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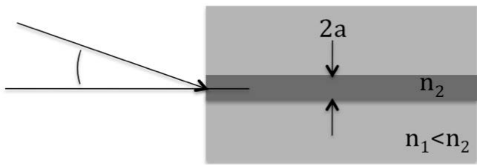
Laser: Theory and Modern Applications
ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE (EPFL)

Exercise Sheet 8: Shot noise, Johnson noise, heterodyne detection



8.1 *Fiber Optics* 1. Derive the numerical aperture of the fiber such that light is guided by total internal reflection (TIR) in the core of index n_2 surrounded by the cladding of index n_1, i.e., derive the following relation:

$$NA \approx \sqrt{2\Delta \cdot n_1} \tag{1}$$

where $\Delta = \frac{n_2-n_1}{n_1}$.

2. Calculate the numerical aperture of a silica fiber with index $n_2 = 1.46$ and $\Delta = 10^{-2}$.

3. The V parameter is defined by the relation:

$$V = \frac{2\pi a}{\lambda_0} NA \tag{2}$$

Single mode propagation is satisfied for $V < 2.405$. Calculate the diameter of the silica fiber core at the telecommunication wavelength of 1.5 μm that ensures single mode performance. Do the same for the visible wavelength 0.5 μm.

A solid core surrounded by air can guide light by TIR, so why aren't single-mode fibers of this type used in practice?

4. Estimate the length of the pre-form of fused silica with a diameter of 10 cm necessary to produce 1000 km of 8/125 fibers.

5. **Double clad fiber**
The absorption efficiency of pump light in the off-centered core of a double clad fiber of length L is given by:

$$\eta = 1 - e^{-F \cdot \frac{S_{core}}{S_{total}} \cdot \alpha \cdot L}$$

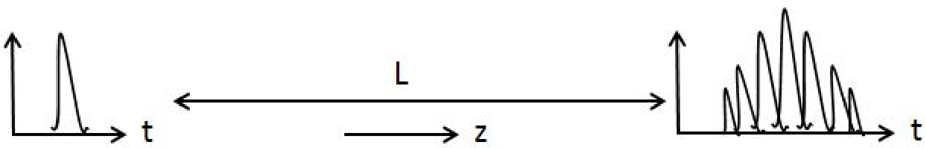
For the off-center double clad with an offset of 15 μm and core diameter 12 μm, clad diameter 400 μm, the confinement factor F is found to be 0.8. The absorption of the pump is 55 dB/m (typical value).

Compute the effective absorption coefficient and the absorption efficiency for a fiber length of 1 m, 5 m, 10 m, 20 m and 50 m.
hint: For the definition of dB and relation with absorption, see script on p.128.

8.2 ***Modal Dispersion in a Multimode Fiber*** The propagation constant of propagating modes

$$\beta_{lm} = \sqrt{n_1^2 k_0^2 - \frac{(l+2m)^2 \pi^2}{4a^2}} \tag{3}$$

1. Express β_{lm} as a function of the number of modes M and simplify the relation for $\Delta \ll 1$.
2. Derive the group velocity $v_{lm} = \frac{d\omega}{d\beta_{lm}}$ (assume $n(\omega)$ independent of ω) and calculate the minimum and maximum group velocity by noting that the minimum value of $(l+2m)$ is 2 and the maximum is $\sqrt{M}$.
3. A pulse entering a multimode fiber will propagate in all the discrete modes of the multimode fiber. The initial pulse will widen as a result of the different arrival times.



Derive the pulse broadening after propagating a length L in the fiber.
4. Calculate the pulse broadening after 100 km of fiber, $\Delta = 0.01$ and $n = 1.46$.
5. Calculate the pulse broadening for the gradient index fiber where $v_{min} = \frac{c}{n_1}\left(1 - \frac{\Delta^2}{2}\right)$ and $v_{max} = \frac{c}{n_1}$.
6. Which fiber type: step index or gradient index, delivers the least time broadening?

8.3 ***Even Mode Ansatz of an Asymmetric Waveguide*** The even mode ansatz of an asymmetric waveguide is

$$E_y = A \cdot e^{-p(|x|-d)} \cdot e^{-j\beta z} \quad |x| \geq d,\ n_1 \text{ index} \tag{4}$$
$$E_y = B \cdot \cos(hx) \cdot e^{-j\beta z} \quad |x| \leq d,\ n_2 \text{ index} \tag{5}$$

Verify that the condition

$$\beta^2 = k_0^2 n_1^2 + p^2 \quad \text{and} \quad \beta^2 = k_0^2 n_2^2 - h^2 \tag{6}$$

is satisfied when E_y is a solution of the wave equation

$$\frac{\partial^2}{\partial x^2} E_y(x,z) + (k_0^2 n_1^2 - \beta^2) E_y(x,z) = 0 \quad |x| \geq d \tag{7}$$
$$\frac{\partial^2}{\partial x^2} E_y(x,z) + (k_0^2 n_2^2 - \beta^2) E_y(x,z) = 0 \quad |x| \leq d \tag{8}$$