Laser: Theory and Modern Applications
ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE (EPFL)

Exercise Sheet 9: Phase-locking, Short Laser Pulses

Questions

1. Phase-locked oscillators

Assume N phase-locked optical modes of equal amplitudes, such that the total resulting electric field is given by:

$$E(t) = \sum_{j=-(N-1)/2}^{(N-1)/2} E_0 e^{i[(\omega_0+\omega_R j)\cdot t+\phi_j]}$$

where ω_R is the mode spacing, t is time, and ω_0 is the central frequency.

(a) First (using $\phi_j = 0$), derive an expression for the pulse duration and use the geometric series:

$$\sum_{j=0}^{N} x^j = \frac{1-x^{N+1}}{1-x}$$

(b) Second (using $\phi_j = 0$), derive the time separating subsequent pulses.

(c) Derive the frequency of the first mode of the Fourier spectrum. What does it mean when the first component coincides exactly with zero frequency?

(d) Include a phase $\phi_j = j \cdot \Delta\phi$ that varies linearly with mode index j. Show that this still yields pulses.

(e) Assuming ω_0 is a multiple of ω_R, show that the presence of a varying phase from pulse to pulse shifts the position of the first component in the Fourier spectrum (known as the carrier-envelope offset frequency) to:

$$\omega_0 = \omega_R \cdot \frac{\Delta\phi}{2\pi}$$

2. Shortest Laser Pulses from a Ti:Sa and He:Ne Lasers

Compare the shortest pulse that can be generated with a Ti:Sa laser (with a gain bandwidth allowing a 50 nm wide spectrum centered at 800 nm) to a Helium-Neon laser (with a gain bandwidth of 1.7 GHz around a 634 nm wavelength).

3. Gaussian Laser Pulses

Assume you have a laser pulse that consists of an infinite set of frequencies:

$$E(t) = \sum_{n=-\infty}^{\infty} E_n e^{i\omega_n t + i\phi_n}$$

where the frequencies are given by $\omega_n = \omega_0 + n \cdot \omega_R$. Assume that the amplitudes follow a Gaussian distribution:

$$E_n = E_0 \exp\left[-2\ln(2)\left(\frac{n\omega_R}{\Delta\omega_0}\right)^2\right]$$

where ω_R is the repetition rate (the spacing between adjacent modes) in the spectrum of the pulsed laser source, and $\Delta\omega_0$ is the 3-dB bandwidth of the pulsed laser source spectrum. Assume the phases obey $\phi_n = 0$.

(a) Derive an expression for the intensity of the output pulse spectrum, $I(t) \propto |E(t)|^2$. Convert the sum over the Fourier components into an integral over n. Recall that the Fourier transform of a Gaussian yields another Gaussian.

(b) Show that the pulse duration given by the full width at half maximum (FWHM) is:

$$\tau_p = \frac{4\ln(2)}{\Delta\omega_0}$$