Transmission Matrices

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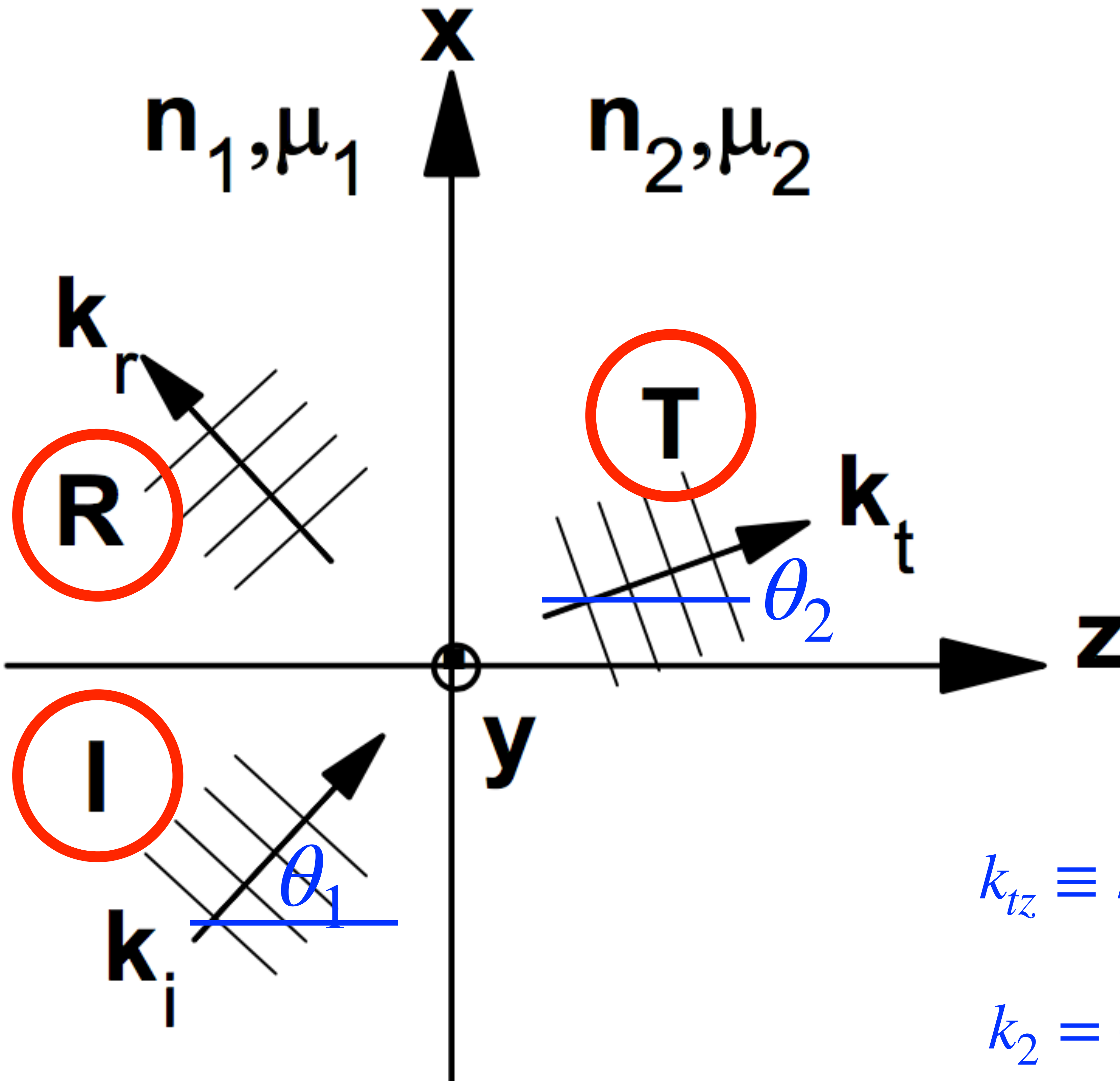
SCATTERING

Single Interface



$$k_{iz} \equiv k_1 cos\theta_1$$

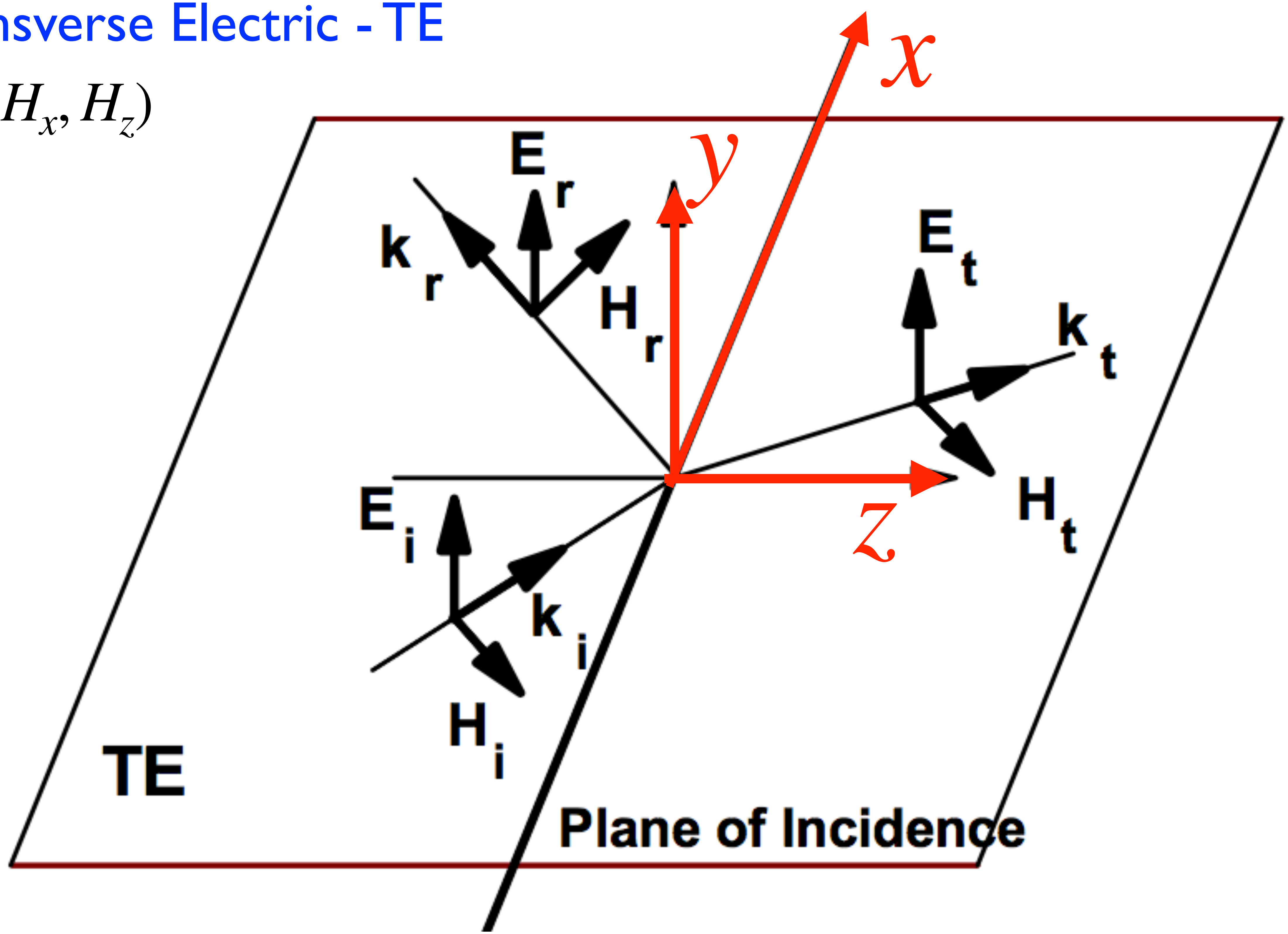
$$k_1 = \frac{2\pi n_1}{\lambda}$$

$$k_{tz} \equiv k_2 cos\theta_2$$

$$k_2 = \frac{2\pi n_2}{\lambda}$$

Transverse Electric - TE

(E_y, H_x, H_z)



Fresnel Equations

Transverse Electric - TE polarization (E_y, H_x, H_z)

$$\overrightarrow{E_i} = E_{yi}\,\widehat{y}\,e^{i\overrightarrow{k}_i\cdot\vec{r}}$$

$$\overrightarrow{H_i} = (H_{xi}\,\widehat{x} + H_{zi}\,\widehat{z})e^{i\overrightarrow{k}_i\cdot\vec{r}}$$

Incident Electromagnetic Plane Wave

$$k_{iz} \equiv k_1 cos\theta_1$$

$$r_{TE} \equiv \frac{E_{yr}}{E_{yi}} = \frac{k_{iz} - k_{tz}}{k_{iz} + k_{tz}}$$

Reflection coefficient

$$k_1 = \frac{2\pi n_1}{\lambda}$$

$$k_{tz} \equiv k_2 cos\theta_2$$

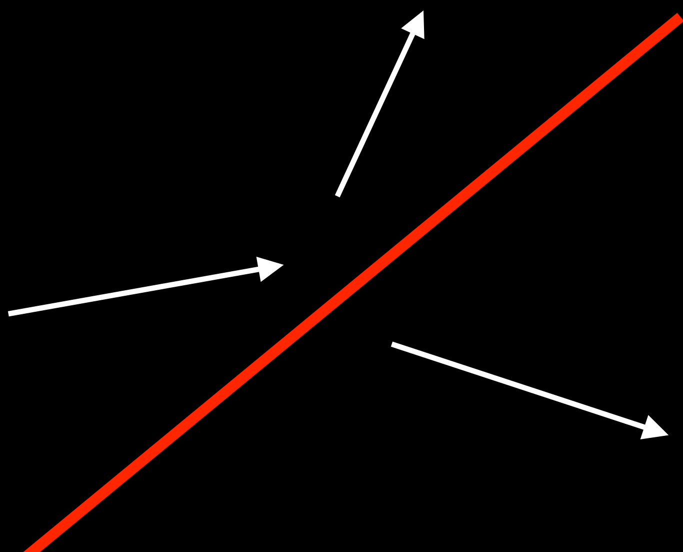
$$k_2 = \frac{2\pi n_2}{\lambda}$$

$$t_{TE} \equiv \frac{E_{yt}}{E_{yi}} = \frac{2k_{iz}}{k_{iz} + k_{tz}}$$

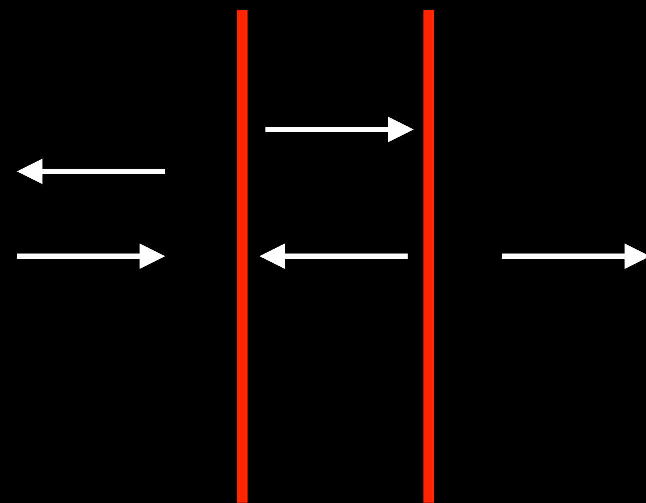
Transmission coefficient

Scattering elements

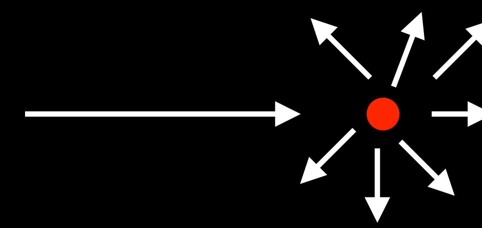
SINGLE INTERFACE



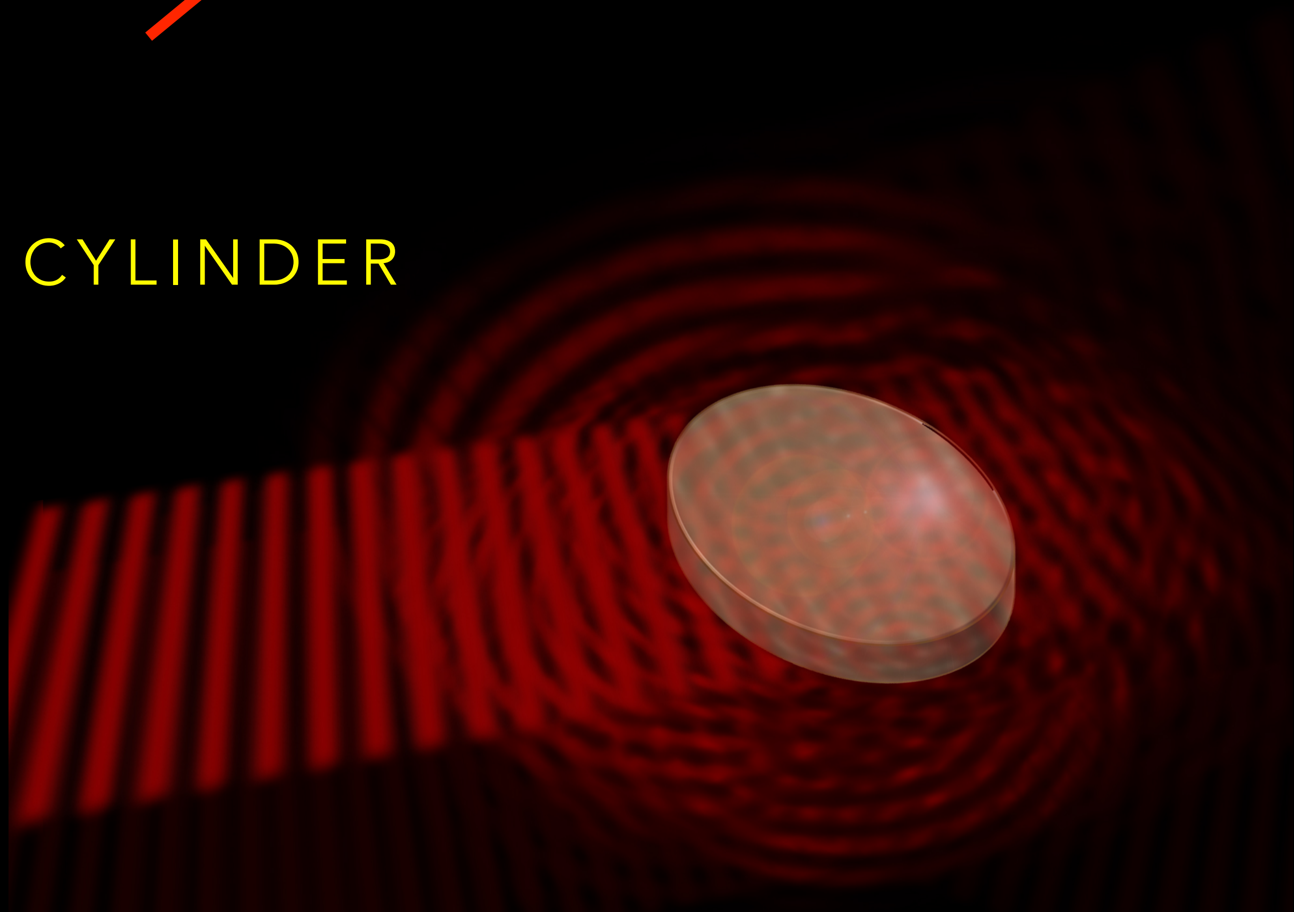
DOUBLE INTERFACE



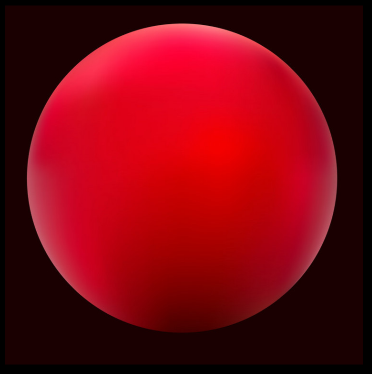
POINT SCATTERER



CYLINDER



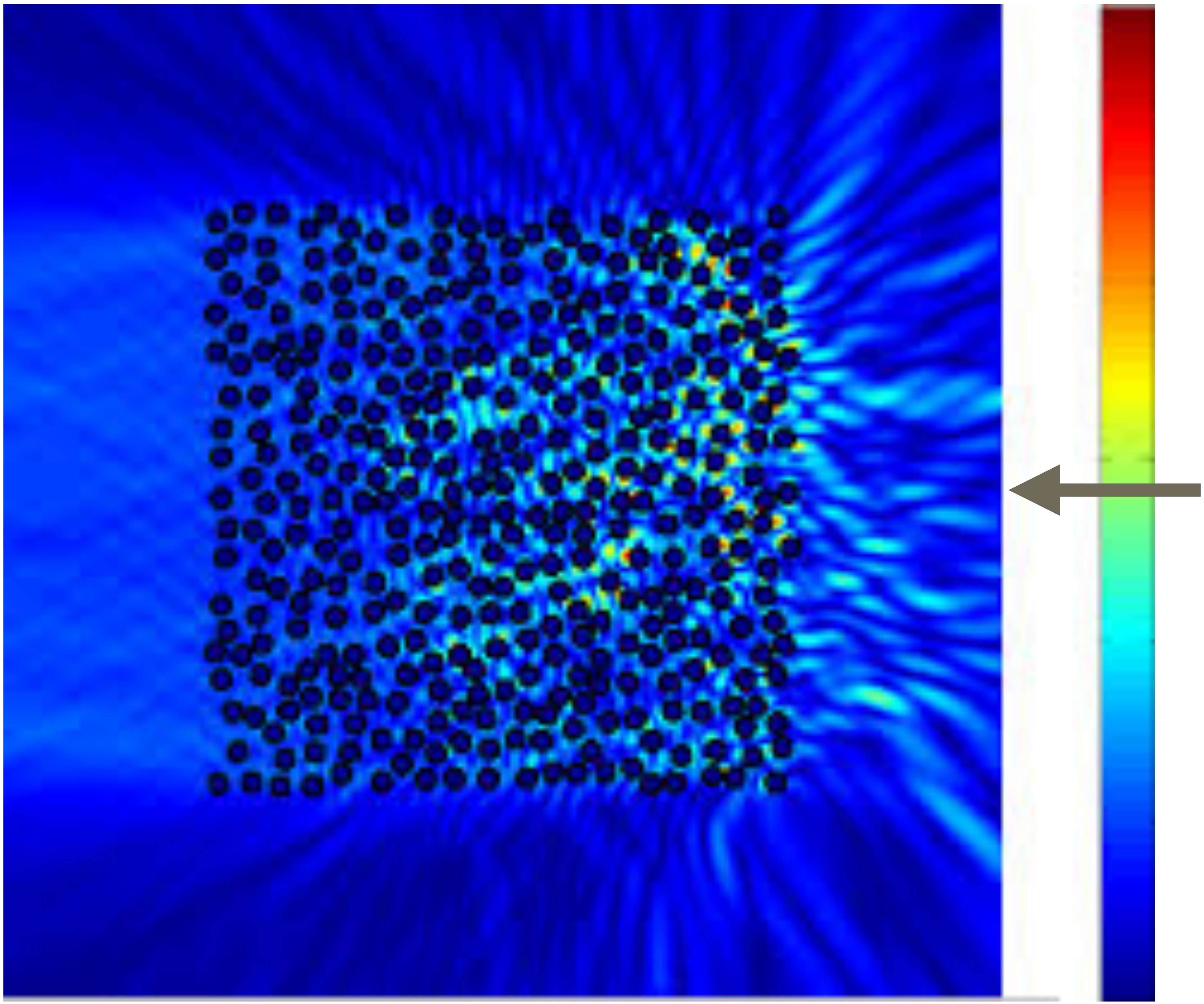
SPHERE



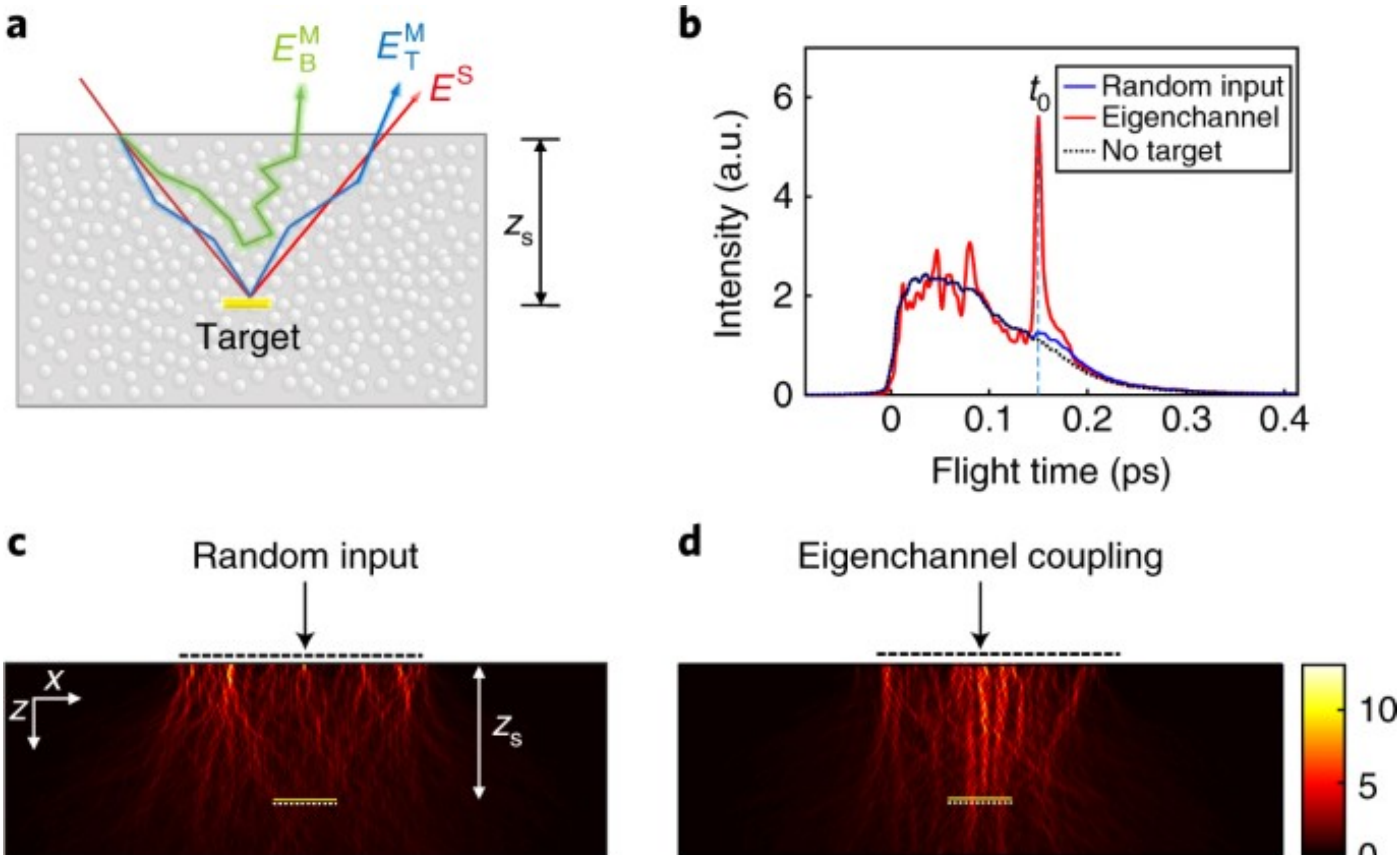
COMPLEX MEDIA

Into the Labyrinth

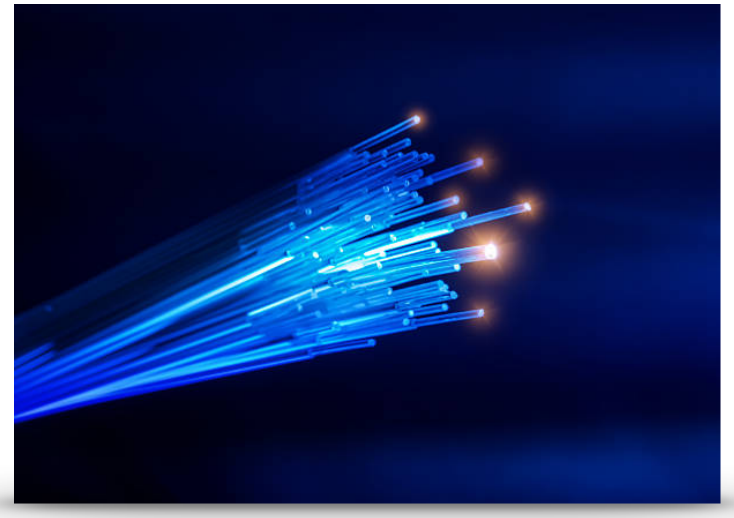
Multiple scattering

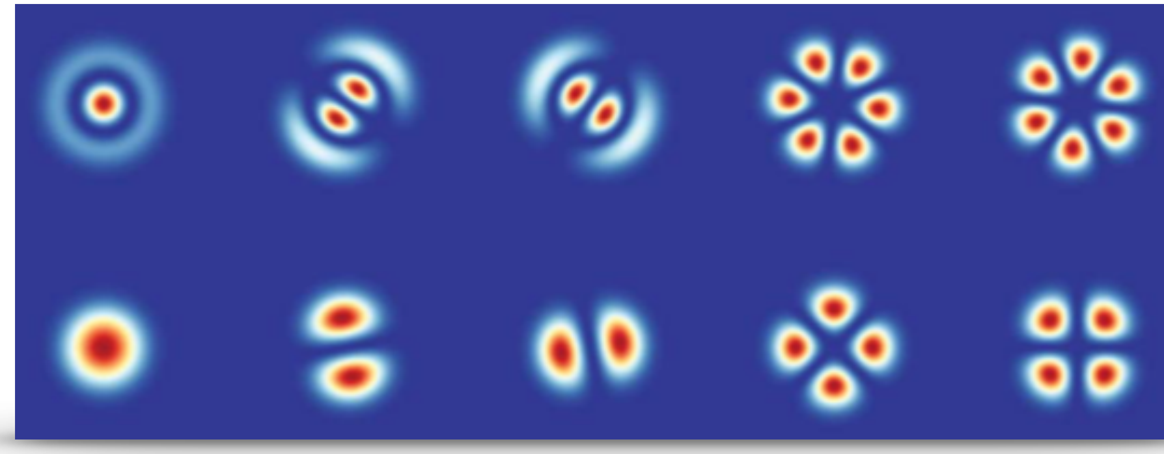


Penetrating Disorder

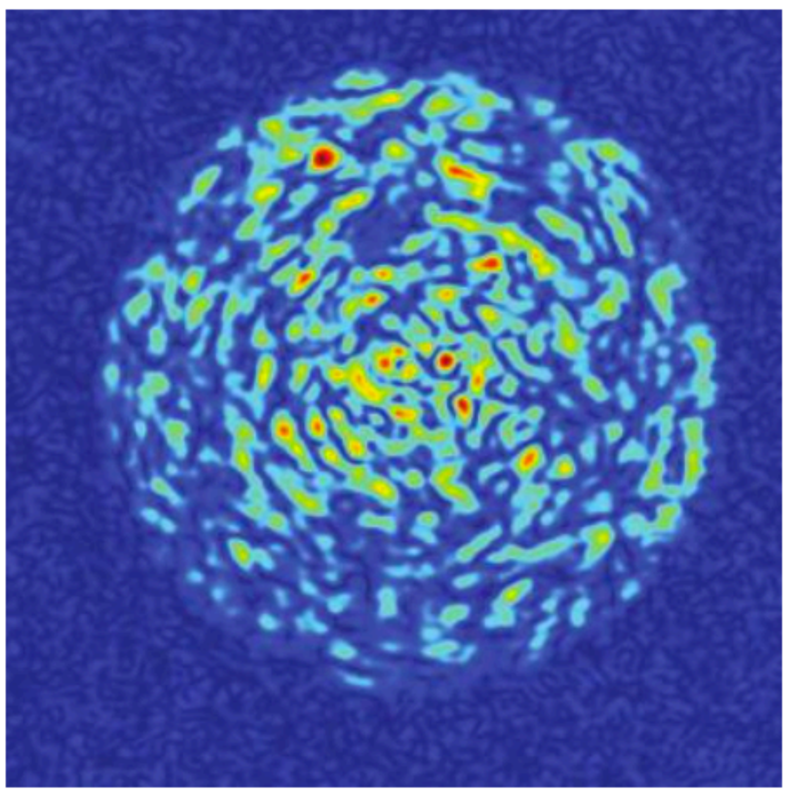


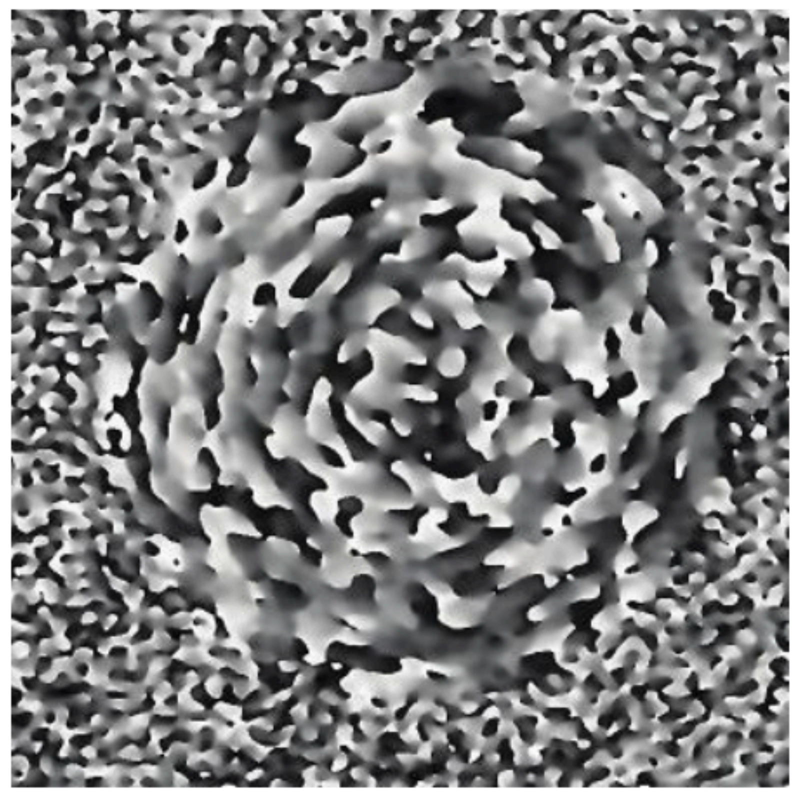
Focusing by phase conjugation

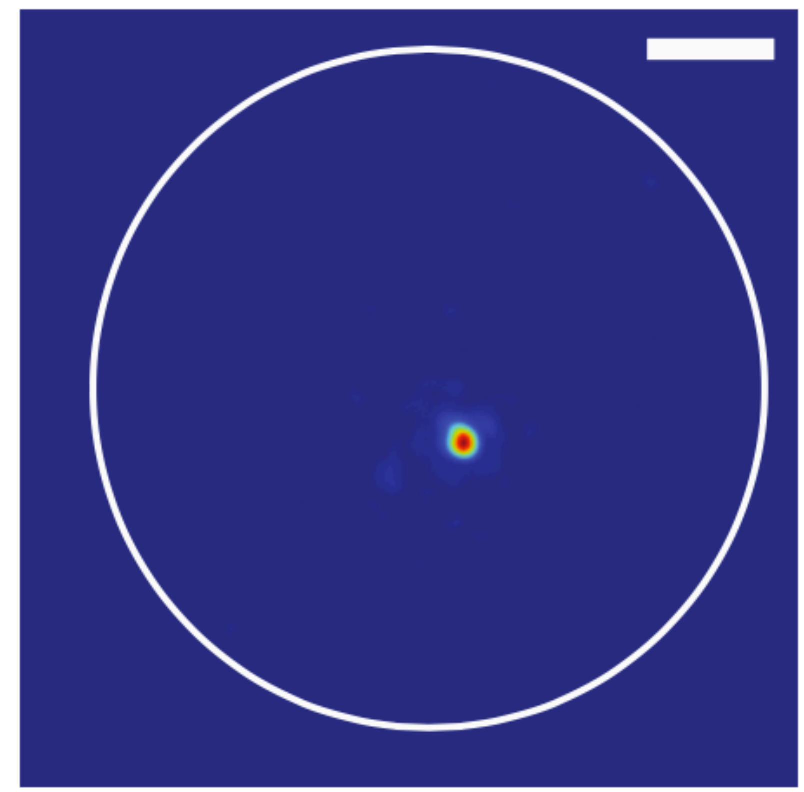




In many applications we have multimode operation







Input Intensity Input Phase Output Intensity

IN Papadopoulos, S Farahi, C Moser, D Psaltis, Optics Express 20, 10583 (2012).

Waves through complex media

The fog clears

A technique has been developed to image a fluorescent object hiding behind a light-scattering screen without the need for a detector behind the screen. The approach could find applications in imaging biological tissue. SEE LETTER P.232

DEMETRI PSALTIS
& IOANNIS N. PAPADOPOULOS

A golfer faced with the problem of hitting a ball out of the woods after an errant shot sometimes makes a brave choice: she aims straight for the trees, swinging the club as hard as possible in the hope that the ball will bounce off the trees and miraculously emerge from the woods. On page 232 of this issue, Bertolotti *et al.*[1] describe a technique for imaging objects through light-scattering media, such as fog and human tissue, that overcomes a challenge that is in some ways similar to this one.

Consider light from a torch passing through a human hand. Information about the shapes of the bones, or even the cells, that make up the hand is thought to be encrypted in this transmitted light (Fig. 1), but a simple device such as a lens cannot be used to image the hand's interior. Numerous attempts have been made to retrieve the shapes of objects that hide behind or are within media that transmit and scatter light. Some of the photons that travel through a light-scattering medium do so without interacting with any of the medium's constituent matter. Such 'ballistic' photons exit the medium a little earlier than their non-ballistic counterparts, which bounce off the matter as they pass through the scattering medium. If the ballistic photons alone are captured in a detector, the blurring effects of scattering can be avoided[2]. However, for

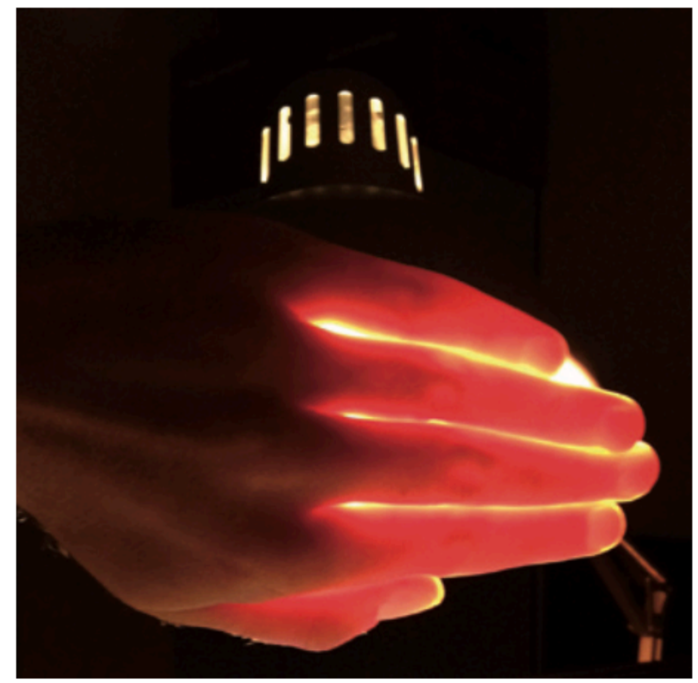
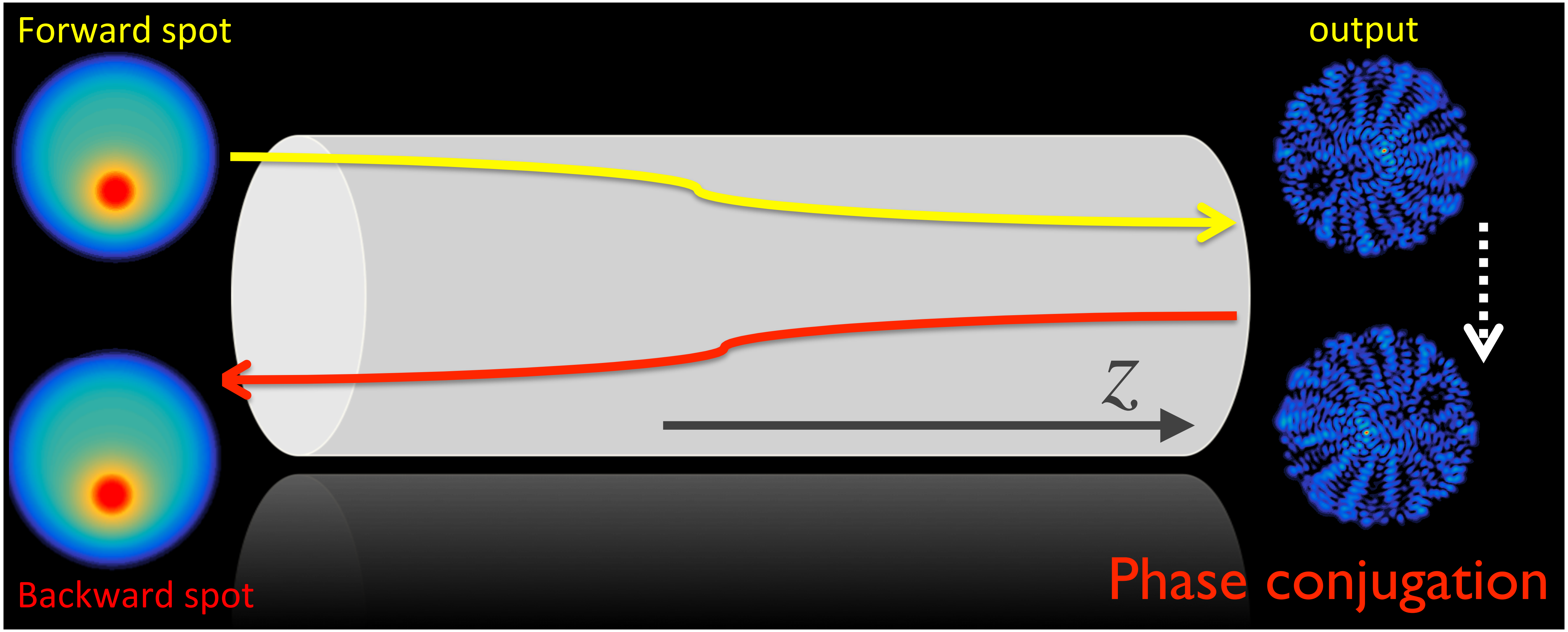


Figure 1 | Seeing through. A light source placed behind a human hand emits photons that travel through the hand. Information about the hand's interior, such as its bones or cells, is encoded in the light's field but cannot be directly retrieved because it is scrambled by the scattering properties of human tissue. Bertolotti *et al.*[1] propose a method for retrieving such information.

Phase Conjugation and Focusing

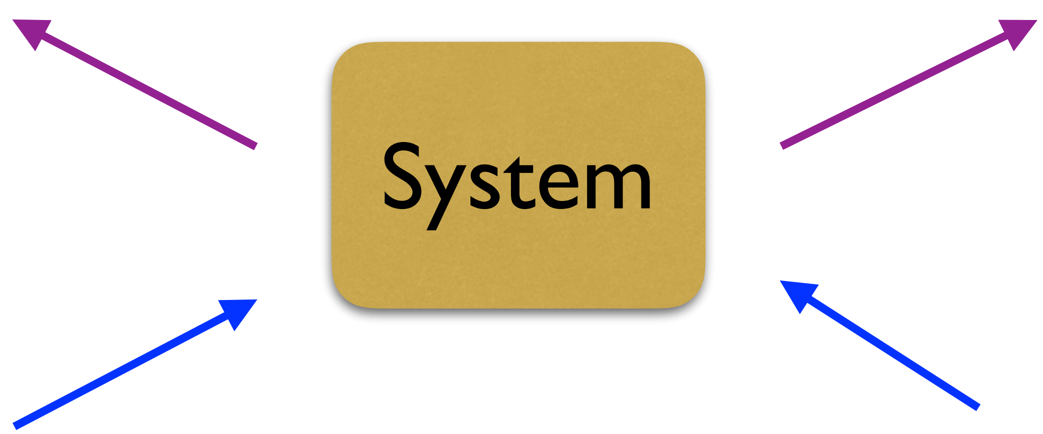


Time reversal - Phase conjugation

TRANSMISSION MATRIX - GENERAL

Scattering Matrix

$$\underbrace{\boxed{\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}}}_{\text{Outgoing waves}} = S\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \equiv \underbrace{\boxed{\begin{pmatrix} r_L & t \\ t & r_R \end{pmatrix}}}_{\text{Scattering matrix}}\underbrace{\boxed{\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}}}_{\text{Incoming waves}}$$



Energy conservation - Unitarity: $S^\dagger = S$

Reciprocity - Symmetry: $S^T = S$

Transmission Matrix

Scattering matrix

$$\boxed{\begin{pmatrix}u_1\\u_2\end{pmatrix}} = S\begin{pmatrix}v_1\\v_2\end{pmatrix} \equiv \boxed{\begin{pmatrix} r_L & t\\ t & r_R\end{pmatrix}}\boxed{\begin{pmatrix}v_1\\v_2\end{pmatrix}}$$

Outgoing waves

Incoming waves

Transmission matrix

Transmission Matrix - Multimode Fibers

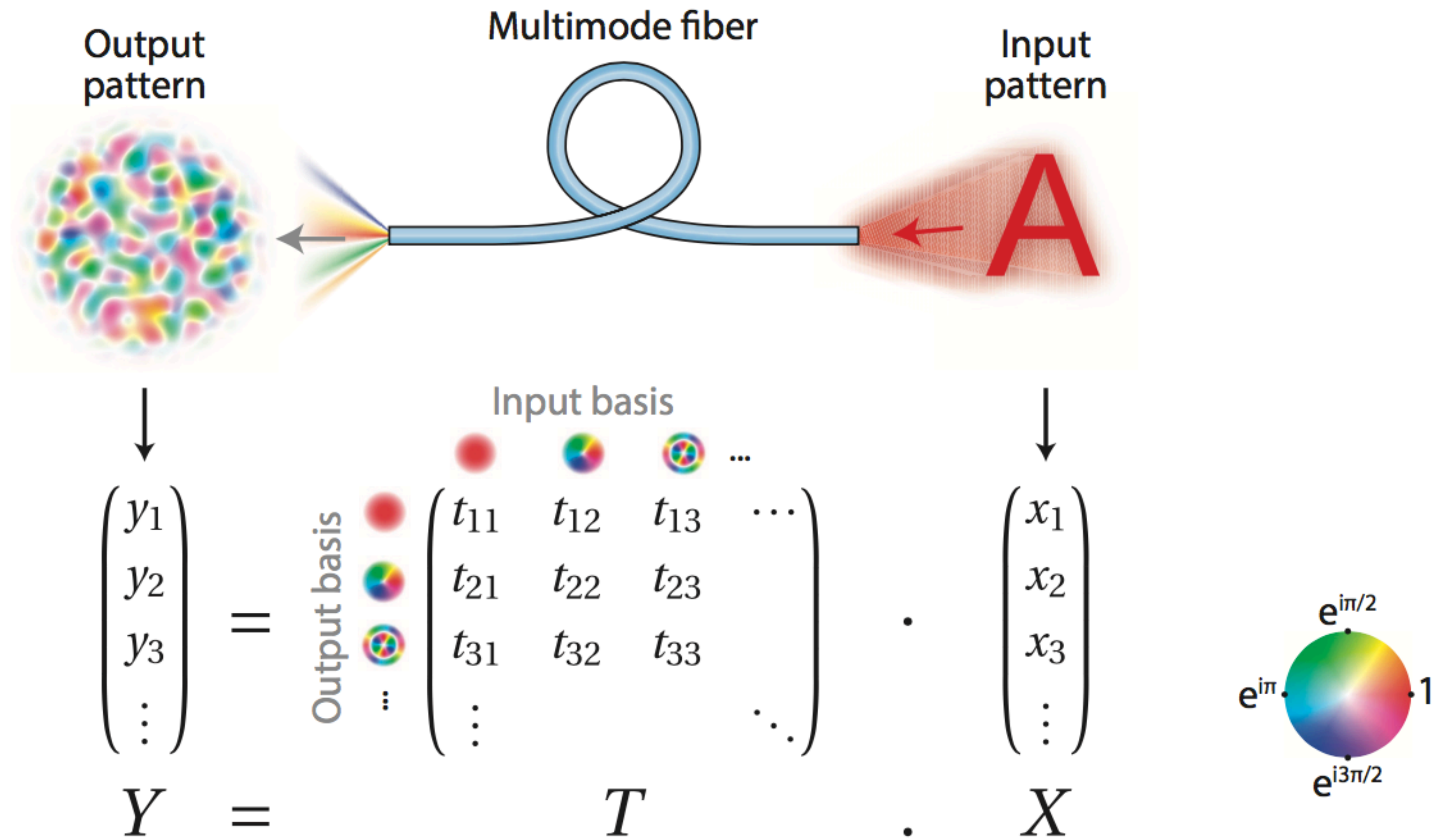
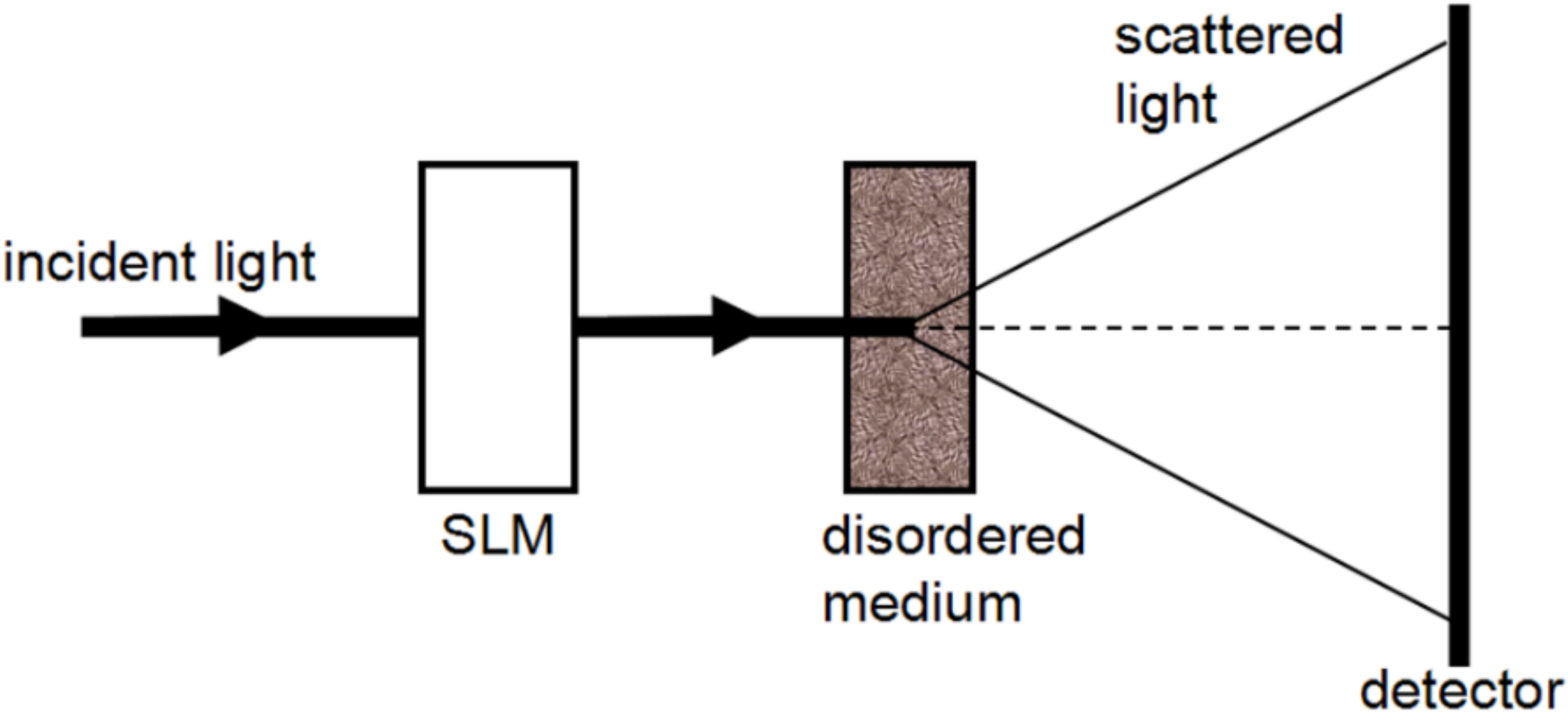


Figure 3.1: The transmission matrix model for multimode fibers. The input pattern X is injected into the fiber, where it undergoes a transformation T, and exits as the output pattern Y. X and Y are a vectors of complex coefficients describing the optical fields at input and output respectively in a suitable basis. In this illustration, the input and output bases consist of the fiber modes, but other choices are possible too. T is the transmission matrix relating the input coefficients X and the output coefficients Y.

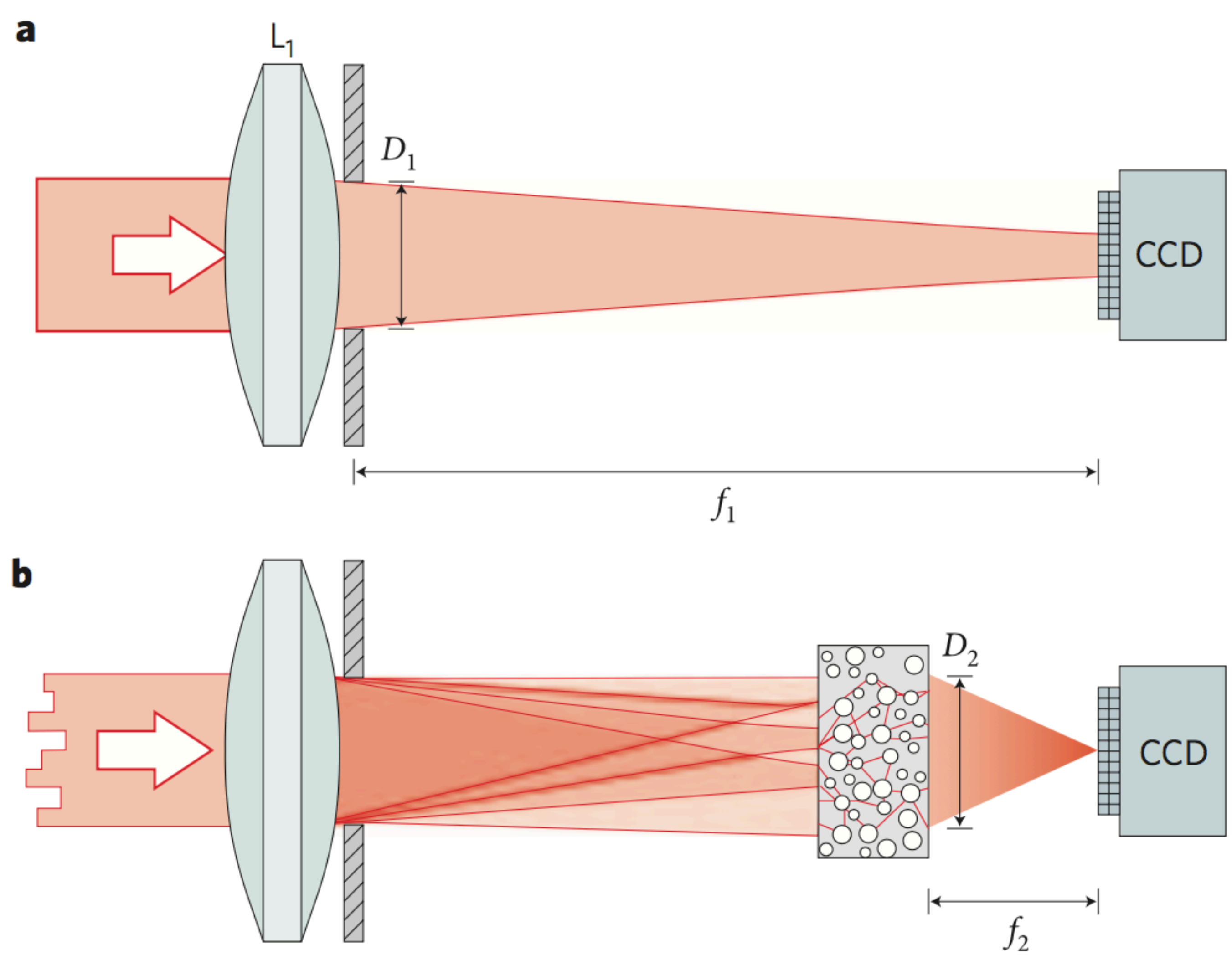
Transmission Matrix - Disordered media



LETTERS
PUBLISHED ONLINE: 14 FEBRUARY 2010 | DOI: 10.1038/NPHOTON.2010.3
nature photonics

Exploiting disorder for perfect focusing

I. M. Vellekoop[1†], A. Lagendijk[2] and A. P. Mosk[1*]

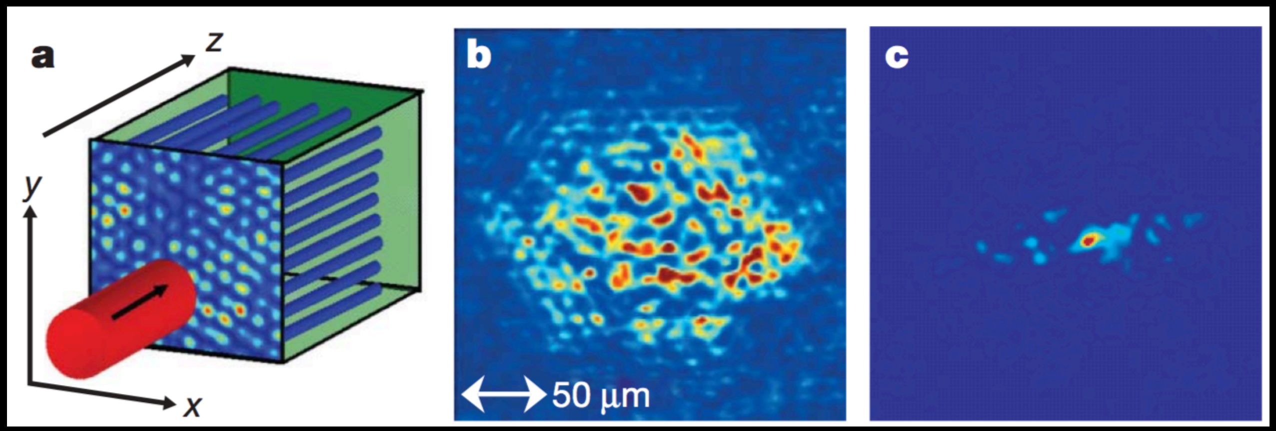


Disordered Photonics

Phys. Rev.109, 1492 (1958)

Anderson localization
Random Matrix Theory

Condensed Matter Physics



Nature 446, 52 (2007)
PRL 100, 013906 (2008)

Random Lasers
Perfect transmission Eigenchannels
Wavefront shaping techniques
Imaging though turbid media

Disordered Photonics

nature photonics

FOCUS | REVIEW ARTICLES
PUBLISHED ONLINE: 27 FEBRUARY 2013 | DOI: 10.1038/NPHOTON.2013.30

Anderson localization of light

Mordechai Segev[1,2*], Yaron Silberberg[3] and Demetrios N. Christodoulides[4]

REVIEW ARTICLES | FOCUS
PUBLISHED ONLINE: 27 FEBRUARY 2013 | DOI: 10.1038/NPHOTON.2013.29

Disordered photonics

Diederik S. Wiersma

Wavefront shaping

Review

www.the-innovation.org

Wavefront shaping: A versatile tool to conquer multiple scattering in multidisciplinary fields

Zhipeng Yu,[1,2,13] Huanhao Li,[1,2,13] Tianting Zhong,[1,2,13] Jung-Hoon Park,[3,13] Shengfu Cheng,[1,2] Chi Man Woo,[1,2] Qi Zhao,[1,2] Jing Yao,[1,2] Yingying Zhou,[1,2] Xiazi Huang,[1,2] Weiran Pang,[1,2] Hansol Yoon,[4] Yuecheng Shen,[5] Honglin Liu,[2,6] Yuanjin Zheng,[7] YongKeun Park,[4,8,9,*] Lihong V. Wang,[10,11,*] and Puxiang Lai[1,2,12,*]

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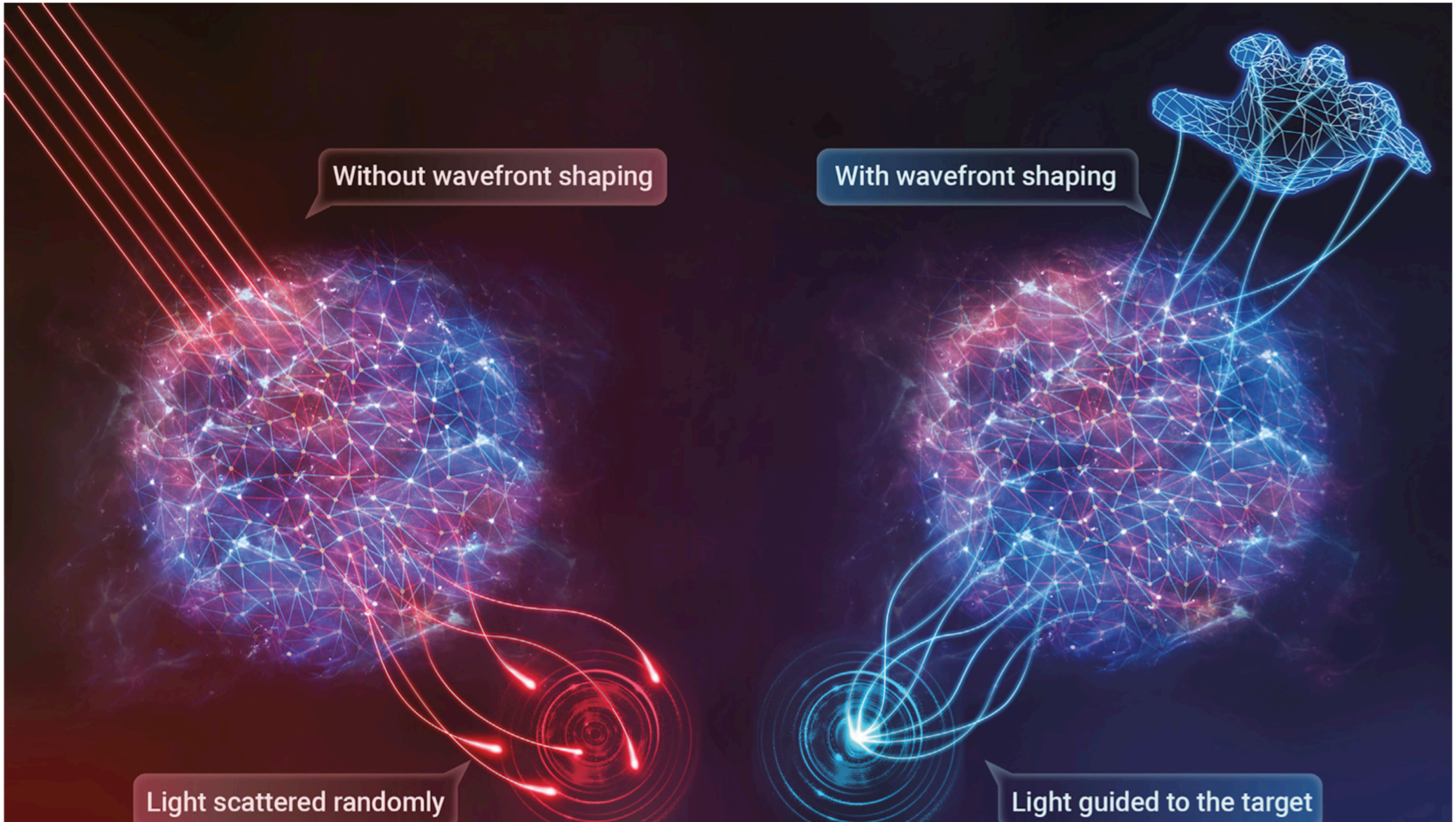
GRAPHICAL ABSTRACT



Transmission Matrix - Disordered media

Selected for a Viewpoint in *Physics*
PRL **104,** 100601 (2010) PHYSICAL REVIEW LETTERS week ending 12 MARCH 2010

Measuring the Transmission Matrix in Optics: An Approach to the Study and Control of Light Propagation in Disordered Media

S. M. Popoff, G. Lerosey, R. Carminati, M. Fink, A. C. Boccara, and S. Gigan
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(Received 27 October 2009; revised manuscript received 11 January 2010; published 8 March 2010)

We introduce a method to experimentally measure the monochromatic transmission matrix of a complex medium in optics. This method is based on a spatial phase modulator together with a full-field interferometric measurement on a camera. We determine the transmission matrix of a thick random scattering sample. We show that this matrix exhibits statistical properties in good agreement with random matrix theory and allows light focusing and imaging through the random medium. This method might give important insight into the mesoscopic properties of a complex medium.

Transmission Matrix - Disordered media

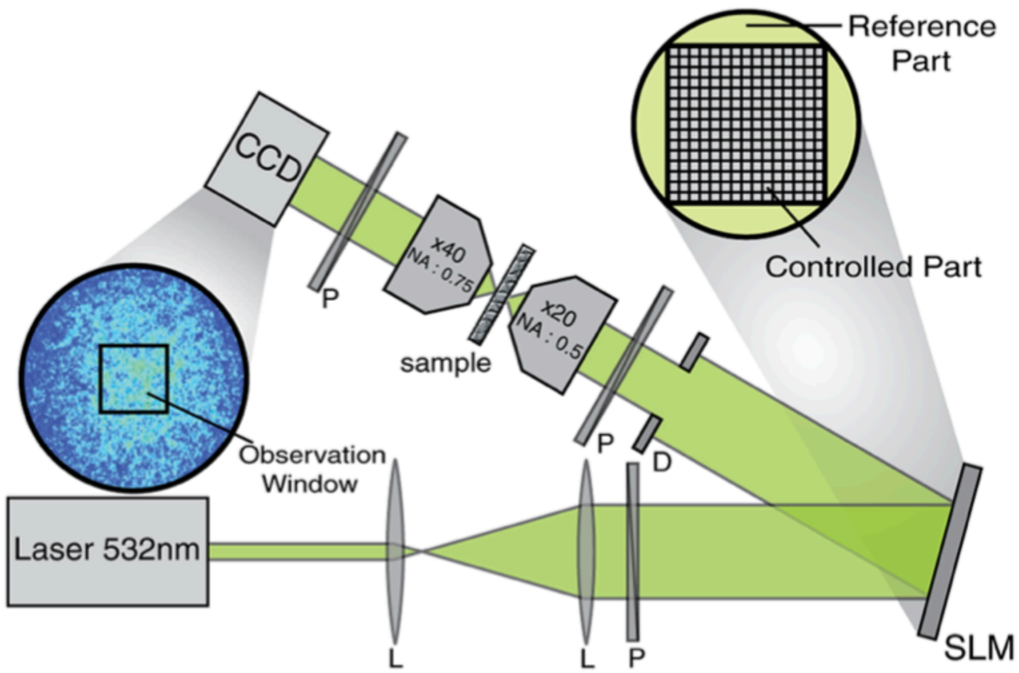

FIG. 1 (color online). Schematic of the apparatus. The laser is expanded and reflected off a SLM. The phase-modulated beam is focused on the multiple-scattering sample and the output intensity speckle pattern is imaged by a CCD camera: lens (L), polarizer (P), diaphragm (D).

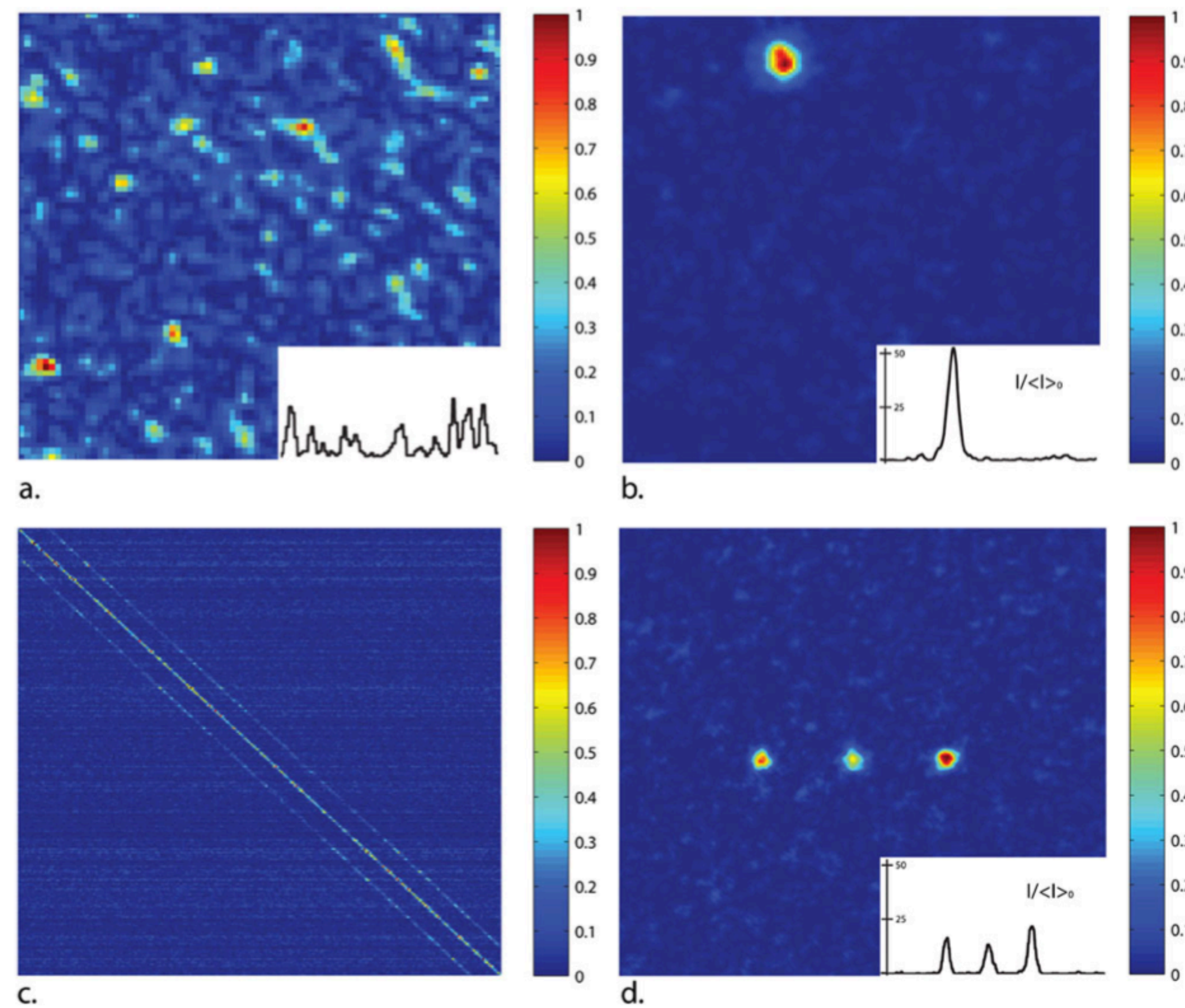

FIG. 2 (color online). Experimental results of focusing. (a) Initial aspect of the output speckle. (b) We measure the TM for 256 controlled segments and use it to perform phase conjugation. (c) Norm of the focusing operator $O_{\mathrm{norm}}^{\mathrm{foc}}$. (d) Example of focusing on several points. (The insets show intensity profiles along one direction.)

Transmission Matrix - Disordered media

APS | Physics

VIEWPOINT

The information age in optics: Measuring the transmission matrix

Elbert G. van Putten and Allard P. Mosk

Complex Photonic Systems, Faculty of Science and Technology and MESA+ Institute for Nanotechnology, University of Twente, P.O. Box 217, 7500 AE Enschede, The Netherlands

March 8, 2010 • *Physics* 3, 22

The transmission of light through a disordered medium is described in microscopic detail by a high-dimensional matrix. Researchers have now measured this transmission matrix directly, providing a new approach to control light propagation.

Transmission Matrix - Disordered media

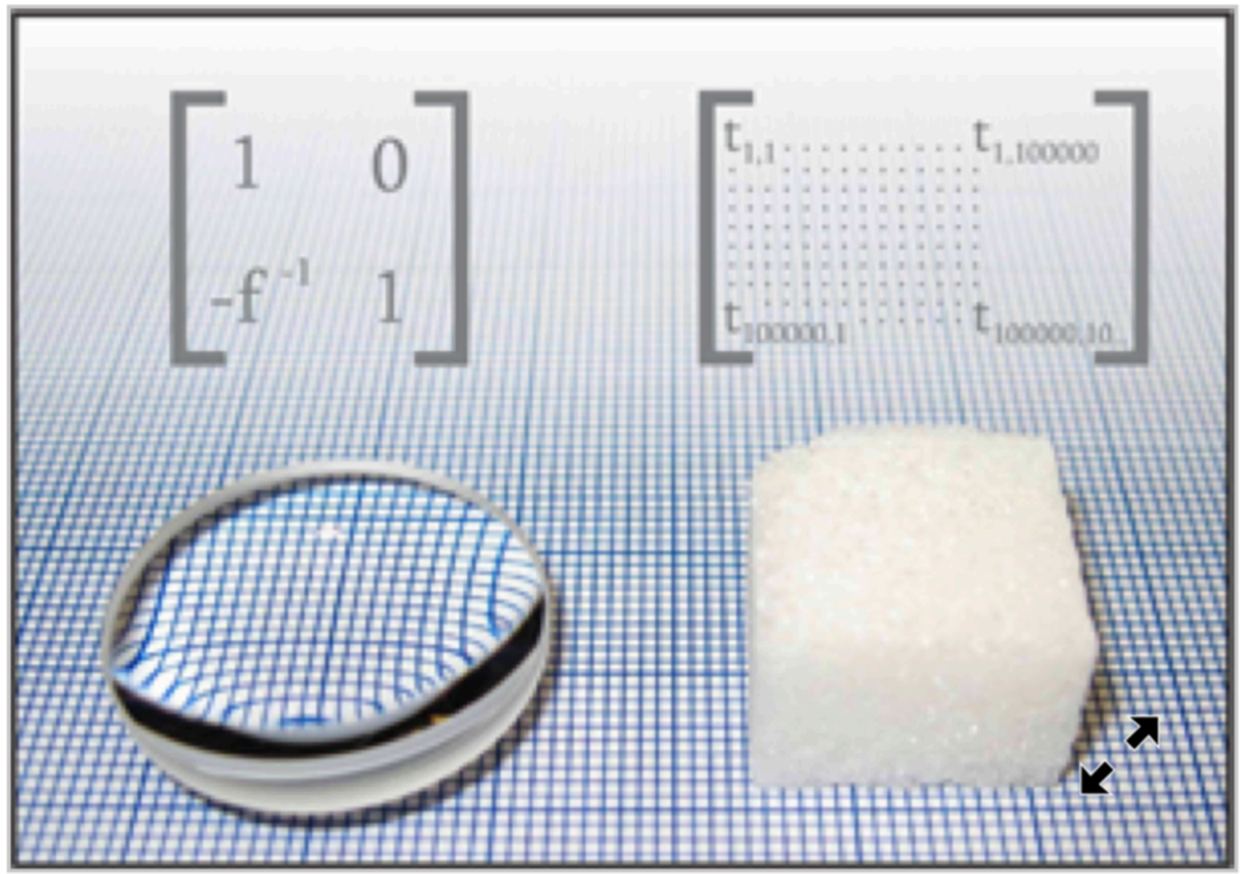

Figure 1: Two optical elements fully characterized by their transmission matrix, which relates the incident wave front to the transmitted one. In the case of a thin lens, the transformation of the wave front is described by a 2×2 matrix operating on a vector describing the wave front curvature [27]. For more complex elements such as a sugar cube the transmission matrix operates in a basis of transversal modes, which is very large. Full knowledge of the transmission matrix enables disordered materials to focus light as lenses.

Transmission Matrix - Disordered media

ARTICLES

Optical phase conjugation for turbidity suppression in biological samples

ZAHID YAQOOB[1†], DEMETRI PSALTIS[1,2], MICHAEL S. FELD[3] AND CHANGHUEI YANG[1*]

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[2]School of Engineering, Ecole Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland
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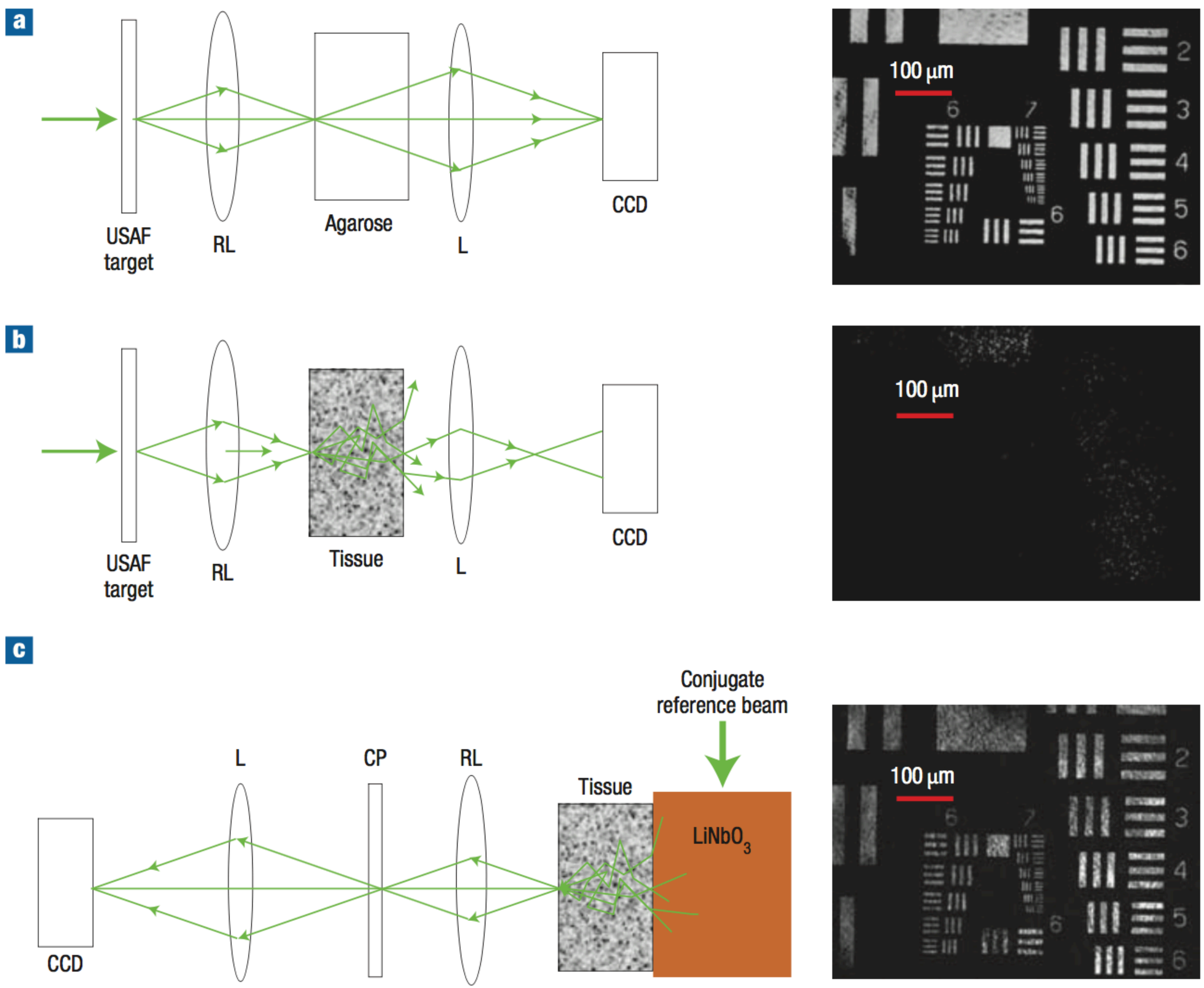
nature photonics | VOL 2 | FEBRUARY 2008 | www.nature.com/naturephotonics

Transmission Matrix - Disordered media



TRANSMISSION MATRIX - PROPERTIES

Transmission Matrix

$$\begin{pmatrix} E_1^{\text{out}} \\ E_2^{\text{out}} \\ E_3^{\text{out}} \\ \vdots \\ E_M^{\text{out}} \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} & t_{13} & \cdots & t_{1N} \\ t_{21} & t_{22} & t_{23} & \cdots & t_{2N} \\ t_{31} & t_{32} & t_{33} & \cdots & t_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ t_{M1} & t_{M2} & t_{M3} & \cdots & t_{MN} \end{pmatrix} \begin{pmatrix} E_1^{\text{in}} \\ E_2^{\text{in}} \\ E_3^{\text{in}} \\ \vdots \\ E_N^{\text{in}} \end{pmatrix}.$$

Output amplitudes　　Transmission matrix　　Input amplitudes

Linear Transformation

$$E_m^{\text{out}} = \sum_{n=1}^{N} t_{mn} E_n^{\text{in}},$$

Matrices and Linear Transformation

Theorem of Linear Algebra

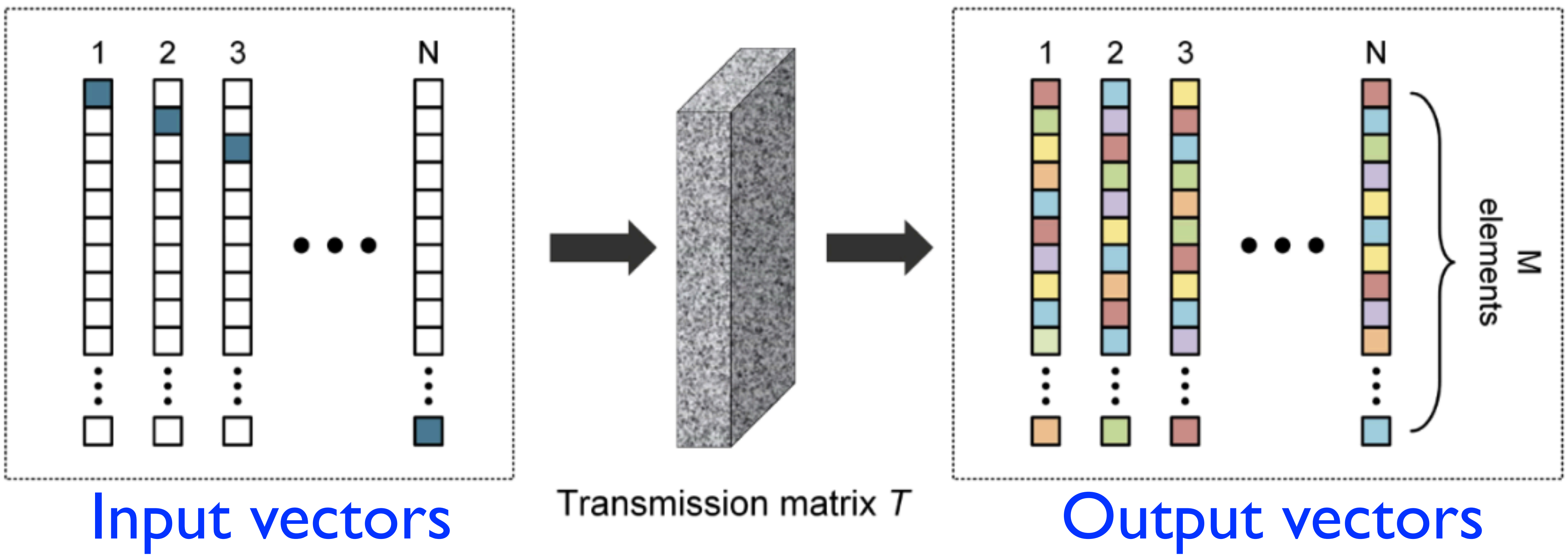
Let $T : \mathbb{R}^n \mapsto \mathbb{R}^m$ be a linear transformation. Then the matrix A satisfying $T\left(\vec{x}\right) = A\vec{x}$ is given by

$$A = \begin{bmatrix} | & & | \\ T\left(\vec{e}_1\right) & \cdots & T\left(\vec{e}_n\right) \\ | & & | \end{bmatrix}$$

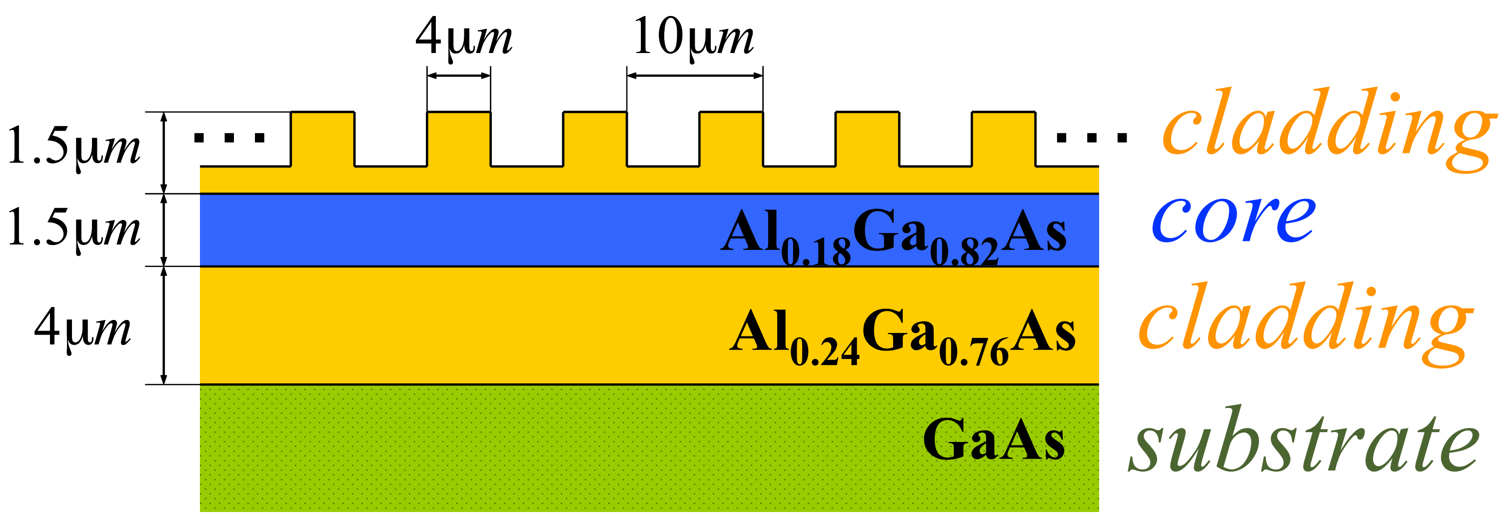
where $\vec{e}_i$ is the i^{th} column of I_n, and then $T\left(\vec{e}_i\right)$ is the i^{th} column of A.

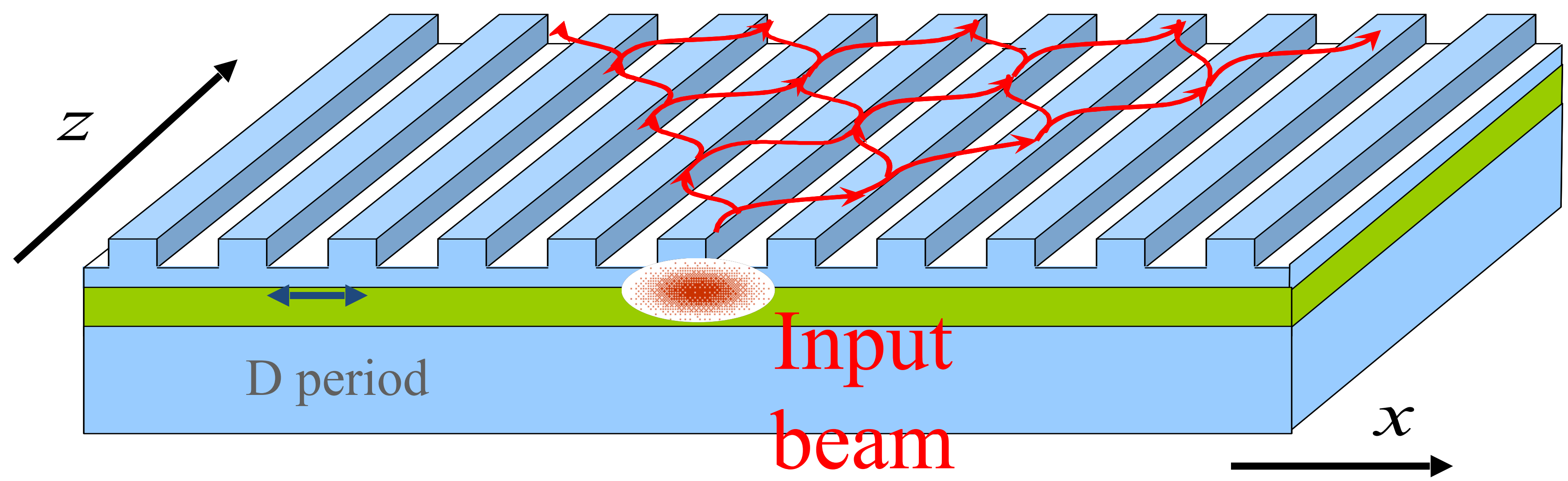
Every linear transformation is related to a matrix, given the bases of the two vector spaces that the linear map is defined

Transmission Matrix - Linear Transformation



Simplest Example - Waveguide Array





$$\boxed{i\frac{dc_n}{dz} + \kappa(c_{n+1} + c_{n-1}) + V_n c_n = 0}$$

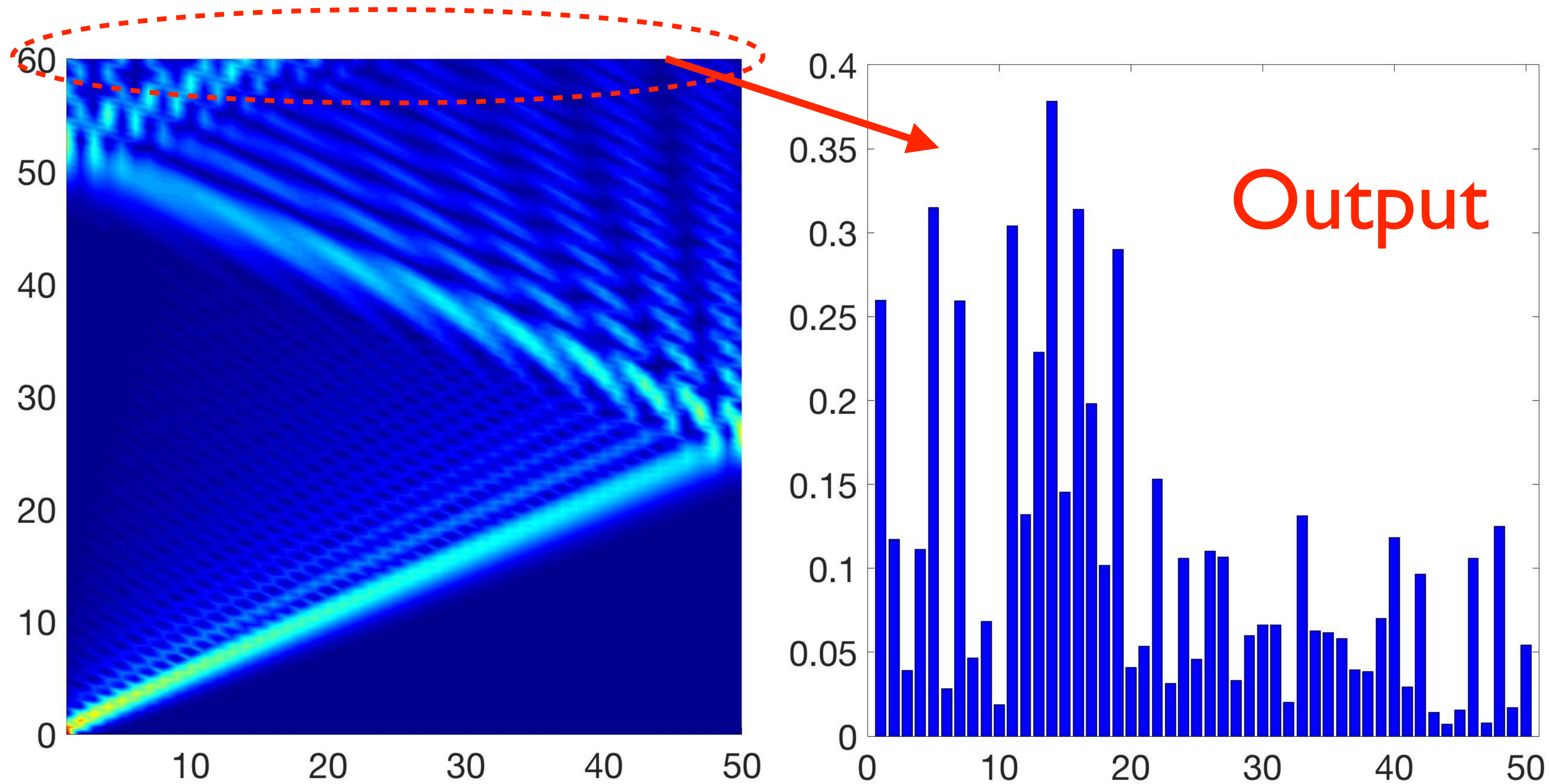
coupling

Discrete Diffraction Equation

Coupled mode Theory (tight-bind approximation)

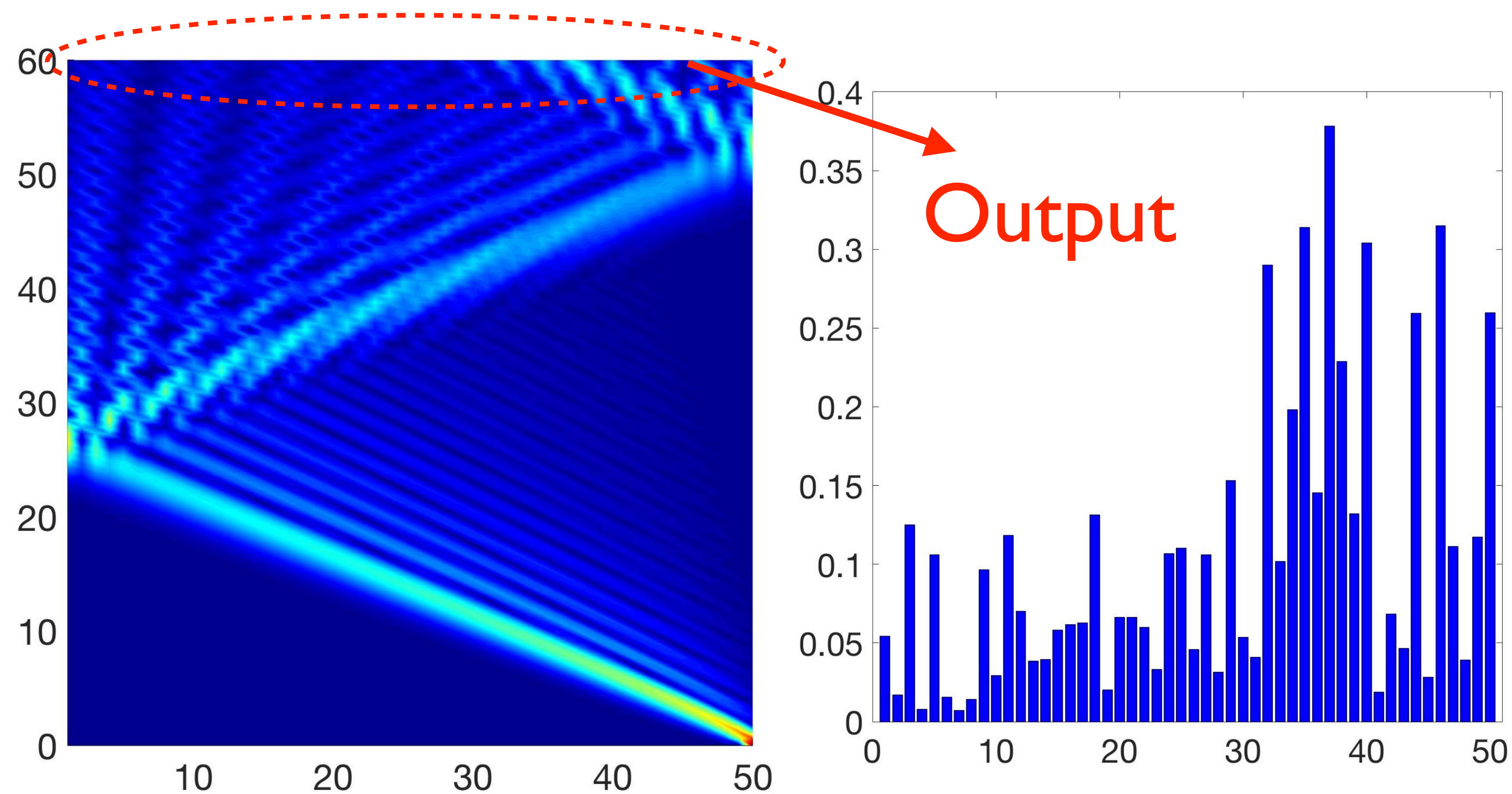
Constructing the transmission matrix

Input
$$|u_1> = \begin{bmatrix} 1 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$



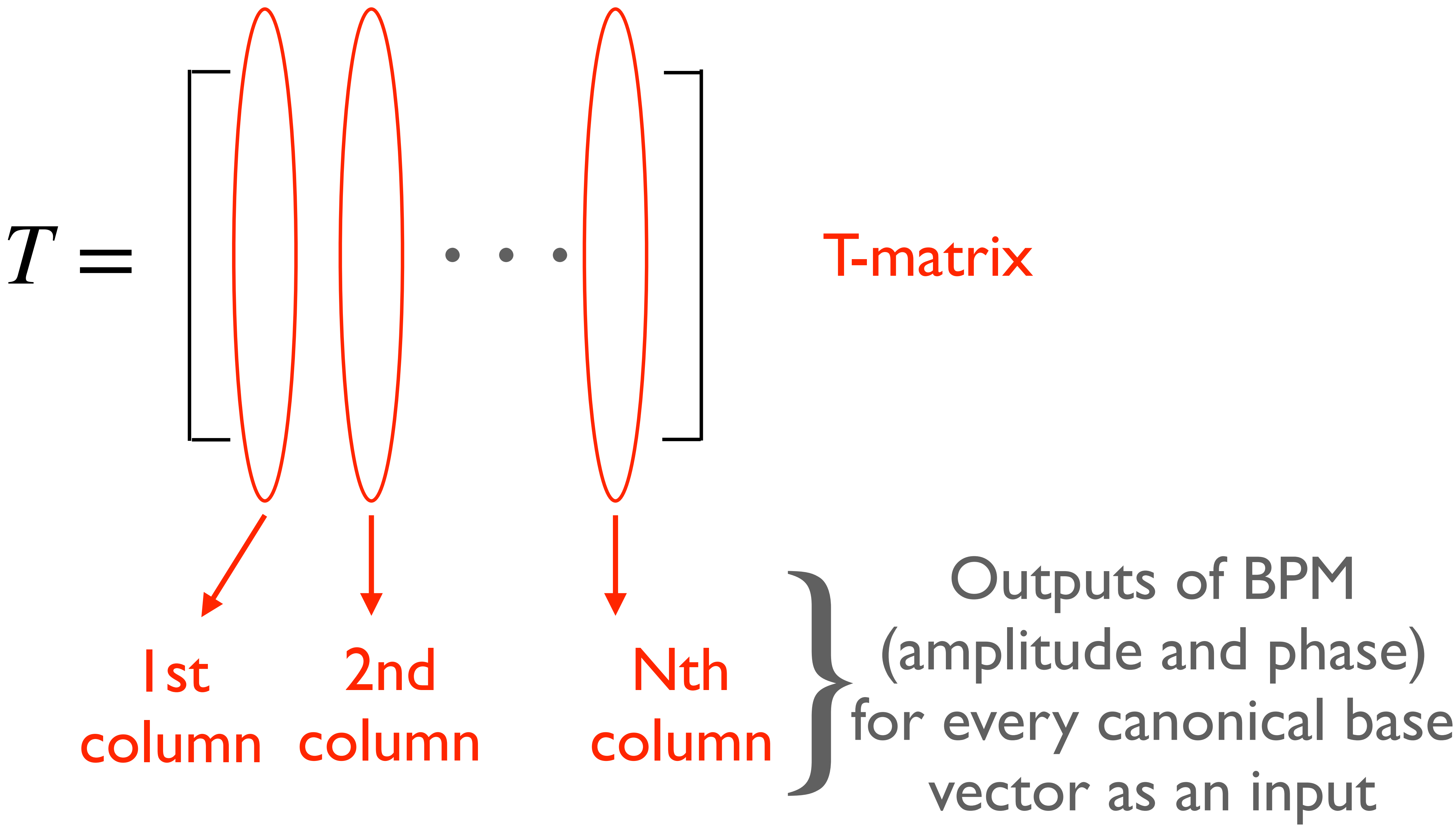
1st column of T-matrix

Input
$$|u_N> = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix}$$



Nth column of T-matrix

Constructing the transmission matrix



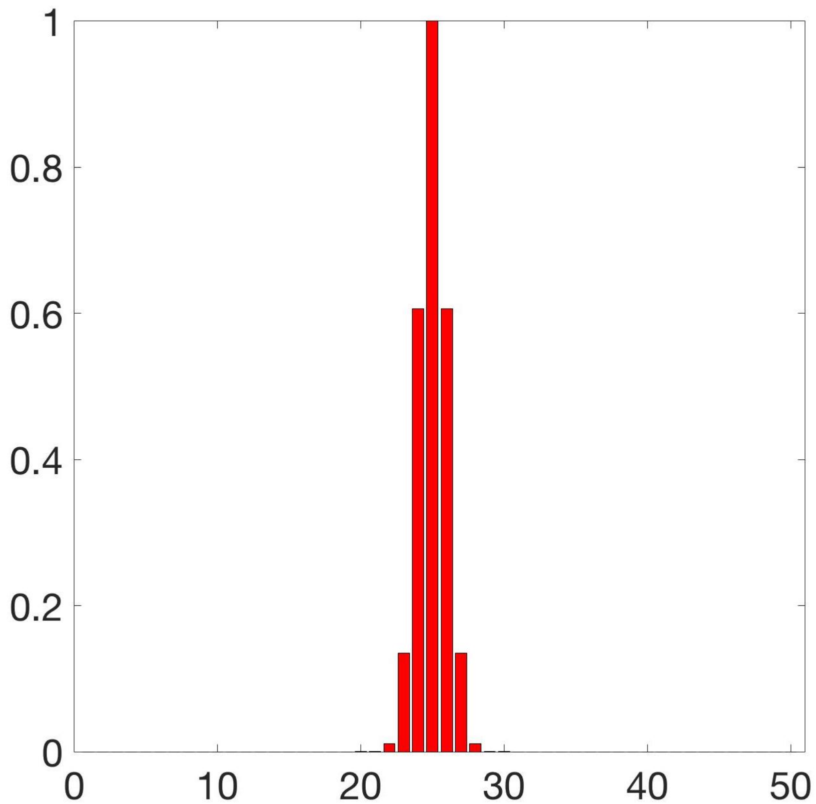
Focusing by inversion

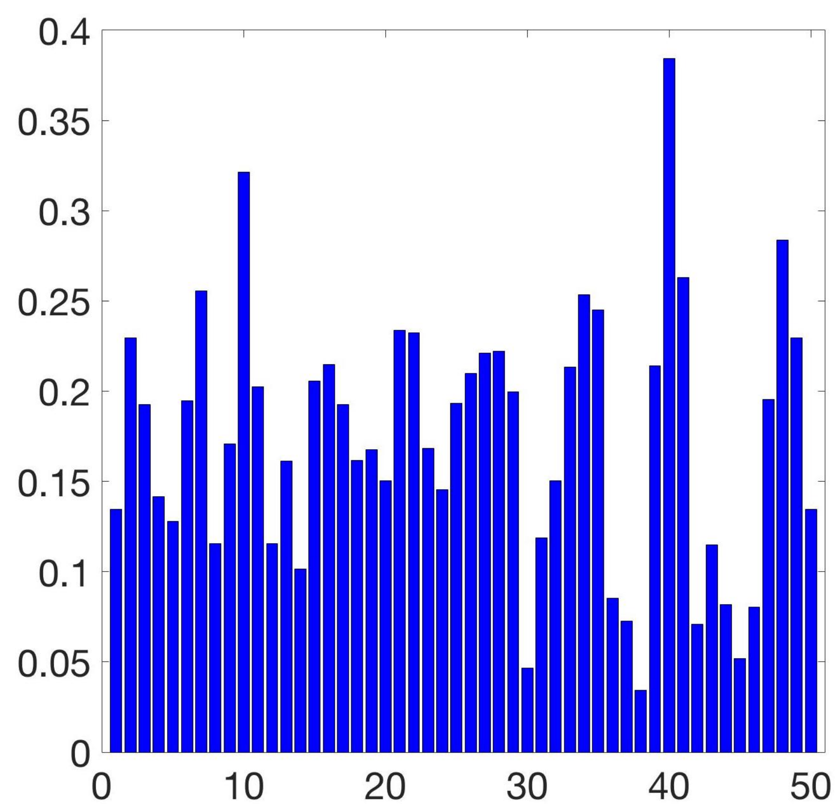
$$T|\psi_{in}> = |\psi_{out}> \Rightarrow$$

$$\boxed{T^{-1}|\psi_{out}> = |\psi_{in}>}$$

Desired Focused spot

Calculate input





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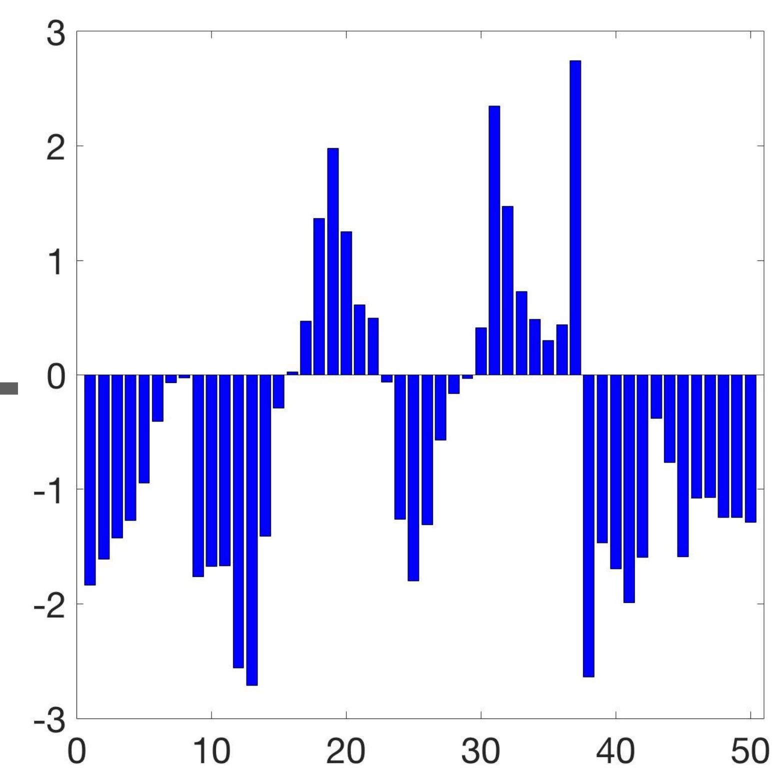

Amplitude

Phase

Focusing by inversion