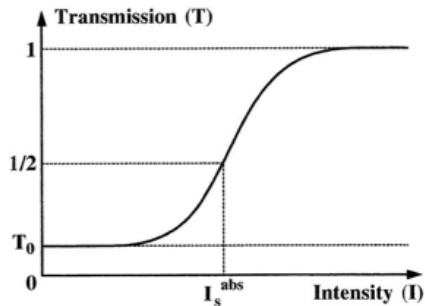


Fig. 3.12. Round-trip pulse in a laser cavity including saturable absorber and amplifying medium



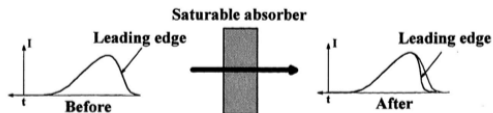


Fig. 3.15. Illustration of pulse shape modification after crossing a saturable absorber

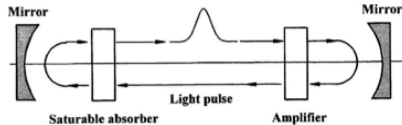


Fig. 3.12. Round-trip pulse in a laser cavity including saturable absorber and amplifying medium

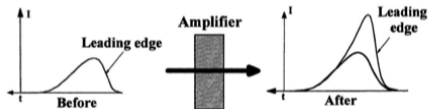
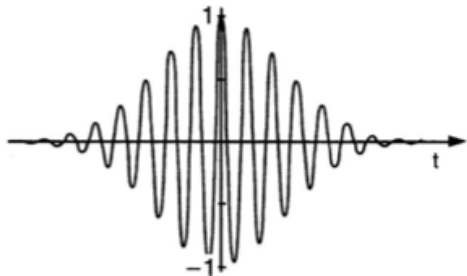
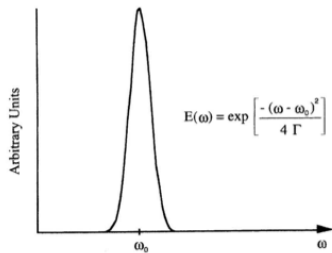


Fig. 3.16. Illustration of pulse shape modification after crossing an amplifying medium

One Gaussian Pulse



$$E_P(t) = E_0 e^{-\Gamma t^2 + i\omega_0 t}$$



$$\tilde{E}_P(\omega) = E_0 \sqrt{\frac{\pi}{\Gamma}} \exp\left(\frac{-(\omega - \omega_0)^2}{4\Gamma}\right)$$

Per-pulse change of phase:

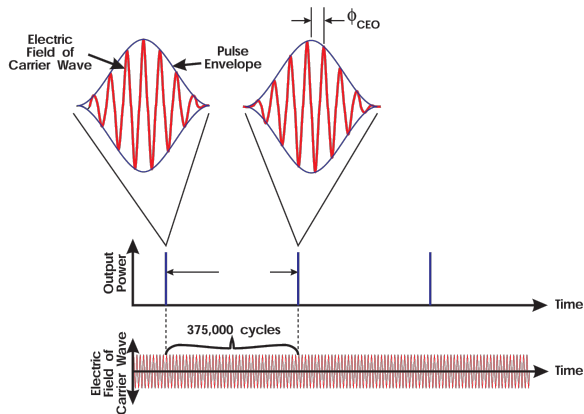
$$\begin{aligned}\Delta\phi &= \beta_0 L - \omega_0 T_{\text{rep}} \\ &= \beta_0 L - \frac{\omega_0 L}{v_g} \\ &= \omega_0 L \left(\frac{\beta_0}{\omega_0} - \frac{1}{v_g} \right) \\ &= \frac{\omega_0 L}{c} (n_0 - n_g)\end{aligned}$$

The group-index

$$n_g = n_0 + \omega_0 \left. \frac{dn}{d\omega} \right|_{\omega_0}$$

Phase change per time:
the carrier-envelope offset frequency

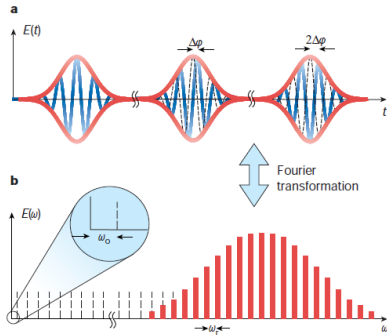
$$\omega_{\text{CEO}} = \frac{\Delta\phi}{T_{\text{rep}}}$$



$$E_T(t) = \sum_{m=0}^{N-1} E_P(t - mT_{\text{rep}}) e^{-i\omega_{\text{CO}} t}, \quad \omega_{\text{CO}} = \frac{\Delta\phi}{T_{\text{rep}}}$$

$$\tilde{E}_T(\omega) = \text{FT} [E_T(t)] = \sum_{m=0}^{N-1} \int_{-\infty}^{\infty} E_P(t - mT_{\text{rep}}) e^{-i\omega_{\text{CO}} t} e^{-i\omega t} dt$$

Translation property:
unreadable (duality)



$$\begin{aligned} I_T(\omega) &= I_P(\omega + \omega_{\text{CO}}) \frac{1 - \cos(2N\pi(\omega + \omega_{\text{CO}})/\omega_{\text{rep}})}{1 - \cos(2\pi(\omega + \omega_{\text{CO}})/\omega_{\text{rep}})} \\ &= \tilde{E}_P(\omega + \omega_{\text{CO}}) \frac{1 - e^{-iNT_{\text{rep}}(\omega + \omega_{\text{CO}})}}{1 - e^{-iT_{\text{rep}}(\omega + \omega_{\text{CO}})}} \end{aligned}$$

$$\omega_{\text{rep}} = \frac{2\pi}{T_{\text{rep}}}$$

$$I_T(\omega) = I_P(\omega + \omega_{\text{CO}}) \frac{1 - \cos(2N\pi(\omega + \omega_{\text{CO}})/\omega_{\text{rep}})}{1 - \cos(2\pi(\omega + \omega_{\text{CO}})/\omega_{\text{rep}})}$$

$$\lim_{N \rightarrow \infty} I_T(\omega) = I_P(\omega + \omega_{\text{CO}}) \sum_{m=-\infty}^{\infty} \delta(\omega + \omega_{\text{CO}} - m\omega_{\text{rep}})$$

A frequency comb with offset ω_{CO}

Starting from offset frequency comb:

$$\tilde{E}_T(\omega) = \tilde{E}_P(\omega) \sum_{m=-\infty}^{\infty} \delta(\omega + \omega_{CO} - m\omega_{rep})$$

FT of multiplication = convolution of FT:

$$E_T(t) = \text{FT} [\tilde{E}_P(\omega)] \# \text{FT} \left[\sum_{m=-\infty}^{\infty} \delta(\omega + \omega_{CO} - m\omega_{rep}) \right]$$

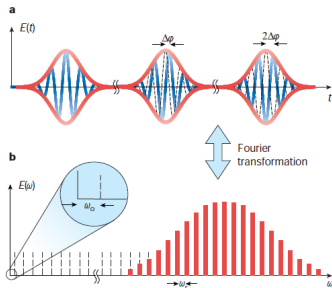
FT of Dirac Comb = Dirac Comb:

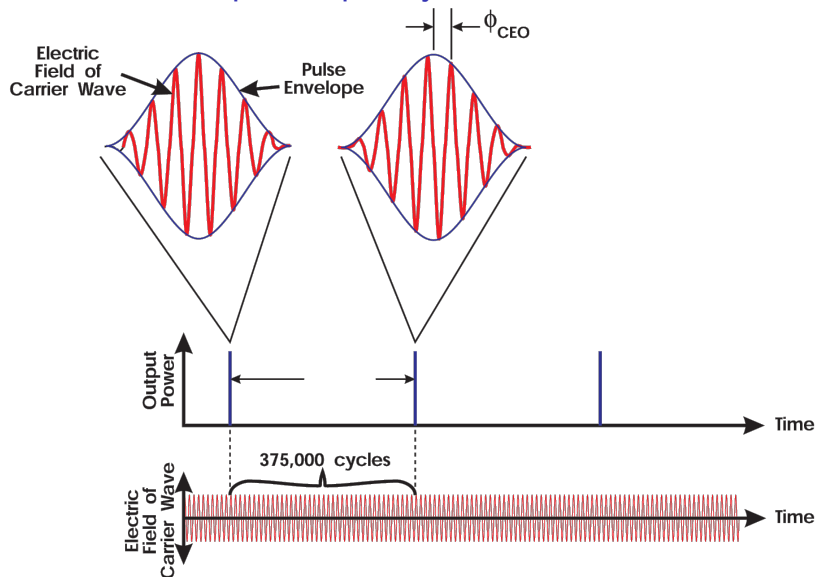
$$= E_P(t) \# \left[\sum_{m=-\infty}^{\infty} \delta(t - mT_{rep}) e^{-i\omega_{CO}t} \right]$$

Convolution with Dirac Comb = “Replicating Property”

$$= \sum_{m=-\infty}^{\infty} E_P(t - mT_{rep}) e^{-im\Delta\phi}$$

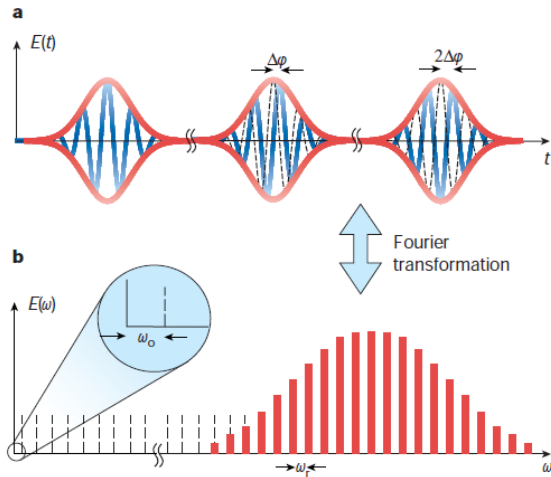
Back to our original pulse-train





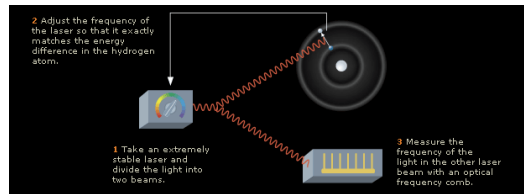
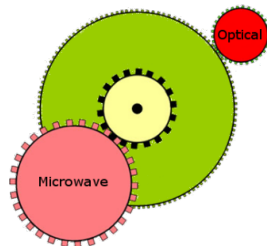
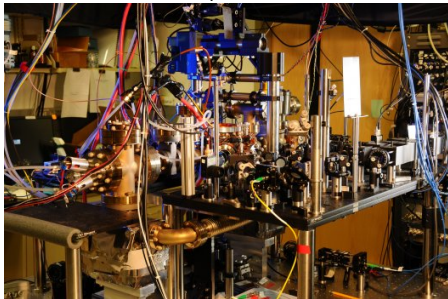
$$\nu_{CEO} = \frac{d\Phi}{dt} = \frac{\Delta\Phi}{2\pi T}$$

$$\nu_m = m f_{rep} + \nu_{CEO}$$

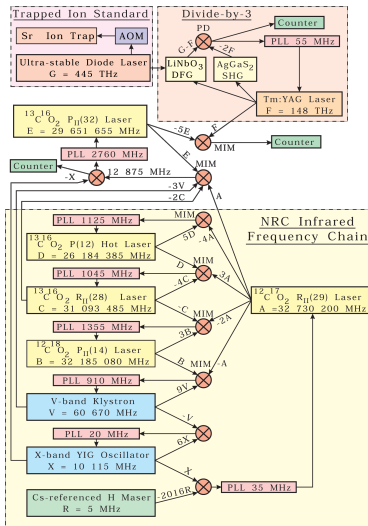


Applications of ultra-short pulses: Optical Clocks

Optical atomic clocks (NIST, Boulder, Colorado)



Counting the cycles of light



First Phase-Coherent Frequency Measurement of Visible Radiation

H. Schnitz, B. Lipphardt, J. Helmcke, F. Riehle, and G. Zinner
Physikalisch-Technische Bundesanstalt (PTB), D-38115 Braunschweig, Germany
(Received 10 August 1995)

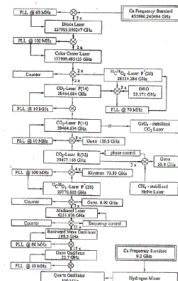
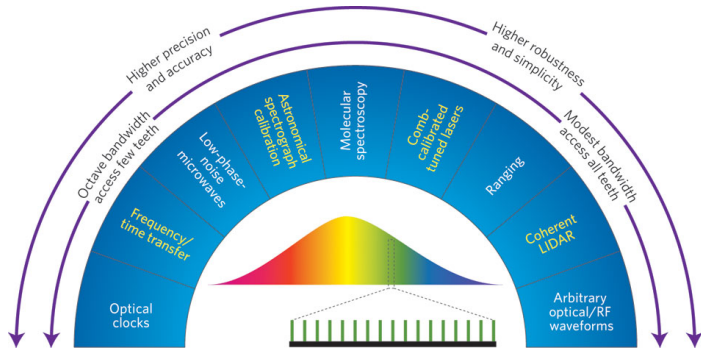
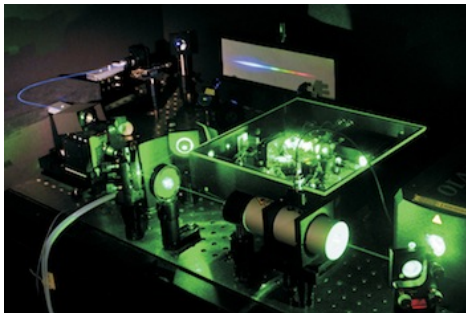
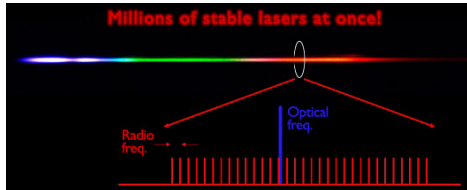
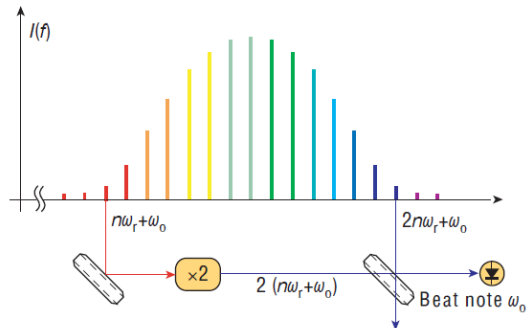


FIG. 1. PTB's frequency chain in the Ca intercombination line (PLL = phase locked loop, details are given in the text).

Applications of frequency combs

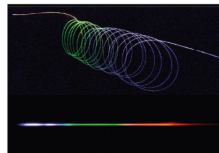
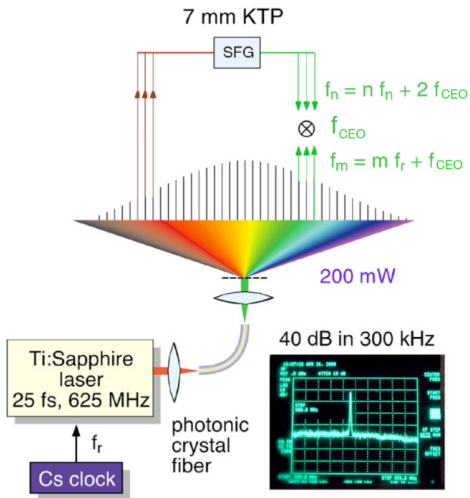




$$f_n = n f_{\text{rep}} + f_0$$

$$2f_n - f_{2n} = 2(n f_{\text{rep}} + f_0) - (2n f_{\text{rep}} + f_0) = f_0$$

Measuring the carrier envelope frequency



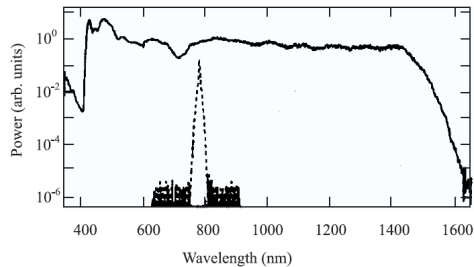
„Rainbow Fiber“

(Lucent Technologies, 1999)



“Photonic Crystal Fiber”

J.C. Knight, W.J. Wadsworth, P. St. Russell
University of Bath, UK



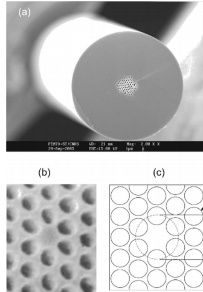
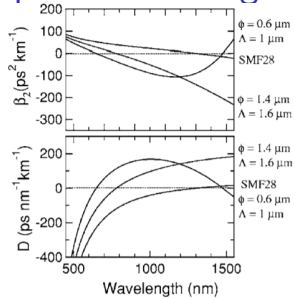


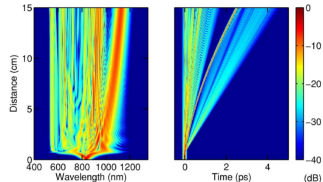
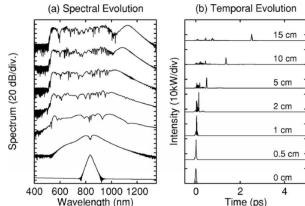
FIG. 2. Calculated GVD parameter β_2 (top) and D (bottom) for PCFs with parameters as shown: $\Lambda=1 \mu\text{m}$, $\phi=0.6 \mu\text{m}$; $\Lambda=1.6 \mu\text{m}$, $\phi=1.4 \mu\text{m}$ [that used by Ranka *et al.* (2000a)]; and standard Corning SMF28 single-mode fiber.

REVIEWS OF MODERN PHYSICS, VOLUME 78, OCTOBER–DECEMBER 2006

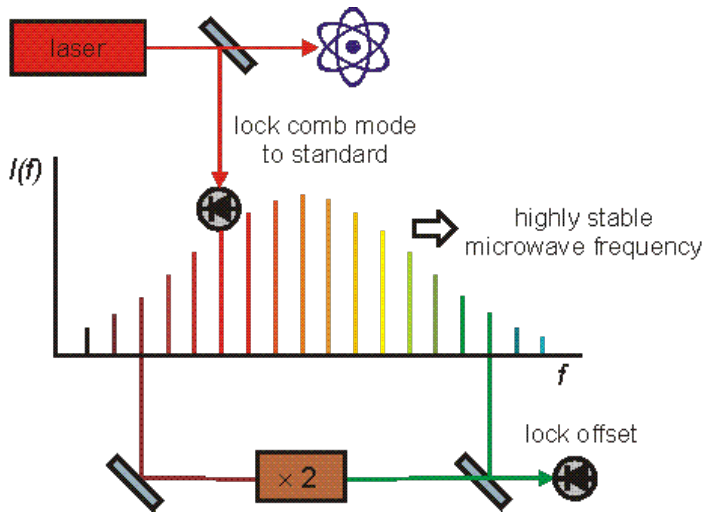
Supercontinuum generation in photonic crystal fiber

Super-continuum generation can be modeled via a generalized Nonlinear Schrödinger equation.

$$\frac{dA}{dz} + \frac{\alpha}{2} A - \sum_{k \geq 2} \frac{i^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k} = i\gamma \left(1 + i\tau \frac{\partial}{\partial T}\right) \cdot \left(A(z, T) \int_{-\infty}^{\infty} R(T') |A(z, T - T')|^2 dT' + i\Gamma_R(z, T)\right)$$



Principle of an atomic clock



An Optical Clock Based on a Single Trapped $^{199}\text{Hg}^+$ Ion

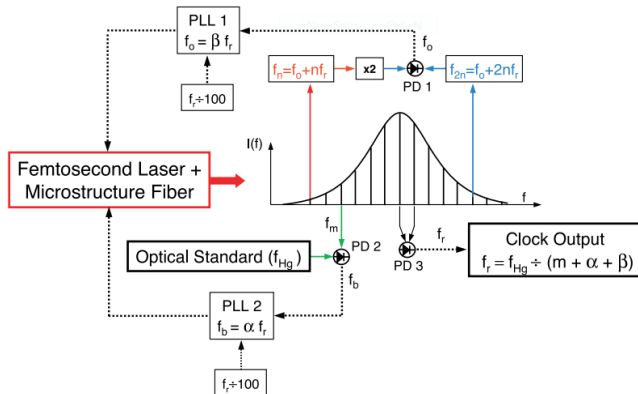
S. A. Diddams,^{1*} Th. Udem,^{1†} J. C. Bergquist,¹ E. A. Curtis,^{1,2}
R. E. Drullinger,¹ L. Hollberg,¹ W. M. Itano,¹ W. D. Lee,¹
C. W. Oates,¹ K. R. Vogel,¹ D. J. Wineland¹

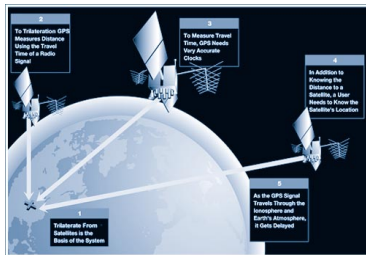


A clockwork based on a mode-locked femtosecond laser provides output pulses at a 1-gigahertz rate that are phase-coherently locked to the optical frequency. By comparison to a laser-cooled calcium optical standard, an upper limit for the fractional frequency instability of $7 \cdot 10^{-15}$ is measured in 1 second of averaging – a value substantially better than that of the world's best microwave atomic clocks.

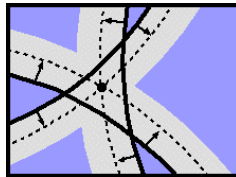
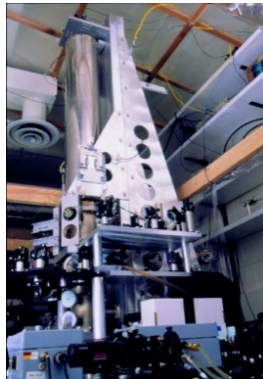
Fractional frequency instability:

$$\sigma_y(\tau) \approx \left\langle \frac{\Delta\nu_{\text{rms}}}{\nu_0} \right\rangle_\tau$$





1999 – NIST-F1 begins operation with an uncertainty of $1.7 \cdot 10^{-15}$, or accuracy to about one second in 20 million years, making it one of the most accurate clocks ever made (a distinction shared with similar standards in France and Germany).

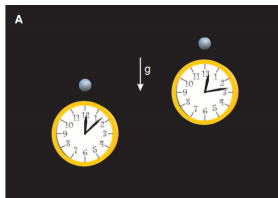


Three spheres are necessary to find position in two dimensions, four are needed in three dimensions.

Optical Clocks and Relativity

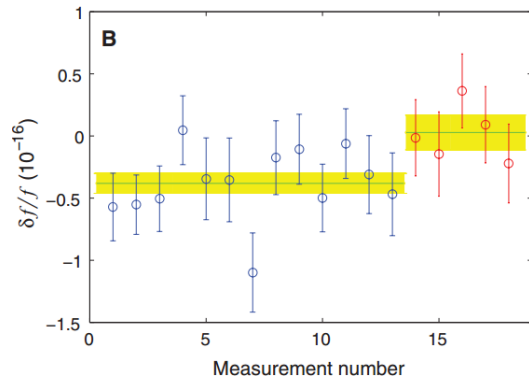
C. W. Chou,* D. B. Hume, T. Rosenband, D. J. Wineland

Observers in relative motion or at different gravitational potentials measure disparate clock rates. These predictions of relativity have previously been observed with atomic clocks at high velocities and with large changes in elevation. We observed time dilation from relative speeds of less than 10 meters per second by comparing two optical atomic clocks connected by a 75-meter length of optical fiber. We can now also detect time dilation due to a change in height near Earth's surface of less than 1 meter. This technique may be extended to the field of geodesy, with applications in geophysics and hydrology as well as in space-based tests of fundamental physics.



$$\sigma_y(\tau) \approx \left\langle \frac{\Delta v_{\text{rms}}}{v_0} \right\rangle_{\tau}$$

Gravitational time dilation at the scale of daily life.
(A) As one of the clocks is raised, its rate increases when compared to the clock rate at deeper gravitational potential



Autocorrelation techniques

How can a fast temporal profile of a few fs be measured?

Direct electronic means are too slow



All optical means

Table 7.2. Examples of data from photodiodes (Antel)

Model (Si PIN)	Rise time [ps]	FWHM pulse response [ps]	-3dB Bandwidth [GHz]	Responsivity [V/W]	Responsivity (in terms of density) [V/(W/mm ²)]
AR-S1	< 100	< 180	> 3.5	16	4
AR-S2	< 35	< 65	> 10	5	0.018
AR-S3	< 25	< 45	> 14	3.75	0.014

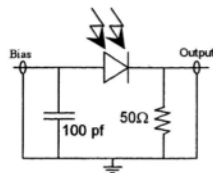
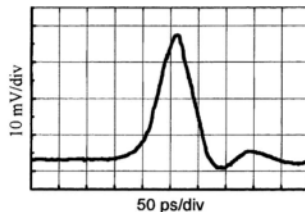
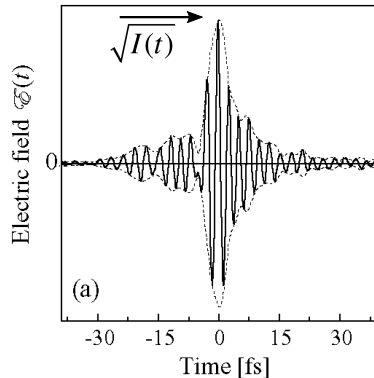


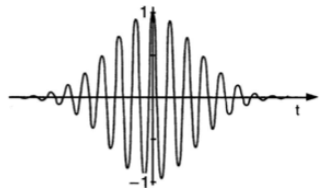
Fig. 7.3. Typical electrical signal given by a photodiode (Antel AR-S2) in response to a femtosecond laser pulse, and the corresponding electric circuit

Neglecting the spatial dependence for now, the pulse electric field is given by:

$$E(t) = \text{Re} \left\{ \underbrace{\sqrt{I(t)}}_{\text{Intensity}} \underbrace{\exp \left\{ i \left[\underbrace{\omega_0}_{\text{Carrier frequency}} t - \underbrace{\phi(t)}_{\text{Phase}} \right] \right\}}_{\text{Phase}} \right\}$$

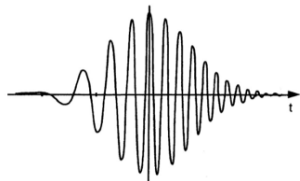


$$E_y = \text{Re} \left(E_0 e^{(-\Gamma t^2 + i\omega_0 t)} \right)$$



Instantaneous frequency:

$$E_y = \text{Re} \left(E_0 e^{[-\Gamma t^2 + i(\omega_0 t - at^2)]} \right)$$



Instantaneous frequency:

The pulse is said to be “chirped”

$$E_0(\omega) = e^{-\frac{(\omega-\omega_0)^2}{4\Gamma}} \quad E(\omega, x) = E_0(\omega)e^{\pm ik(\omega)x}, \quad k(\omega) = \frac{n\omega}{c}$$

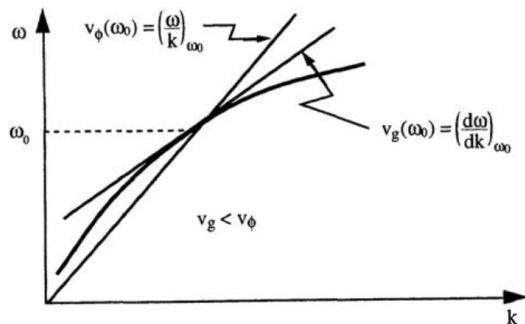
$$k(\omega) = k(\omega_0) + k'(\omega - \omega_0) + \frac{1}{2}k''(\omega - \omega_0)^2 + \dots$$

$$E(\omega, x) = \exp \left[-ik(\omega_0)x - ikx(\omega - \omega_0) - \left(\frac{1}{4\Gamma} + \frac{i}{2}k'' \right) (\omega - \omega_0)^2 \right]$$

$$\varepsilon(t, x) = -\frac{1}{\pi^2} \int_{-\infty}^{+\infty} E(\omega, x) e^{i\omega t} d\omega$$

$$\varepsilon(t, x) = \sqrt{\frac{\Gamma(x)}{\pi}} \exp \left[i\omega_0 \left(t - \frac{x}{\nu_\phi(\omega_0)} \right) \right] \times \exp \left[-\Gamma(x) \left(t - \frac{x}{\nu_g(\omega_0)} \right)^2 \right]$$

$$\nu_{\phi}(\omega_0) = \left(\frac{\omega}{k} \right)_{\omega_0}, \quad \nu_g(\omega_0) = \left(\frac{d\omega}{dk} \right)_{\omega_0}, \quad \frac{1}{\Gamma(x)} = \frac{1}{\Gamma} + 2ik''x$$



$$\nu_{\phi} = \frac{c}{n(\omega)}$$

$$\nu_g = \frac{d\omega}{dk} = \frac{1}{dk/d\omega},$$

$$\frac{dk}{d\omega} = \frac{1}{c} \left(n(\omega) + \omega \frac{dn(\omega)}{d\omega} \right)$$

$$\nu_g \approx \nu_{\phi} \left(1 - \frac{\omega}{n(\omega)} \frac{dn(\omega)}{d\omega} \right)$$

Slowing of light pulses

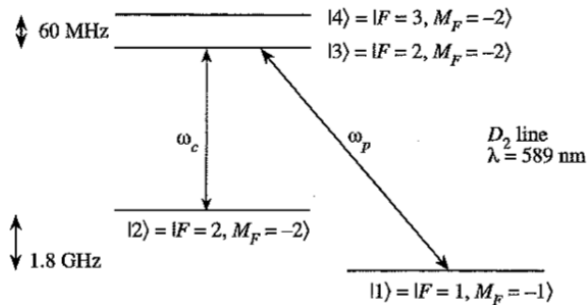
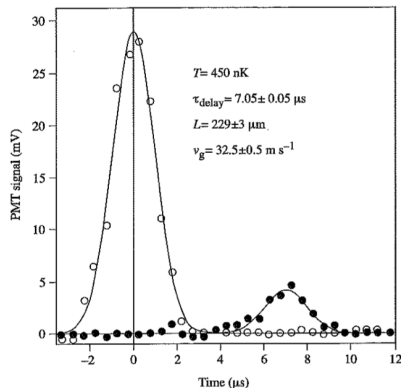


Figure 5.6. Experimental data of Hau *et al* showing the input pulse (open circles) and the pulse transmitted by an ultracold gas of length $229 \pm 3 \mu\text{m}$ (filled circles). The transmitted pulse delay of $7.05 \pm 0.05 \mu\text{s}$ corresponds to a group velocity of $32.5 \pm 0.5 \text{ m s}^{-1}$. From [179], with permission.

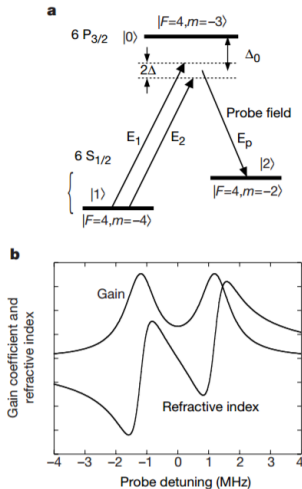
Superluminal group velocity

Gain-assisted superluminal light propagation

L. J. Wang, A. Kuzmich & A. Dogariu

NEC Research Institute, 4 Independence Way, Princeton, New Jersey 08540, USA

Einstein's theory of special relativity and the principle of causality¹⁻⁴ imply that the speed of any moving object cannot exceed that of light in a vacuum (c). Nevertheless, there exist various proposals⁵⁻¹⁸ for observing faster-than- c propagation of light pulses, using anomalous dispersion near an absorption line^{4,6-8}, nonlinear⁹ and linear gain lines¹⁰⁻¹⁸, or tunnelling barriers¹⁹. However, in all previous experimental demonstrations, the light pulses experienced either very large absorption⁷ or severe reshaping^{8,19}, resulting in controversies over the interpretation. Here we use gain-assisted linear anomalous dispersion to demonstrate superluminal light propagation in atomic caesium gas. The group velocity of a laser pulse in this region exceeds c and can even become negative^{16,17}, while the shape of the pulse is preserved. We measure a group-velocity index of $n_g = -310 (\pm 5)$; in practice, this means that a light pulse propagating through the atomic vapour cell appears at the exit side so much earlier than if it had propagated the same distance in a vacuum that the peak of the pulse appears to leave the cell before entering it. The observed superluminal light pulse propagation is not at odds with causality, being a direct consequence of classical interference between its different frequency components in an anomalous dispersion region.

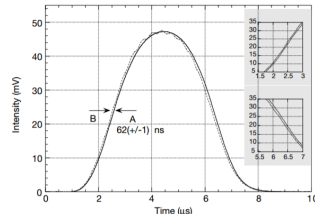


$$\nu_\phi = \frac{c}{n(\omega)}_1$$

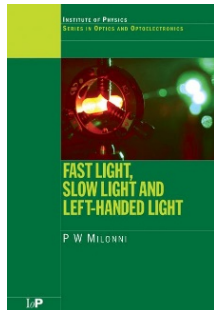
$$\nu_g = \frac{d\omega}{dk} = \frac{c}{dk/d\omega},$$

$$\frac{dk}{d\omega} = \frac{1}{c} \left(n(\omega) + \omega \frac{dn(\omega)}{d\omega} \right)$$

$$\nu_g \approx \nu_\phi \left(1 - \frac{\omega}{n(\omega)} \frac{dn(\omega)}{d\omega} \right)$$



Energy propagation velocity



Energy propagation velocity

$$|S_\omega| = \frac{n}{2\mu c} |E_\omega|^2 = \nu_g u_\omega$$

$$v_E \equiv |S_\omega| / u_\omega$$

Electron oscillator model:

$$m \left(\ddot{r} + \Gamma \dot{r} + \omega_0^2 r \right) = eE$$

$$\nu_E = \frac{c}{n_R + 2\omega n_I / \Gamma} = \frac{\nu_p}{1 + 2\omega n_I / n_R \Gamma}$$

Lossless limit:

$$\nu_E = c \left[n_R + \frac{\omega^2 \omega_p^2 / n_R}{(\omega_0^2 - \omega^2)^2} \right]^{-1} = \frac{c}{n_R + \omega \cdot dn_R / d\omega} = v_g$$

The speed of information in a 'fast-light' optical medium

Michael D. Stenner¹, Daniel J. Gauthier¹ & Mark A. Neifeld²

¹Duke University Department of Physics, and The Fitzpatrick Center for Photonics and Communication Systems, Durham, North Carolina 27708, USA
²Department of Electrical and Computer Engineering, The Optical Sciences Center, University of Arizona, Tucson, Arizona 85721, USA

One consequence of the special theory of relativity is that no signal can cause an effect outside the source light cone, the space-time surface on which light rays emanate from the source¹. Violations of this principle of relativistic causality leads to paradoxes, such as that of an effect preceding its cause². Recent experiments on optical pulse propagation in so-called 'fast-light' media—which are characterized by a wave group velocity v_g exceeding the vacuum speed of light c or taking on negative values³—have led to renewed debate about the definition of the information velocity v_i . One view is that $v_i = v_g$ (ref. 4), which would violate causality, while another is that $v_i = c$ in all situations⁵, which would preserve causality. Here we find that the time to detect information propagating through a fast-light medium is slightly longer than the time required to detect the same information travelling through a vacuum, even though v_g in the medium vastly exceeds c . Our observations are therefore consistent with relativistic causality and help to resolve the controversies surrounding superluminal pulse propagation.

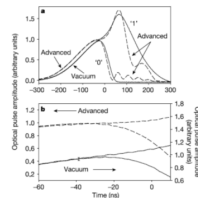
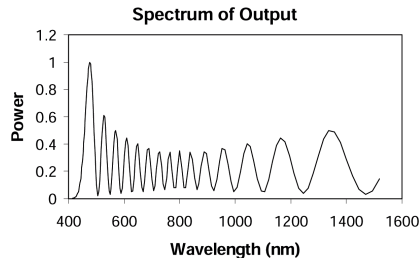
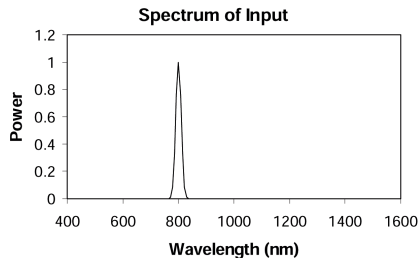
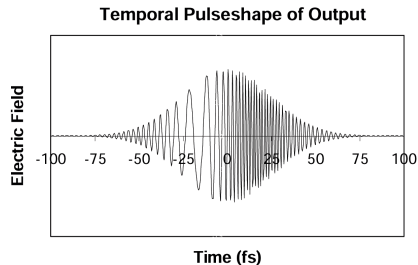
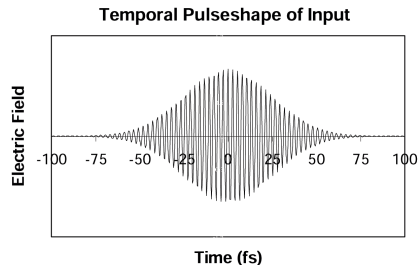


Figure 2 Transmitting information-encoded optical pulses through a fast-light medium. **a**, Transmitting '0' and '1' through the fast-light medium (dashed line) and vacuum (solid line). Each symbol is transmitted separately through the medium and vacuum, where each curve is an average of 50 pulses. **b**, High-resolution plot of part of **a** with an offset for clarity. The amplitude of the advanced and vacuum pulses have been scaled so that their heights would be the same if a gaussian pulse propagated through the system, as in Fig. 1b. The error bar indicates the typical standard deviation of the pulse amplitudes. From **a**, it is seen that the fast-light medium advances the early part of the pulses during the smooth turn on, identically to that observed for the full gaussian-shaped pulses shown in Fig. 1b. Most important is the observation that both symbols are the same for early times so that it is not possible to distinguish between them. Hence, no information can yet be conveyed to a receiving party at the end of the communication channel.

Autocorrelation techniques

Example of pulse distortion by traveling through glass



What can give rise to chirped pulses?

$$\delta\omega(t) = \omega(t) - \omega_0 = -\frac{\omega_0 n_2}{2c} x \frac{\partial I(t)}{\partial t}$$

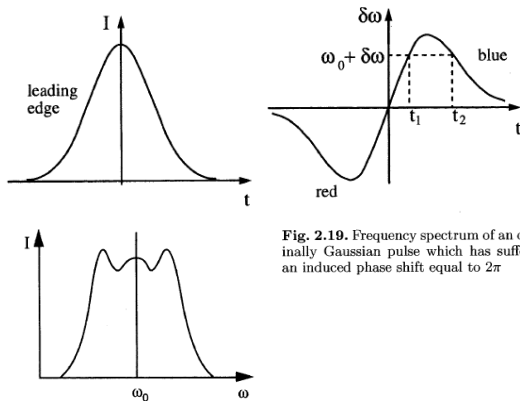
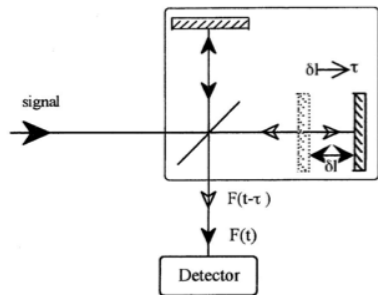


Fig. 2.19. Frequency spectrum of an originally Gaussian pulse which has suffered an induced phase shift equal to 2π

All optical Pulse characterization techniques are based on time to space transformations.
1 ps $\leftrightarrow$ 300 μm in air

7.4.2 & 7.5 (Ruliere, "femtosecond laser pulses")
How to measure characteristics of laser pulses

$$I_1(\tau) = \int_{-\infty}^{+\infty} |E(t) + E(t - \tau)|^2 dt$$
$$I_1(\tau) \propto 2 \int I(t) dt + 2G(\tau)$$



$$I_2(\tau) = \int_{-\infty}^{+\infty} \left| \left\{ E(t)e^{i[\omega t + \Phi(t)]} + E(t-\tau)e^{i[\omega(t-\tau) + \Phi(t-\tau)]} \right\}^2 \right|^2 dt$$

$$\begin{aligned} I_2(\tau) = & \int_{-\infty}^{+\infty} |2E^4 + 4E^2(t)E^2(t-\tau) \\ & + 4E(t)E(t-\tau) [E^2(t) + E^2(t-\tau)] \cos[\omega\tau + \Phi(t) - \Phi(t-\tau)] \\ & + 2E^2(t)E^2(t-\tau) \cos[2(\omega\tau + \Phi(t) - \Phi(t-\tau))]| dt \end{aligned}$$

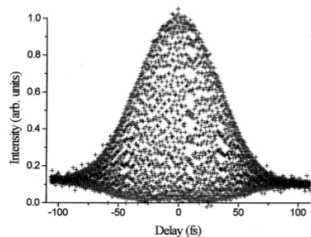


Fig. 7.10. Second-order autocorrelation of some pulse as in Fig. 7.9

$$I_2(\tau = 0) = 2^4 \int E^4(t) dt$$

$$I_2(\tau \rightarrow \infty) = 2 \int E^4(t) dt$$

Fringes are spatially averaged (same effect as fast sweep)

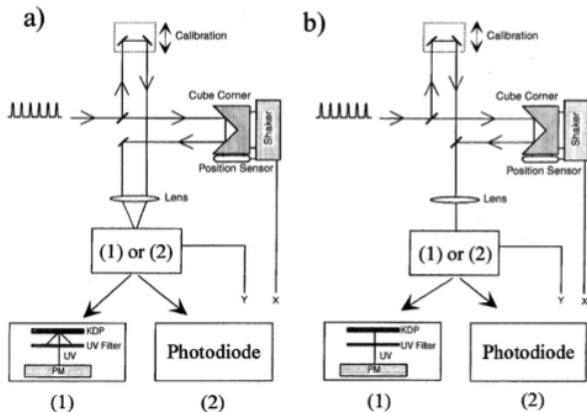
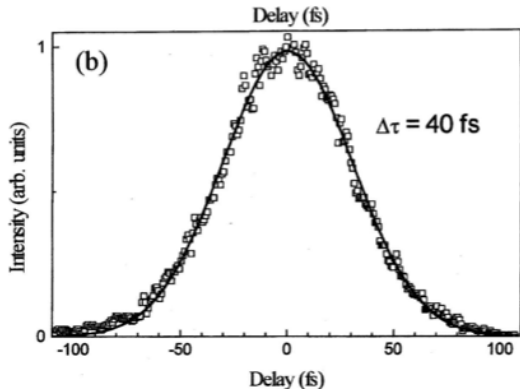
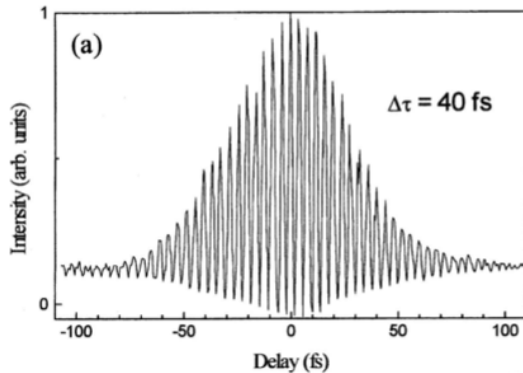


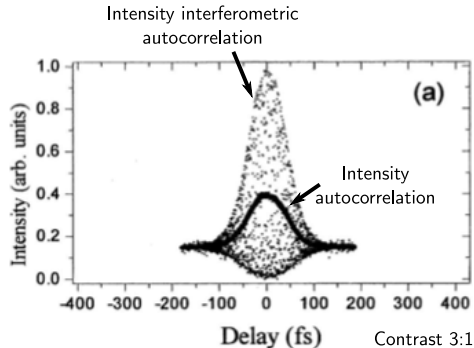
Fig. 7.13. Basic principle of second-order autocorrelator: noncolinear (a) and colinear (b) arrangements

Second order intensity autocorrelation
(non-colinear)



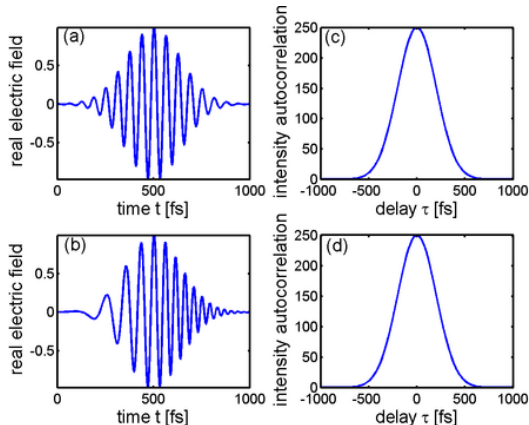
Second order interferometric autocorrelation
(colinear)



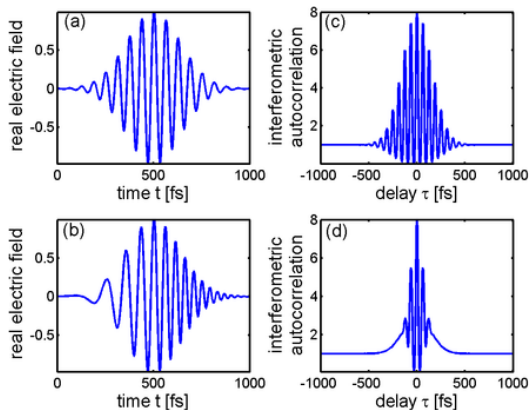


$$I_2(\tau) = \int_{-\infty}^{+\infty} \left| \left\{ E(t)e^{i[\omega t + \Phi(t)]} + E(t-\tau)e^{i[\omega(t-\tau) + \Phi(t-\tau)]} \right\}^2 \right|^2 dt$$

$$G_2(\tau) = \frac{\int_{-\infty}^{+\infty} I(t)I(t-\tau)dt}{\int_{-\infty}^{+\infty} I^2(t)dt}$$



Two **ultrashort pulses** (a) and (b) with their respective intensity autocorrelation (c) and (d). Because the intensity autocorrelation ignores the temporal phase of pulse (b) that is due to the instantaneous frequency sweep (**chirp**), both pulses yield **the same intensity autocorrelation**. The zero in this figure has been shifted to omit this background.

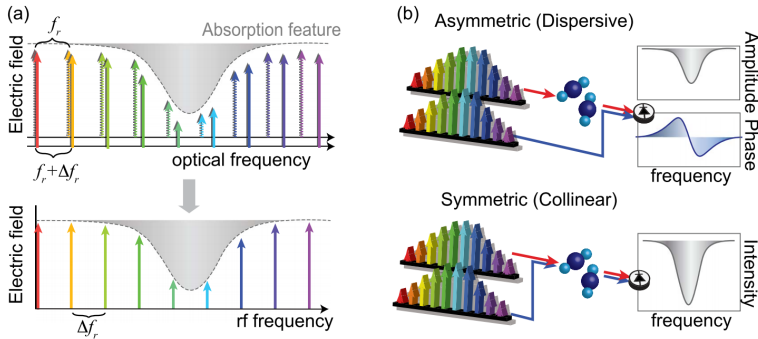


Two **ultrashort pulses** (a) and (b) with their respective interferometric 2nd order autocorrelation (c) and (d). Because of the phase present in pulse (b) due to an instantaneous frequency sweep (**chirp**), the fringes of the autocorrelation trace (d) wash out in the wings. Note the ratio 8:1 (peak to the wings), characteristic of interferometric autocorrelation traces.

Dual-Comb Spectroscopy (DCS)

Can we apply frequency comb for spectroscopy measurements?

Comb light could be coupled into conventional spectrometers. However, since the comb tooth spacing is finer than the resolution of nearly all spectrometers. Its associated frequency resolution and accuracy are lost! → Let's apply Dual-Comb and transfer signal to RF domain!

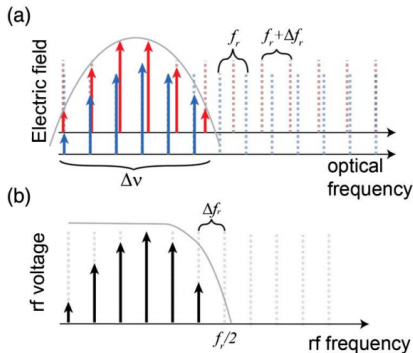


Dual-comb spectroscopy

IAN CODDINGTON,^{1,*} NATHAN NEWBURY,^{1,2} AND WILLIAM SWANN¹

Dual-Comb Spectroscopy (DCS)

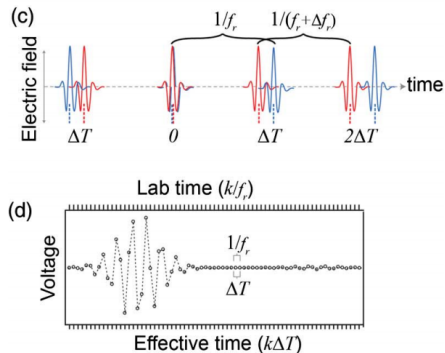
Frequency domain



$$m = f_r / \Delta f_r$$

$$\Delta \nu \leq m f_r / 2 = f_r^2 / (2 \Delta f_r)$$

Time domain



$$\Delta T = \frac{\Delta f_r}{f_r (f_r + \Delta f_r)} \approx \frac{1}{m f_r}$$

The minimum time to resolve the RF comb teeth, and therefore acquire a single spectrum, is simply $\frac{1}{\Delta f_r}$.

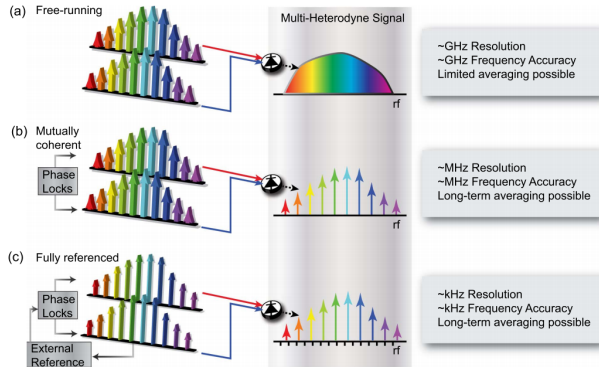
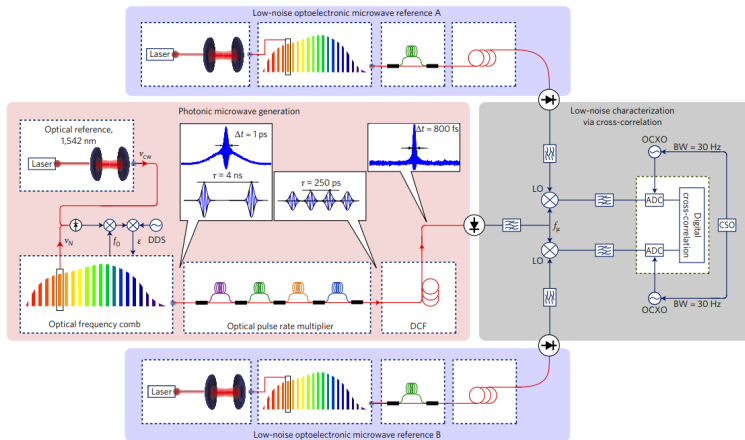


Fig. 4. Three different categories of DCS demonstrations. (a) Free-running combs can yield dual-comb spectra, but with low resolution, low frequency accuracy, and low SNR, since only limited signal averaging is possible; (b) mutually coherent combs can yield comb-tooth-resolved spectra that can be averaged for high SNR; and (c) fully referenced combs yield spectra with simultaneous high resolution, absolute frequency accuracy, and high SNR. Text boxes indicate some general rules of thumb for frequency combs based on mode-locked lasers.



$$\delta \nu_{CW} / \nu_{CW} = \delta f_r / f_r = \delta f_\mu / f_\mu$$

Figure 1 | Experimental set-up for low-noise microwave generation and characterization. The left part of the set-up is used to generate a record low-noise 12 GHz microwave signal from an ultrastable optical reference at 1,542 nm via optical division. The right part is used to characterize the microwave-signal phase noise using a heterodyne digital cross-correlation method. The auxiliary optoelectronic microwave references A and B are also obtained by low-noise optical division of the light from two additional distinct ultrastable laser references at 1,542 nm using two separate optical frequency combs. ADC, analog-to-digital converter; CSO, cryogenic sapphire oscillator; DCF, dispersion compensation fibre; DDS, direct digital synthesizer; OCXO, oven-controlled crystal oscillator.

Fiber laser mode locking

All polarization-maintaining fiber laser architecture for robust femtosecond pulse generation

