Lecture 2

Demetri Psaltis

Optics Laboratory

EPFL - Spring semester 2022

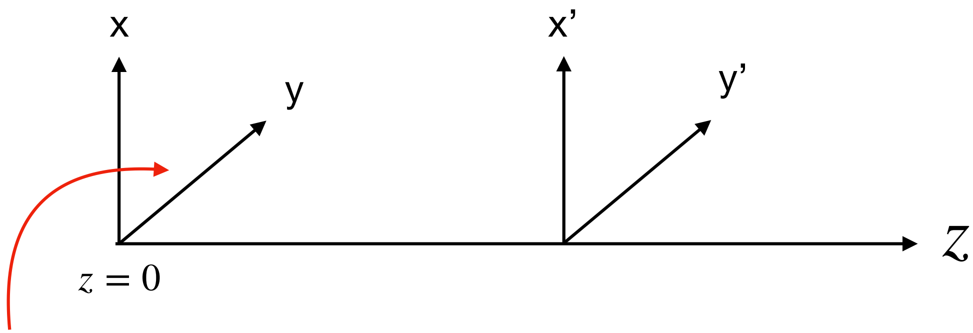
TOPICS

1.	Optical beam propagation; free space	(Feb 25)
2.	Plane waves, lens-less imaging	(March 4)
3.	Fresnel Diffraction, thin transparencies.	(March 11)
4.	Imaging, lenses, eye	(March 18)
5.	Ray optics (YE)	(March 25)
6.	Microscopy	(April 1)
7.	Waveguides, Endoscopy	(April 8)
8.	Midterm ??	(April 29)
9.	Coherence, interferometry, OCT	(May 6)
10.	Periodic structures Professor Makris?	(May 13)_
11.	Holography (Lyncee)	(May 20)
12.	Ptychography, Tomography.	(May 27)
13.	Final (written NOT oral)	(June 3)

Plan for today

1. Review beam propagation method (BPM)
2. Plane waves
3. Angular spectrum
4. Periodic patterns, gratings
5. Imaging without a lens: Talbot imaging
6. Imaging without a lens: Near field
7. Felix: JIULIAcode for grating diffraction
8. Homework

Propagation in Free Space



$$E'(x, y, z = 0) = |E(x, y, z = 0)|\,cos(\omega t + \phi(x, y, z = 0))$$

For z>0 in complex notation:

$$E = |E(x, y, z)|\,e^{-j\phi(x,y,z)}e^{j\omega t}$$

Slow varying envelope:

$$E(x, y, z) = A(x, y, z)\ e^{j\phi(x,y,x)}\ e^{j\omega t}e^{-jkz}$$

Where

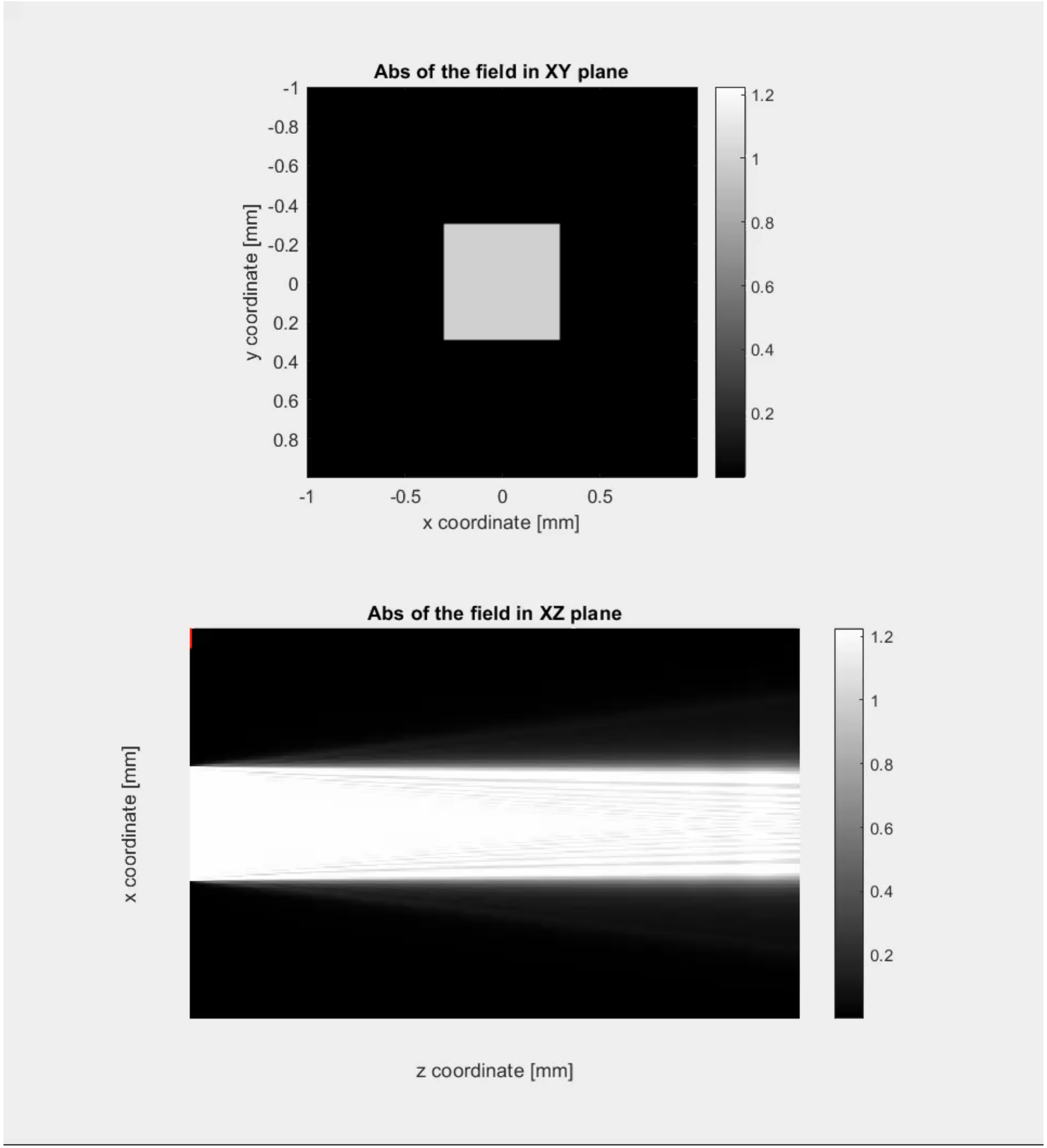
$$A(x, y, z) = |E(x, y, z)|\,e^{j\phi(x,y,z)}e^{+jkz}$$

$$\boxed{\begin{aligned} k &= \frac{2\pi}{\lambda} \\ \omega &= 2\pi\nu \\ c &= \lambda\nu \end{aligned}}$$

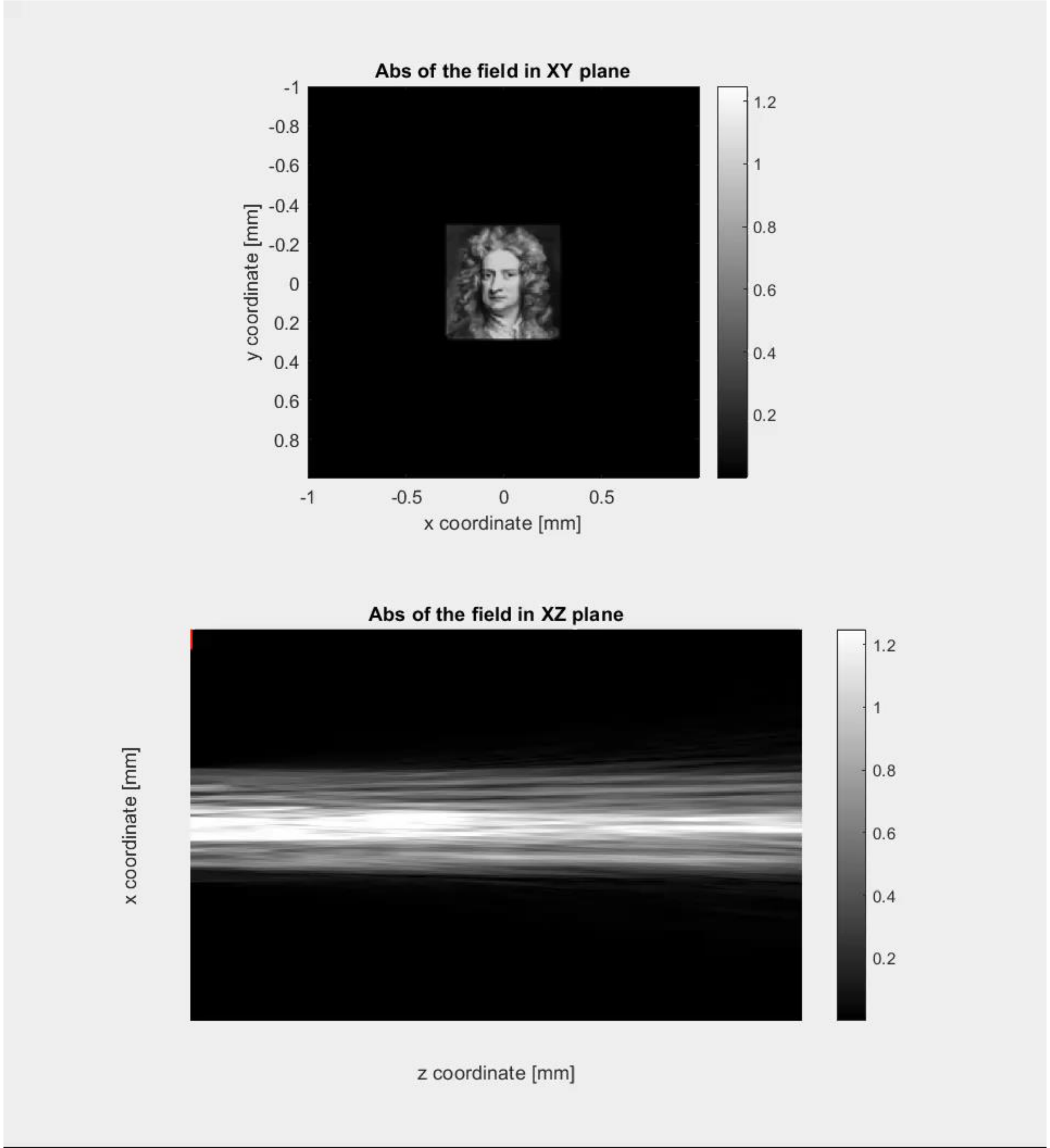
Beam propagation equationn

$$\frac{dA}{dz} = -\frac{j}{2k}\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\right)$$

Diffraction Square aperture



Diffraction Newton



Plane waves

$$\frac{\partial A}{\partial z} = -\frac{j}{2k}(\frac{\partial^2 A +}{\partial x^2}\frac{\partial^2 A}{\partial y^2})$$

$$If\ A(x, y, z = 0) = A_0\ is\ constant\ (no\ x\ or\ y\ dependence)\ then\ \frac{\partial A}{\partial z} = 0$$

$$The\ field\ is\ then\ given\ by\ E(z) = A_0 e^{(j\omega t - kz)}$$

Plane waves

$$\nabla \times \nabla \times \vec{E} = -\mu \frac{\partial(\nabla \times \vec{H})}{\partial t} = -\varepsilon\mu \frac{\partial^2 E}{\partial t^2}$$

$$\nabla \times \nabla \times \vec{E} = -\nabla^2 \vec{E} + \nabla(\nabla \bullet \vec{E}) = -\nabla^2 \vec{E}$$

This gives us the wave equation

$$\nabla^2 \vec{E} = \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2}$$

This is a vector equation. We can write in terms of each of the elements of the vector, for example E_{x}.

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = -\omega^2 \mu\varepsilon E_x$$

Solution to this is the plane wave:

$$E_x(x,y,z,t) = A\cos(\omega t - k_x x - k_y y - k_z z)$$

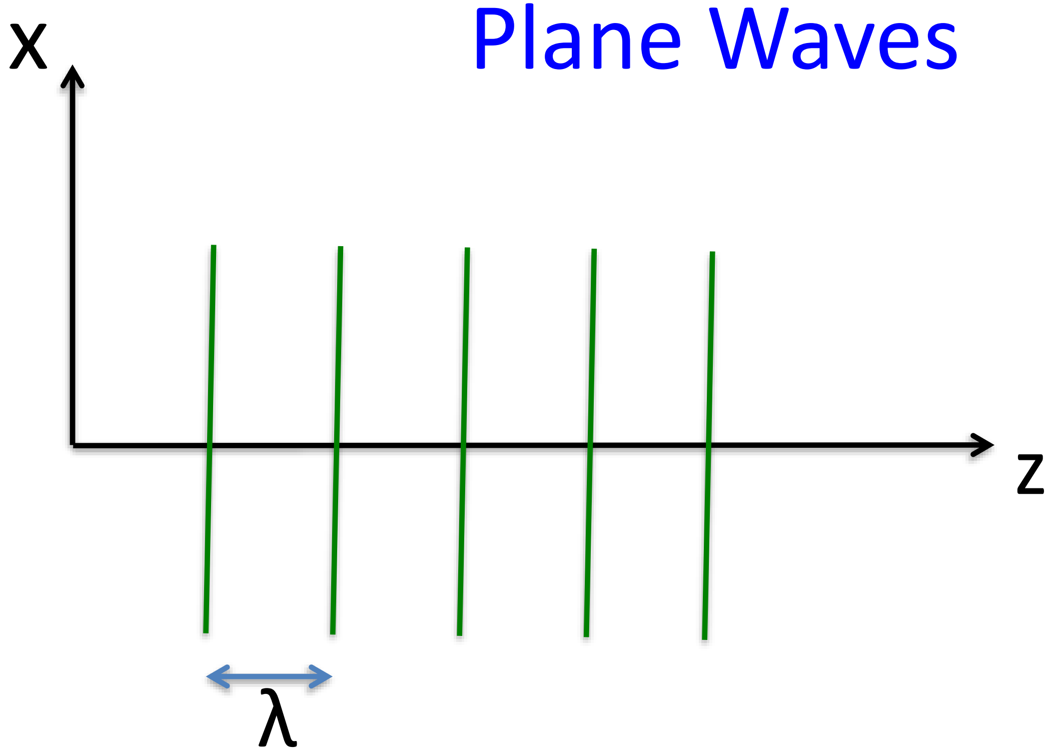
Or in complex notation

$$E_x(x,y,z,t) = Ae^{j(\omega t - k_x x - k_y y - k_z z)}$$

Substituting this back into the wave equation

$$k^2 = k_x^2 \ + k_y^2 + k_z^2 = \omega^2 \mu\varepsilon$$

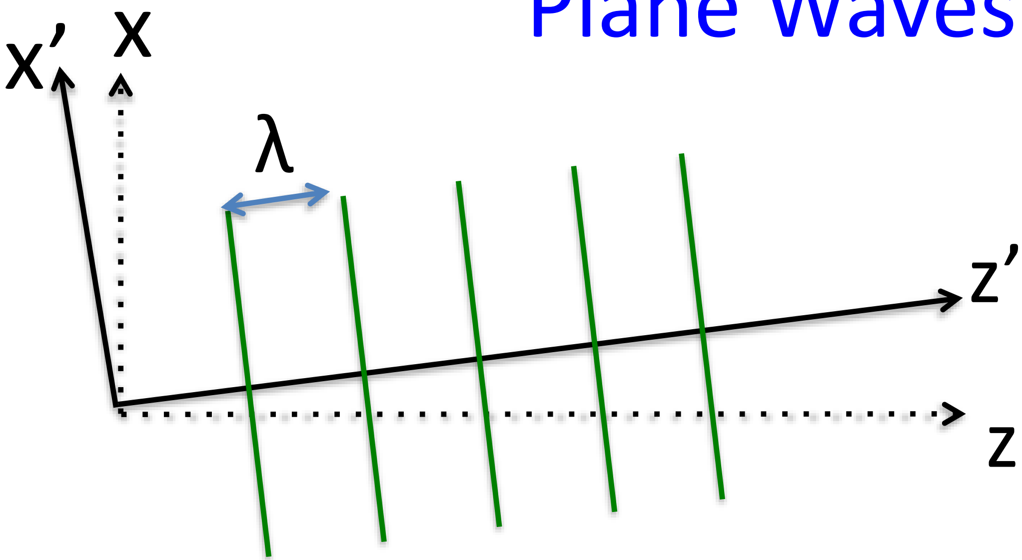
Plane Waves



$$k_x = 0 \text{ Þ } \quad E(x,y,z,t) = Ae^{j(wt - kz)}$$

$$k = 2p / l = w / v$$

Plane Waves



$$k_x \neq 0 \Rightarrow \quad E(x',y',z',t) = Ae^{j(\omega t - kz')}$$

$$z' = x\cos\theta_x + z\cos\theta_z \Rightarrow \; E = Ae^{j(\omega t - k\cos\theta_x x + k\cos\theta_z x)}$$

$$k_x = k\cos\theta_x \qquad k_z = k\cos\theta_z$$

$$k^2 = k_x^2 + k_z^2 = \left(\frac{2\pi}{\lambda}\right)^2 \Rightarrow \; k_z = \sqrt{k^2 - k_x^2}$$

Eigenmodes and eigenvalues?

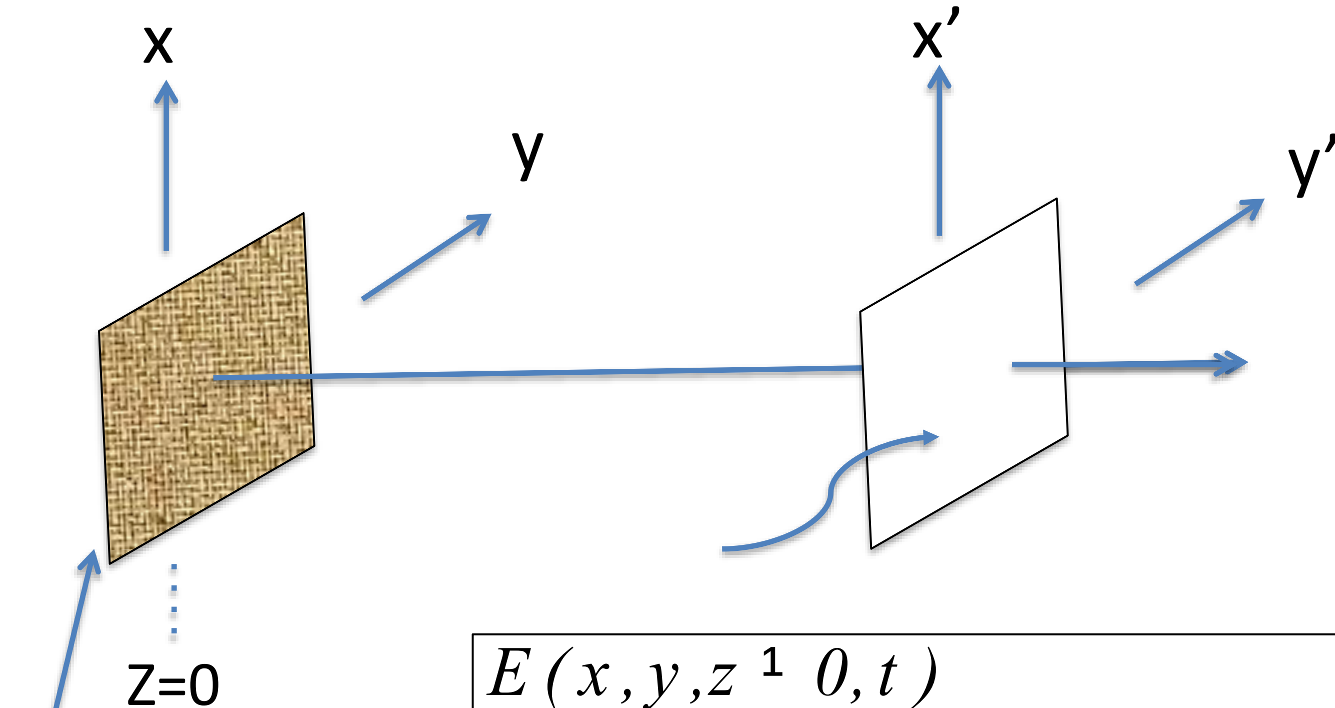
2D Fourier transform

$$F(u,v) = \int f(x,y) e^{-j2\pi(ux+vy)}\,dx\,dy$$

$$f(x,y) = \frac{1}{2} + \frac{1}{2}\cos(2\pi u_0 x + 2\pi v_0 y)$$

$$\Rightarrow \quad F(u,v) = \frac{1}{2}\delta(u,v) + \frac{1}{4}\delta(u \pm u_0, v \pm v_0)$$

Angular spectrum



$$E(x,y,z \neq 0,t) = e^{j\omega t} \iint F(u,v) e^{j(2\pi ux + 2\pi vy + \sqrt{k^2 - (2\pi u)^2 - (2\pi u)^2}\, z)}\, du\, dv$$

$$E(x,y,z=0,t) = Ae^{j\omega t} f(x,y) = Ae^{j\omega t} (2\pi)^2 \iint F(u,v) e^{j2\pi(ux+vy)}\, du\, dv$$

$$k_x = 2\pi u \quad k_y = 2\pi v \quad k_z = \sqrt{k^2 - k_x^2 - k_y^2} = k_z = \sqrt{k^2 - (2\pi u)^2 - (2\pi u)^2}$$

Derivation of BPM

$$E_x(x,y,z,t) = A(x,y,z)e^{j(wt-kz)}$$

A Þ *slow envelope*

$$\frac{\partial^2 E_x}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial A}{\partial x}\right)e^{j\omega t}e^{-jkz} = \frac{\partial^2 A}{\partial x^2}e^{j\omega t}e^{-jkz}$$ same for y

$$\frac{\partial^2 E_x}{\partial z^2} = \frac{\partial}{\partial z}\left(\frac{\partial A}{\partial z}e^{-jkz} - jkAe^{-jkz}\right)e^{j\omega t} = \left(\frac{\partial^2 A}{\partial z^2} - 2jk\frac{\partial A}{\partial z} - k^2 A\right)e^{j\omega t}e^{-jkz}$$

Plugging back into the wave equation and dropping the oscillatory terms that are common to all terms we obtain:

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2} - 2jk\frac{\partial A}{\partial z} - k^2 A = -\omega^2 \mu\varepsilon A$$

Recognizing that

$$-k^2 A = -\omega^2 \mu\varepsilon A$$

and making the approximation that the second derivative in z of the *A* envelope (slowly varying in z) is negligible we obtain

$$2jk\frac{\partial A}{\partial z} \simeq \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}$$

or

$$\boxed{\frac{\partial A}{\partial z} \simeq -\frac{j}{2k}\left(\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2}\right)} \quad \xrightarrow{\textit{FT}} \quad \boxed{\tilde{A}(k_x,k_y,z) = e^{j\left[\frac{k_x^2+k_y^2}{2k}\right]z}\tilde{A}(k_x,k_y,z=0)}$$

Paraxial Approximation

$$e^{-j(k_x x + k_y y + k_z z)}$$

$$k_x^2 + k_y^2 + k_z^2 = k^2 = \left(\frac{2\pi}{\lambda}\right)^2$$

$$e^{-j(k_x x + k_y y + k_z z)} = e^{-j(k_x x + k_y y)} e^{-j\sqrt{k^2 - k_x^2 - k_y^2}\, z}$$

$$= e^{-j(k_x x + k_y y)} e^{-j\left(k - \frac{k_x^2 + k_y^2}{2k}\right) z}$$

Note: $\sqrt{1-x} \approx 1 - \frac{x}{2},\ x \ll 1$

Gratings

Periodic structures

$$\text{Periodic function}: f(x,y) = f(x-\alpha, y-\alpha) \quad \text{period } \alpha$$

$$F(u,v) = \int f(x,y)\, e^{-j2\pi(ux+vy)}\, dxdy$$

$$\int f(x-\alpha, y-\alpha)\, e^{-j2\pi(ux+vy)}\, dxdy = F(u,v)\, e^{-j2\pi(\alpha u+\alpha v)}$$

$$\Rightarrow \; F(u,v) = F(u,v)\, e^{-j2\pi(\alpha u+\alpha v)}$$

$$\Rightarrow \; F(u,v) = 0 \text{ except when } u = n/\alpha \text{ and } v = m/\alpha$$

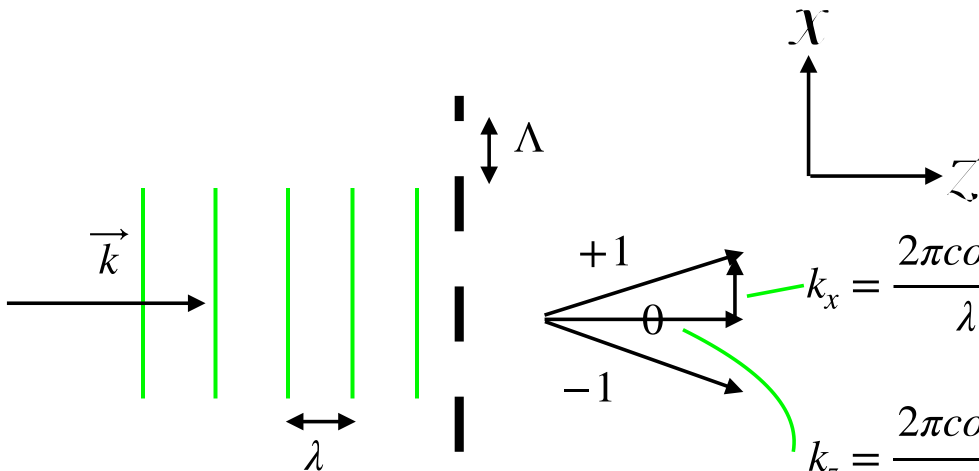
$$i.e.\; F(u,v) = \sum_n \sum_m F_{nm}\, \delta\!\left(u - \frac{n}{\alpha}, v - \frac{m}{\alpha}\right)$$

$$\text{where } F_{nm} = \iint_{\alpha} f(x,y)\, e^{-j2\pi\left(\frac{n}{\alpha}x + \frac{m}{\alpha}y\right)}$$

$$\text{Fourier Series}: f(x,y) = \sum_n \sum_m F_{nm}\, e^{j2\pi\left(\frac{n}{\alpha}x + \frac{m}{\alpha}y\right)}$$

Grating

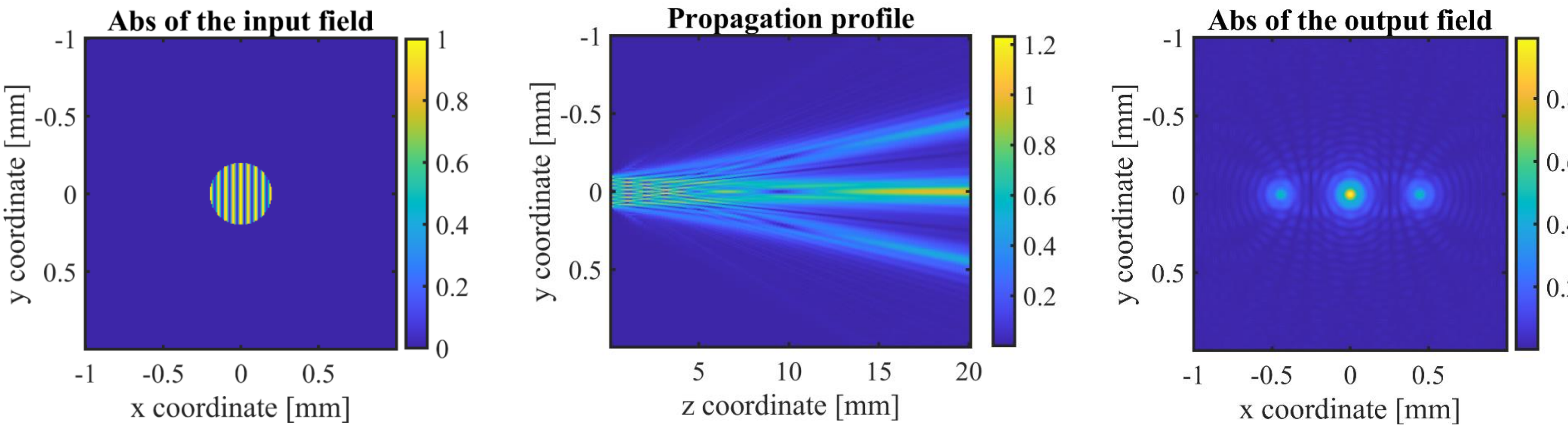
$$k_x^2 + k_y^2 + k_z^2 = \omega^2\mu\epsilon = (\frac{2\pi}{\lambda})^2 = k^2 \qquad k_z = \sqrt{k^2 - (k_x^2 + k_y^2)}$$



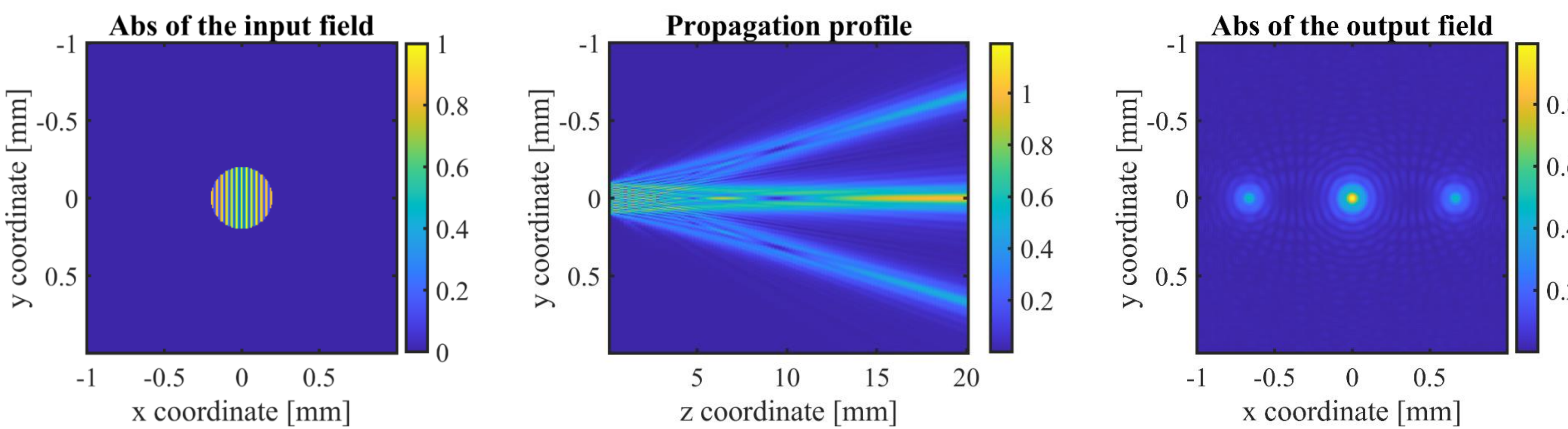
$$k_x = \frac{2\pi cos\theta_x}{\lambda} = \frac{2\pi}{\Lambda}$$

$$k_z = \frac{2\pi cos\theta_z}{\lambda} = \frac{2\pi}{\Lambda}$$

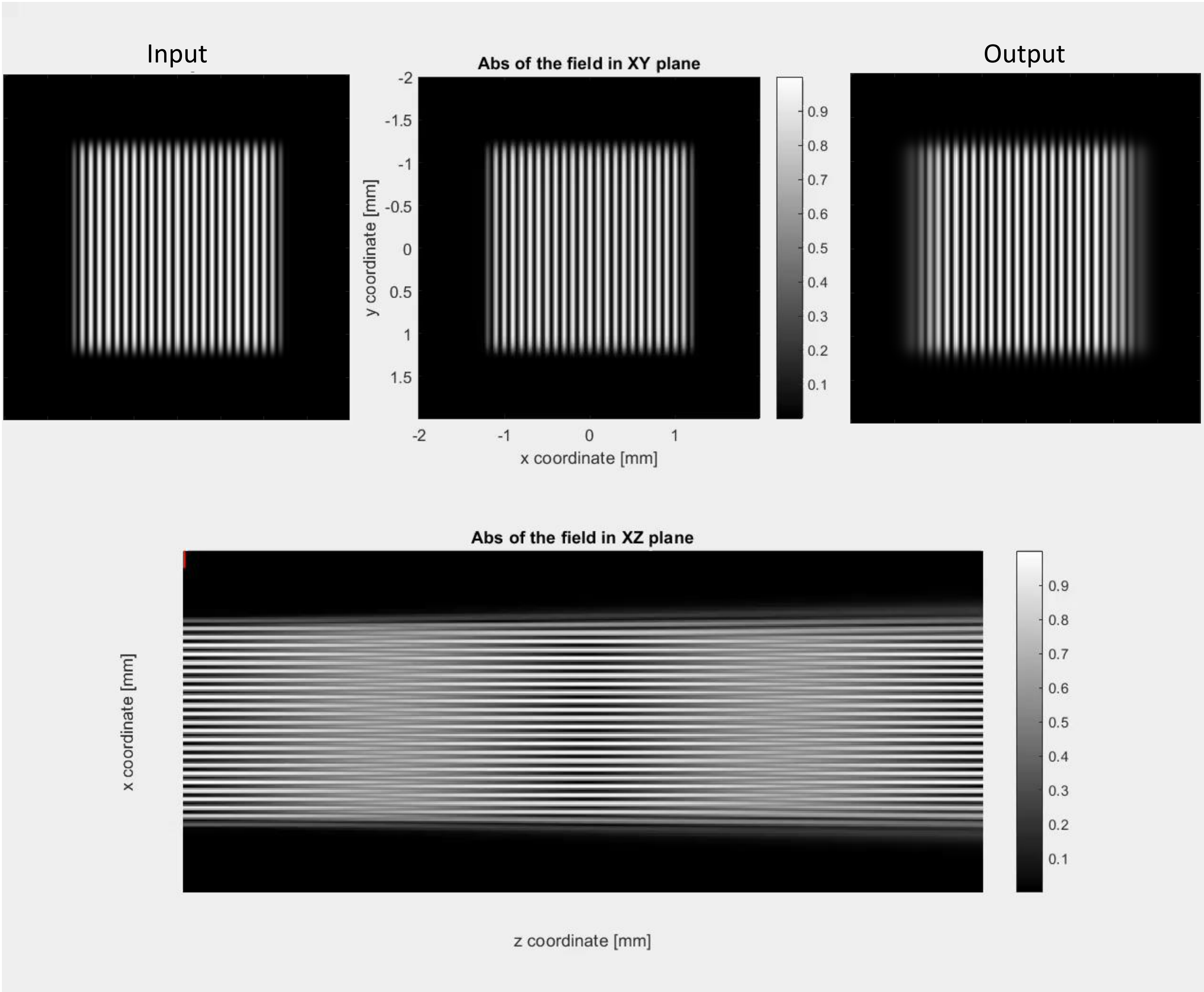
Period: 24 µm



Period: 16 µm



Talbot Effect



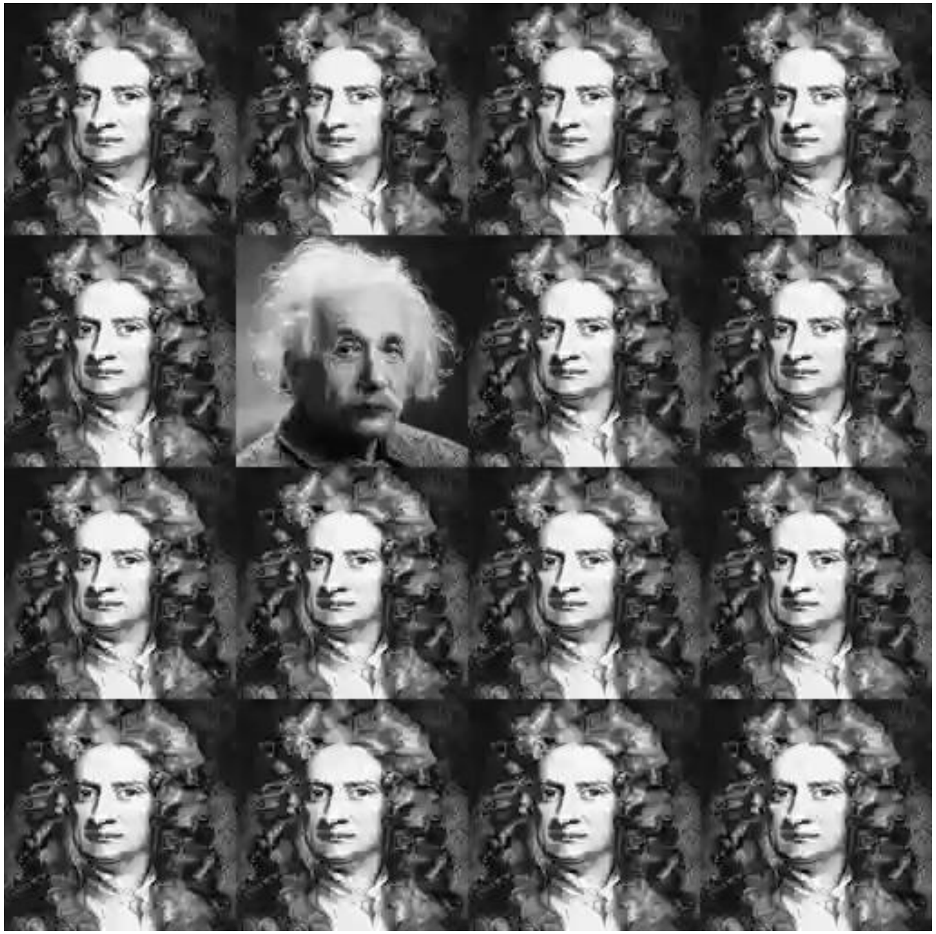
Talbot effect explanation

$$for\ diffraction\ order\ n:\quad \frac{k_x^2}{2k}z = \frac{(2\pi n/a)^2}{2(2\pi/\lambda)}z$$

$$for\ diffraction\ order\ n+1:\quad \frac{k_x^2}{2k}z = \frac{(2\pi(n+1)/a)^2}{2(2\pi/\lambda)}z$$

$$difference: \frac{(\frac{2\pi}{a})^2[(n+1)^2 - n^2]z}{4\pi/\lambda} = \pi\frac{\lambda}{a^2}(2n+1)z$$

$$\Rightarrow\ z_T = \frac{2a^2}{\lambda}$$



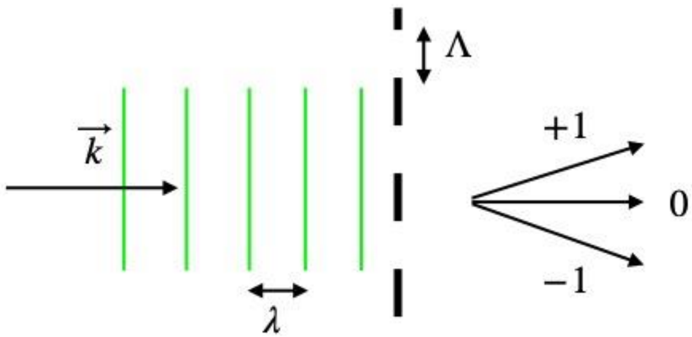
Imaging without lenses

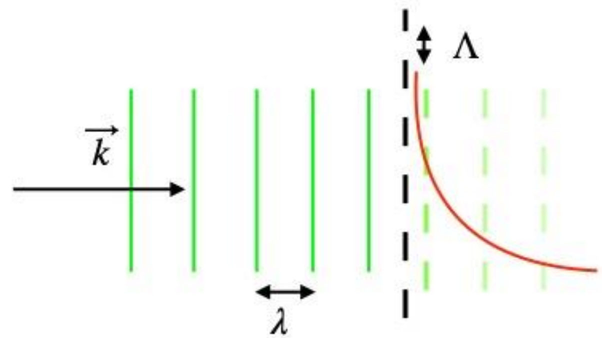
Near field scanning

Evanescent Waves

$$k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \epsilon = (\frac{2\pi}{\lambda})^2 = k^2$$

$$k_z = \sqrt{k^2 - (k_x^2 + k_y^2)}$$





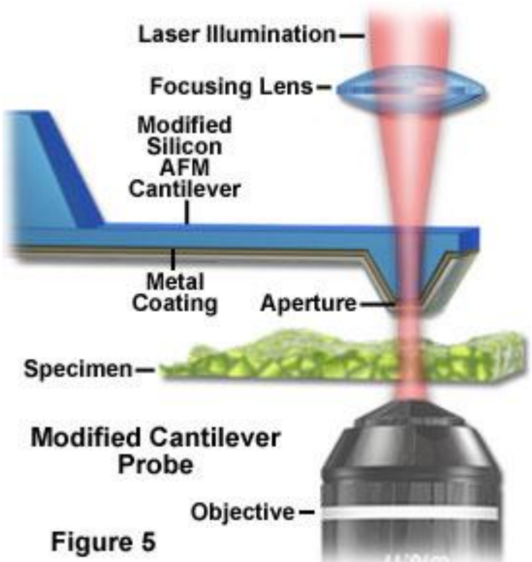
Exponential Decay

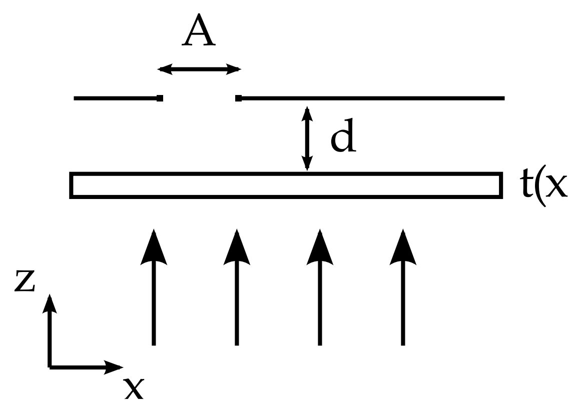
Λ<λ

$$k_x = \frac{2\pi cos\theta_x}{\lambda} = \frac{2\pi}{\Lambda}$$

$$k_z = \sqrt{k^2 - (k_x)^2} = \sqrt{(\frac{2\pi}{\lambda})^2 - (\frac{2\pi}{L})^2}$$

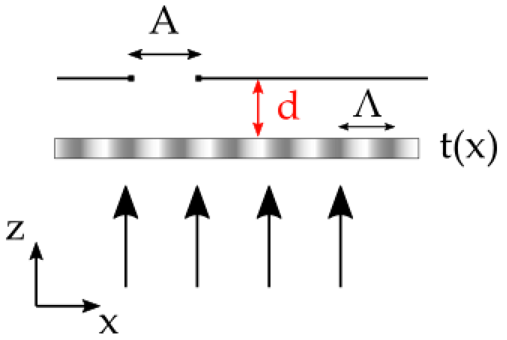
Near-field imaging

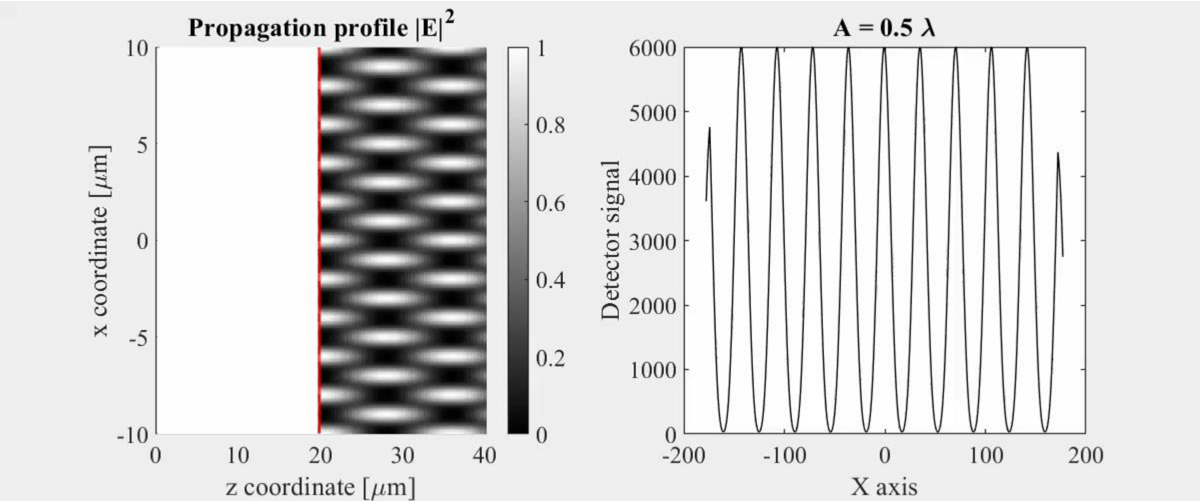




Example 1: Grating

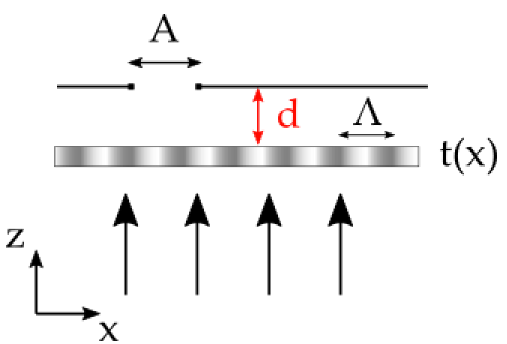
$$t(x) = \frac{1}{2} + \frac{1}{2}\cos\left(2\pi\frac{x}{\Lambda}\right), \Lambda = 4\lambda$$

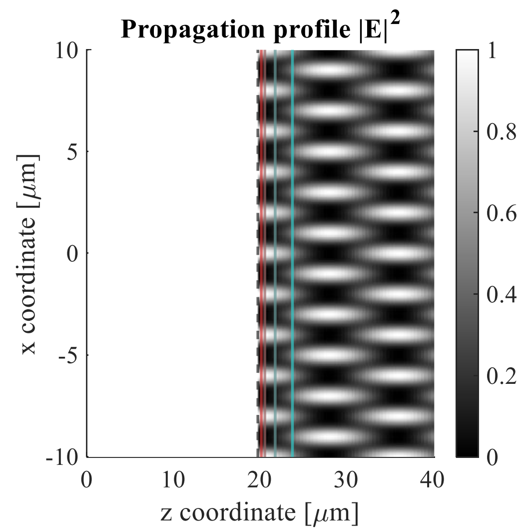


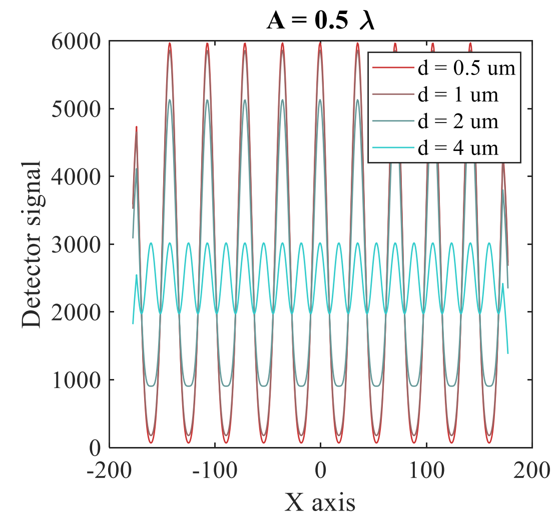


Example 1: Grating

$$t(x) = \frac{1}{2} + \frac{1}{2}\cos\left(2\pi\frac{x}{\Lambda}\right), \Lambda = 4\lambda$$

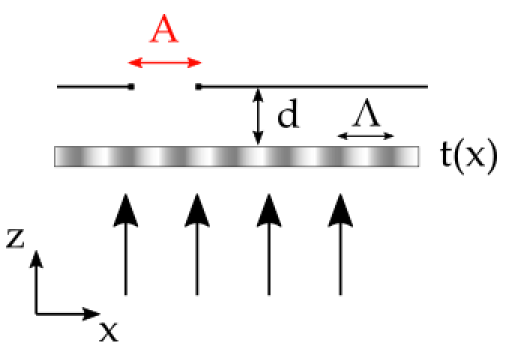


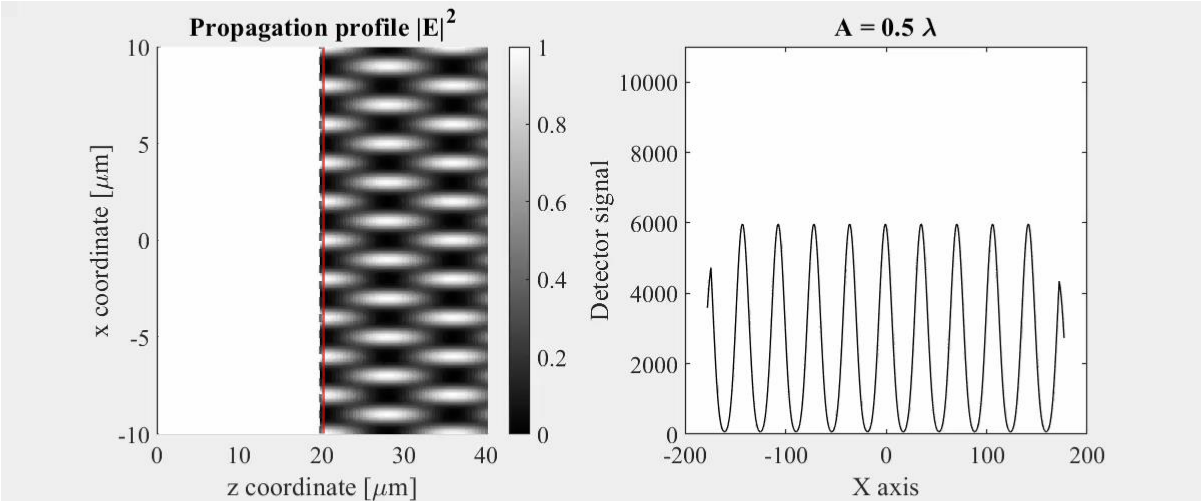




Example 1: Grating

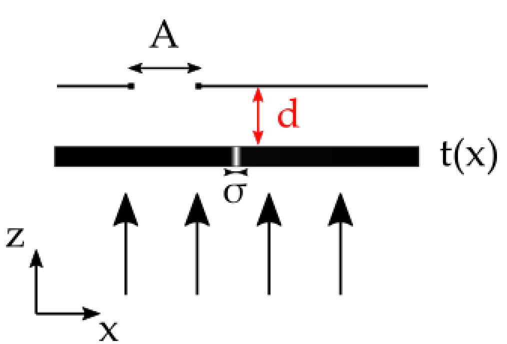
$$t(x) = \frac{1}{2} + \frac{1}{2}\cos\left(2\pi \frac{x}{\Lambda}\right), \Lambda = 4\lambda$$

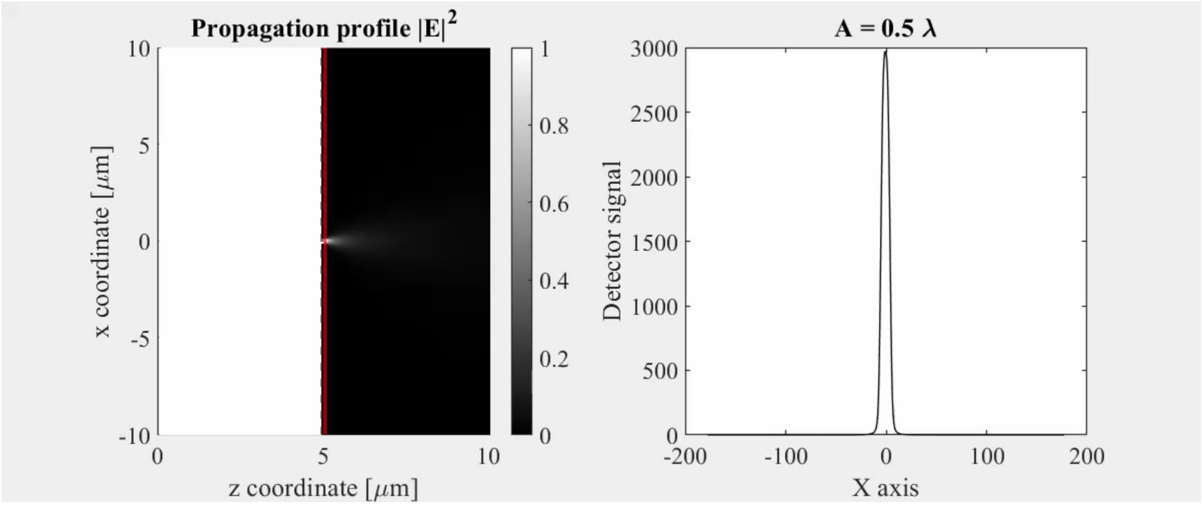




Example 2: Point source
Point Spread Function (PSF)

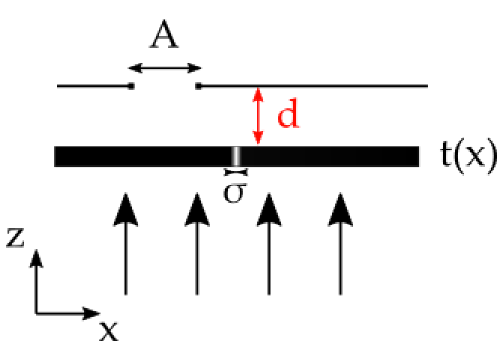
$\sigma = \lambda/2$

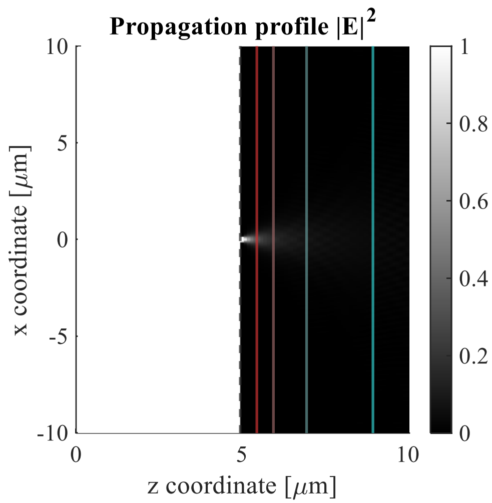


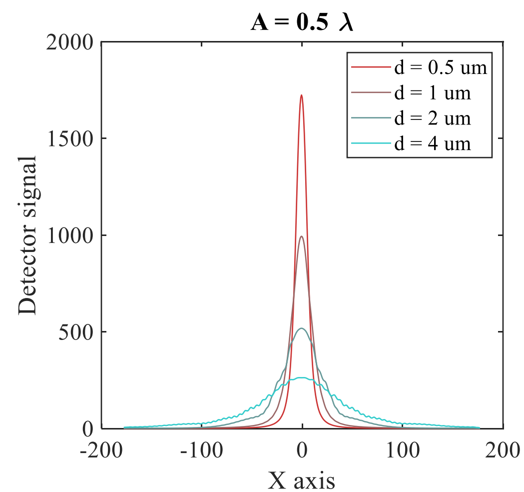


Example 2: Point source

$\sigma = \lambda/2$

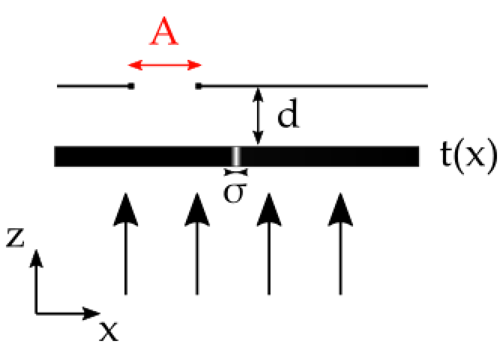


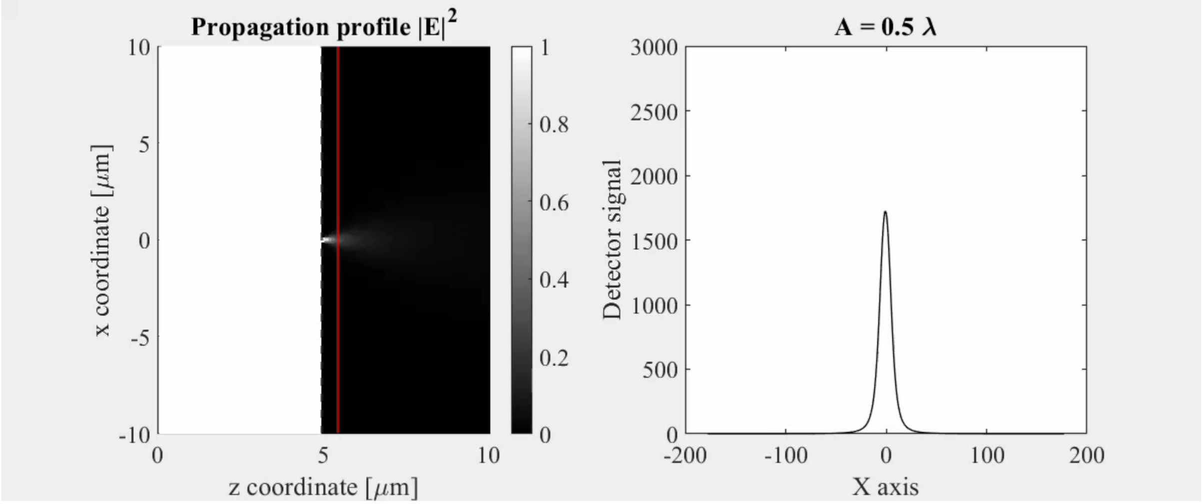




Example 2: Point source

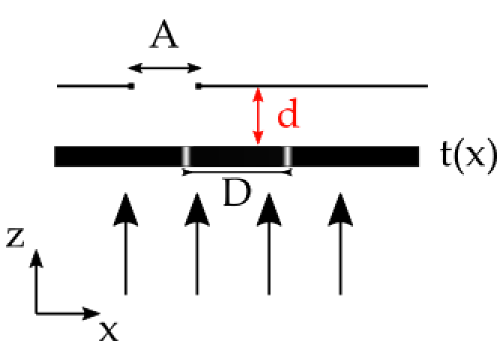
$\sigma = \lambda/2$

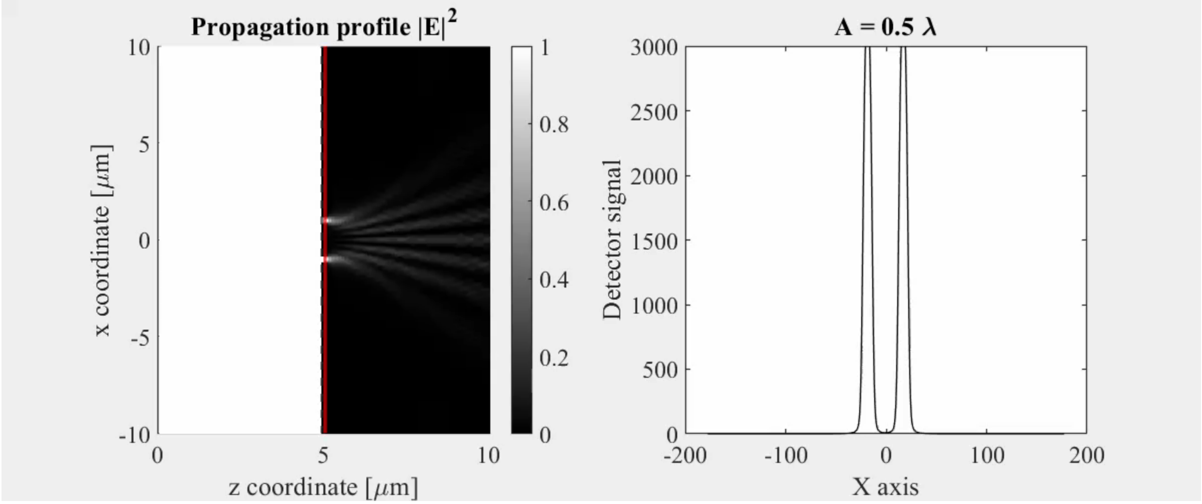




Example 3: Double slit

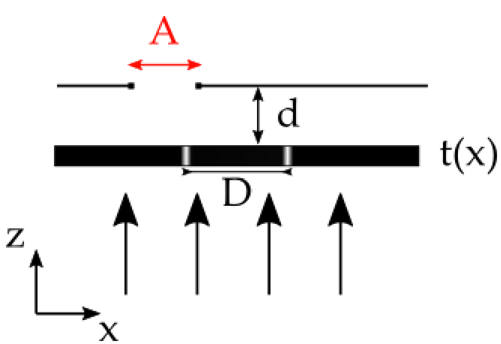
$D = 2\lambda$

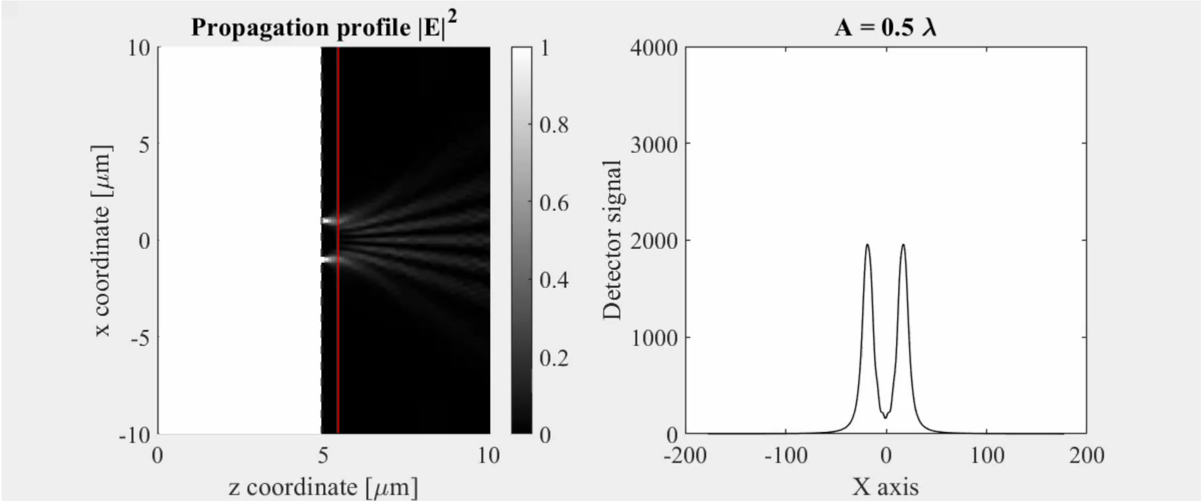




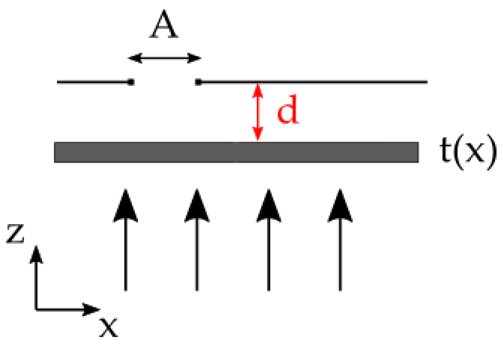
Example 3: Double slit

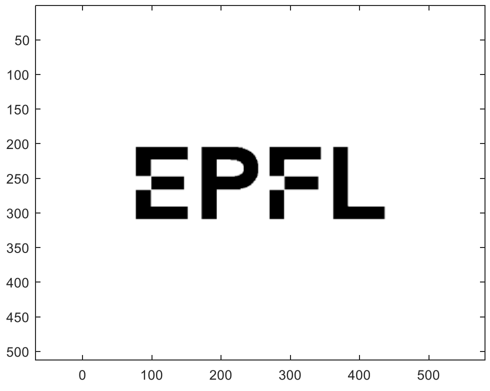
$D = 2\lambda$

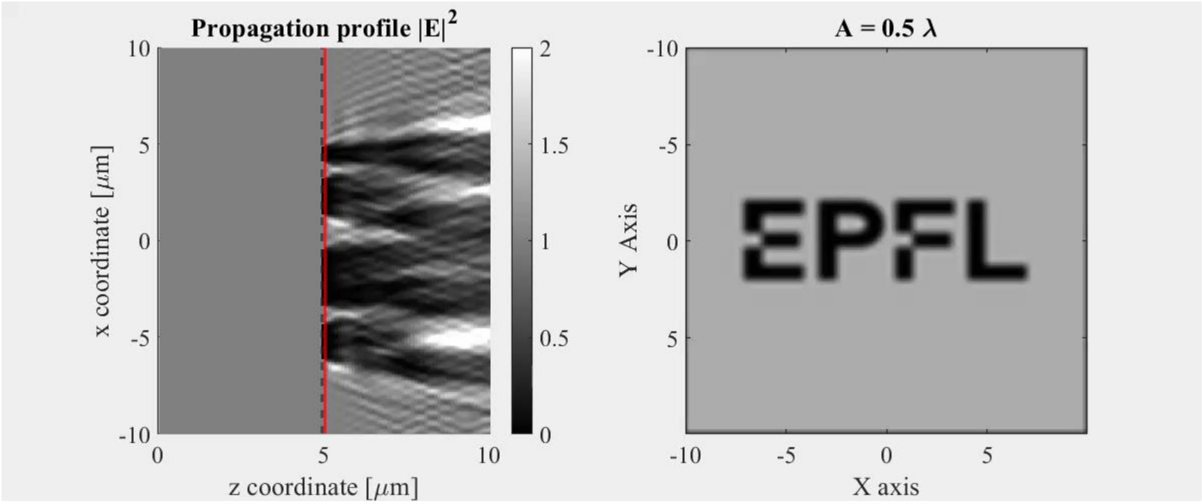




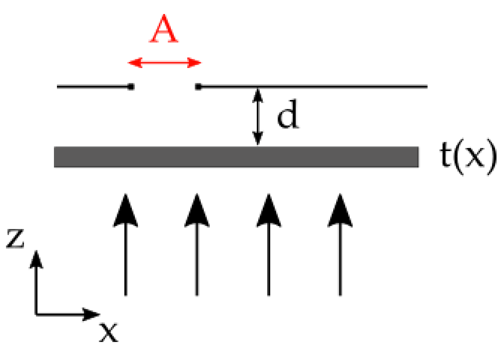
Example 4: Image

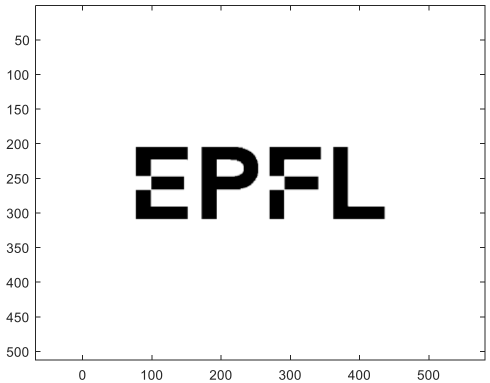


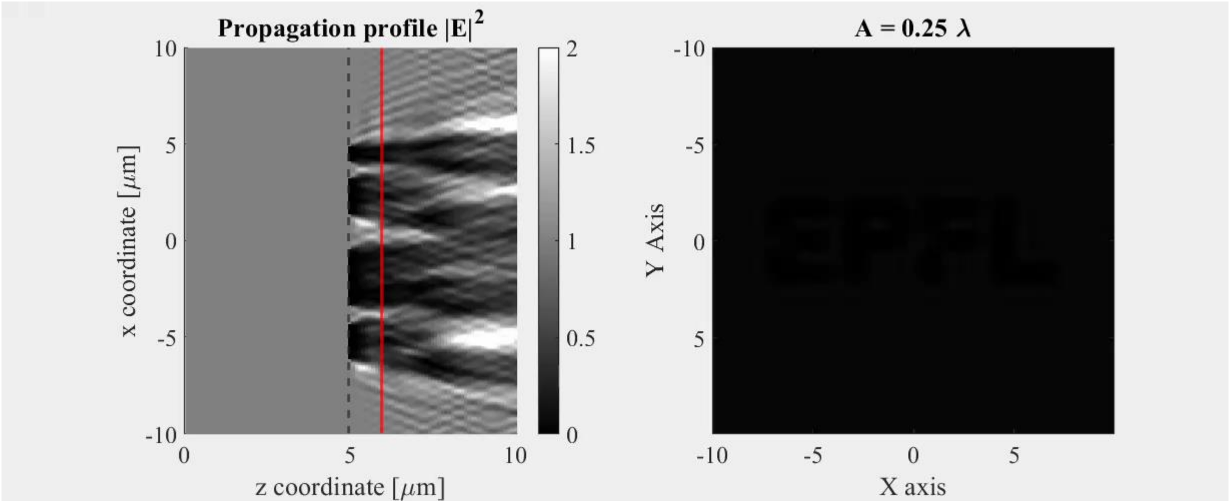




Example 4: Image







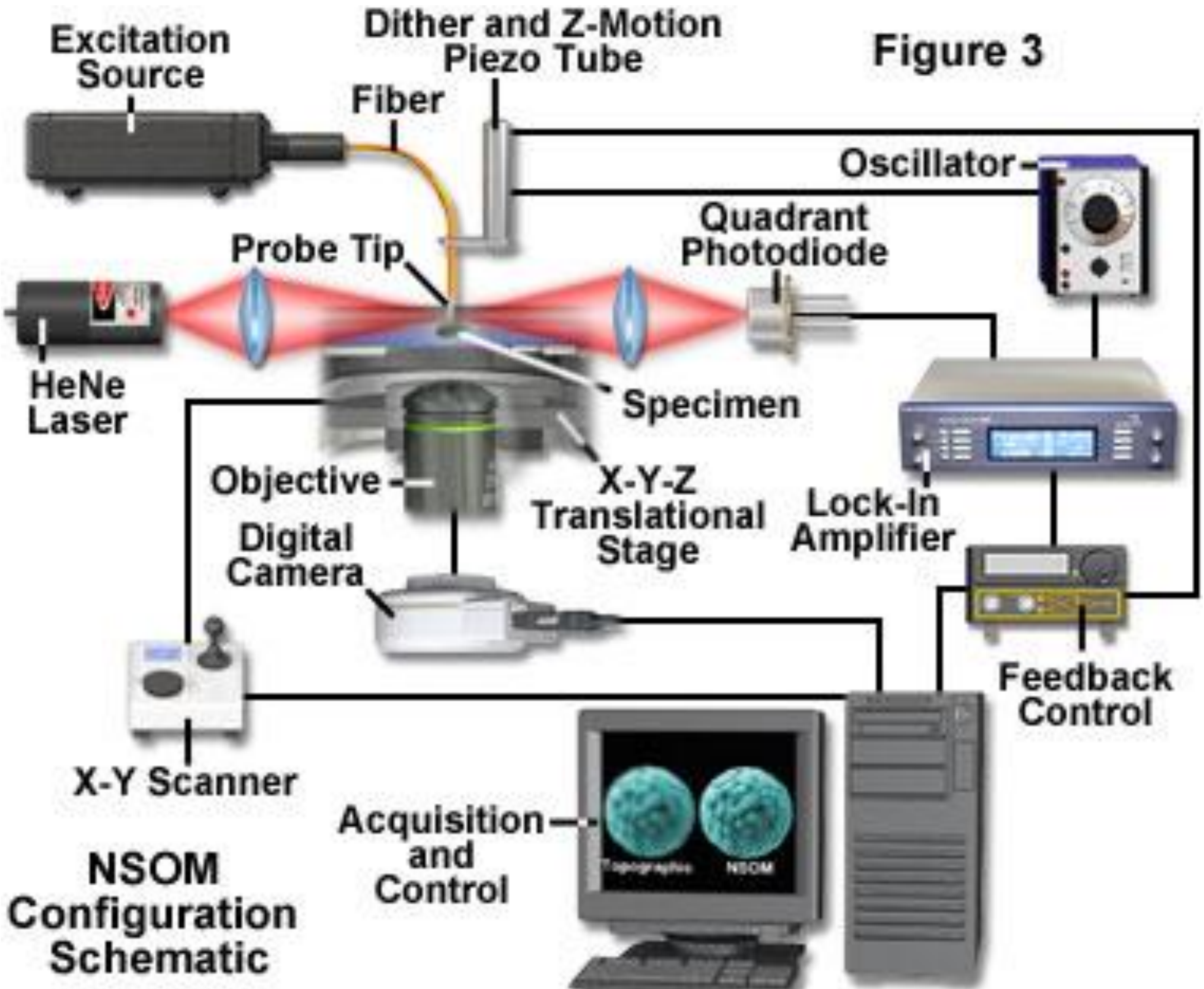
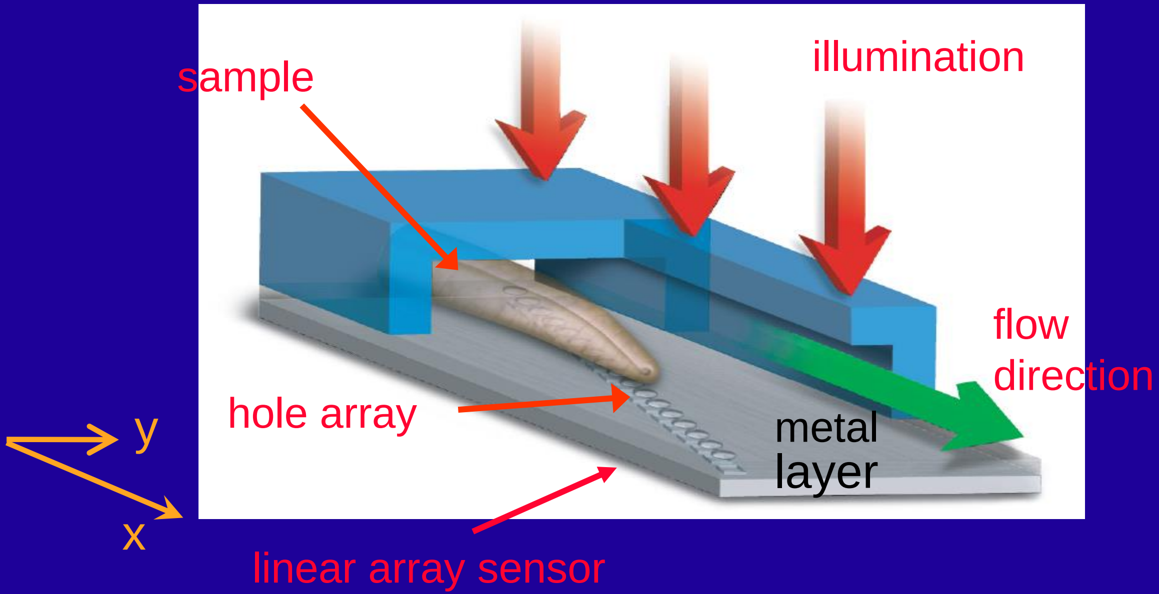
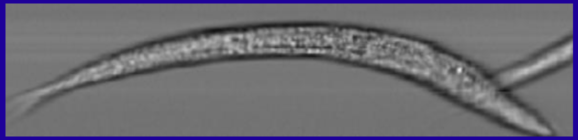


Figure 3
Excitation Source
Dither and Z-Motion Piezo Tube
Fiber
Oscillator
Quadrant Photodiode
Probe Tip
HeNe Laser
Specimen
Objective
X-Y-Z Translational Stage
Lock-In Amplifier
Digital Camera
Feedback Control
X-Y Scanner
Acquisition and Control
Topographics
NSOM
NSOM Configuration Schematic

Optofluidic Microscope (OFM)



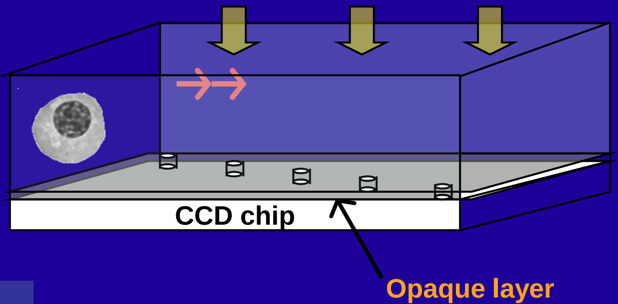


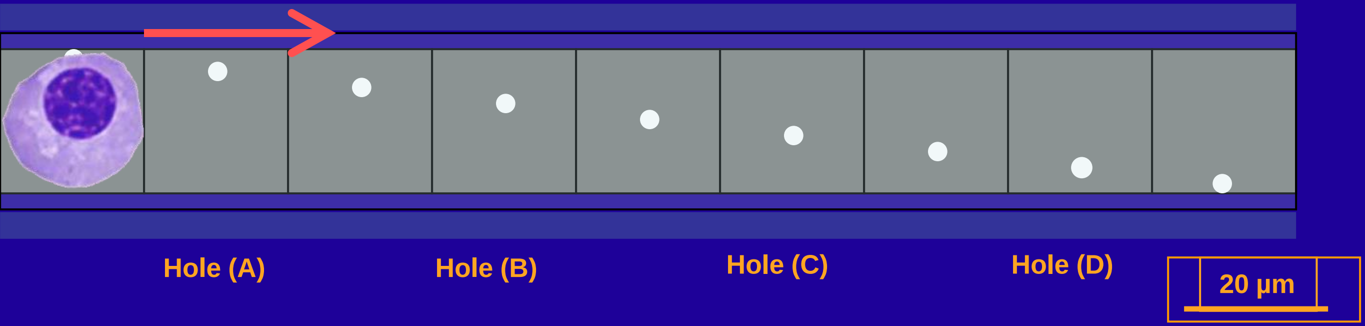
OFM

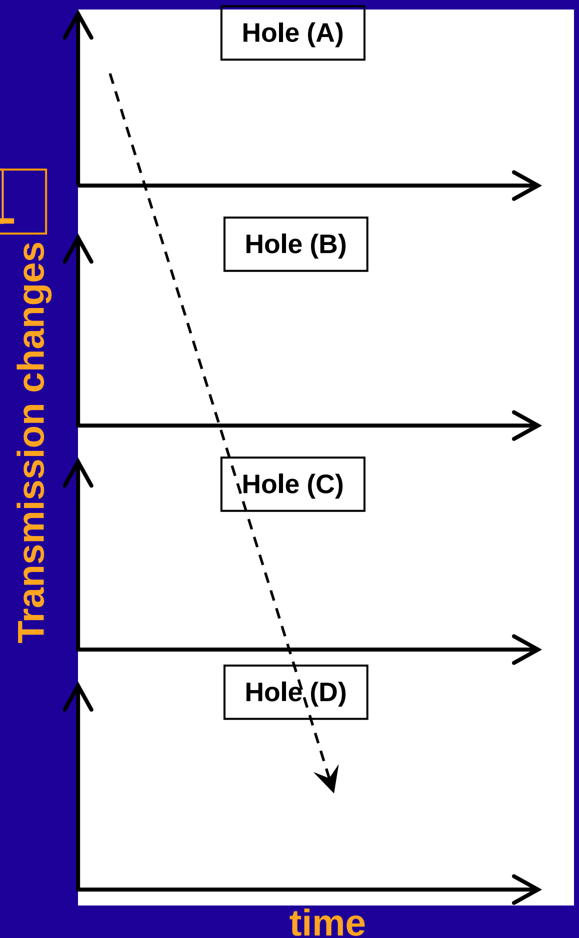
Conventional microscope

Xin Heng, David Erickson, Larry R. Baugh, Zahid Yaqoob, Paul Sternberg, Demetri Psaltis, Changhuei Yang.
'Optofluidic Microscopy: A Method for Implementing High Resolution Optical Microscope On A Chip,'
Lab on a Chip, August (2006)

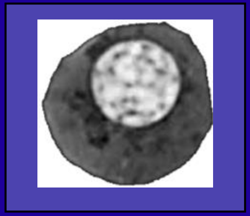
Optofluidic Microscope (OFM)



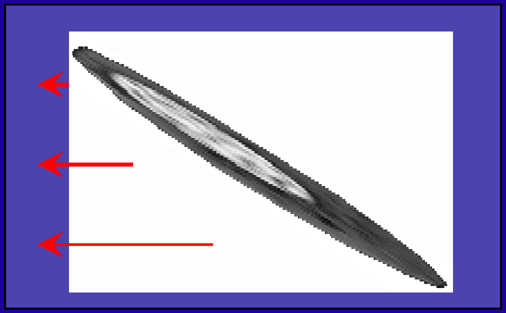




Slanted nanohole 1D array
Improve resolution in both x and y direction !!!



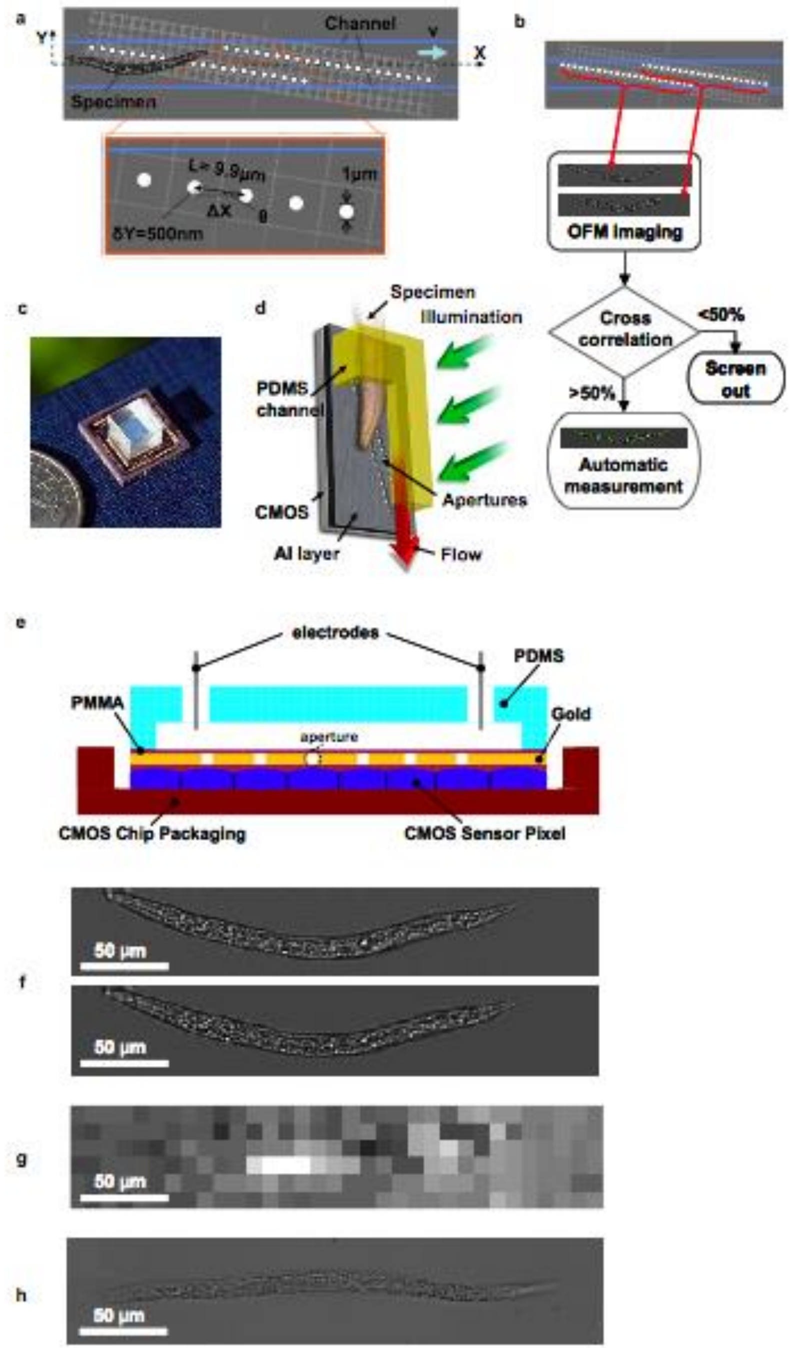
Final !!!



Shift forward linearly

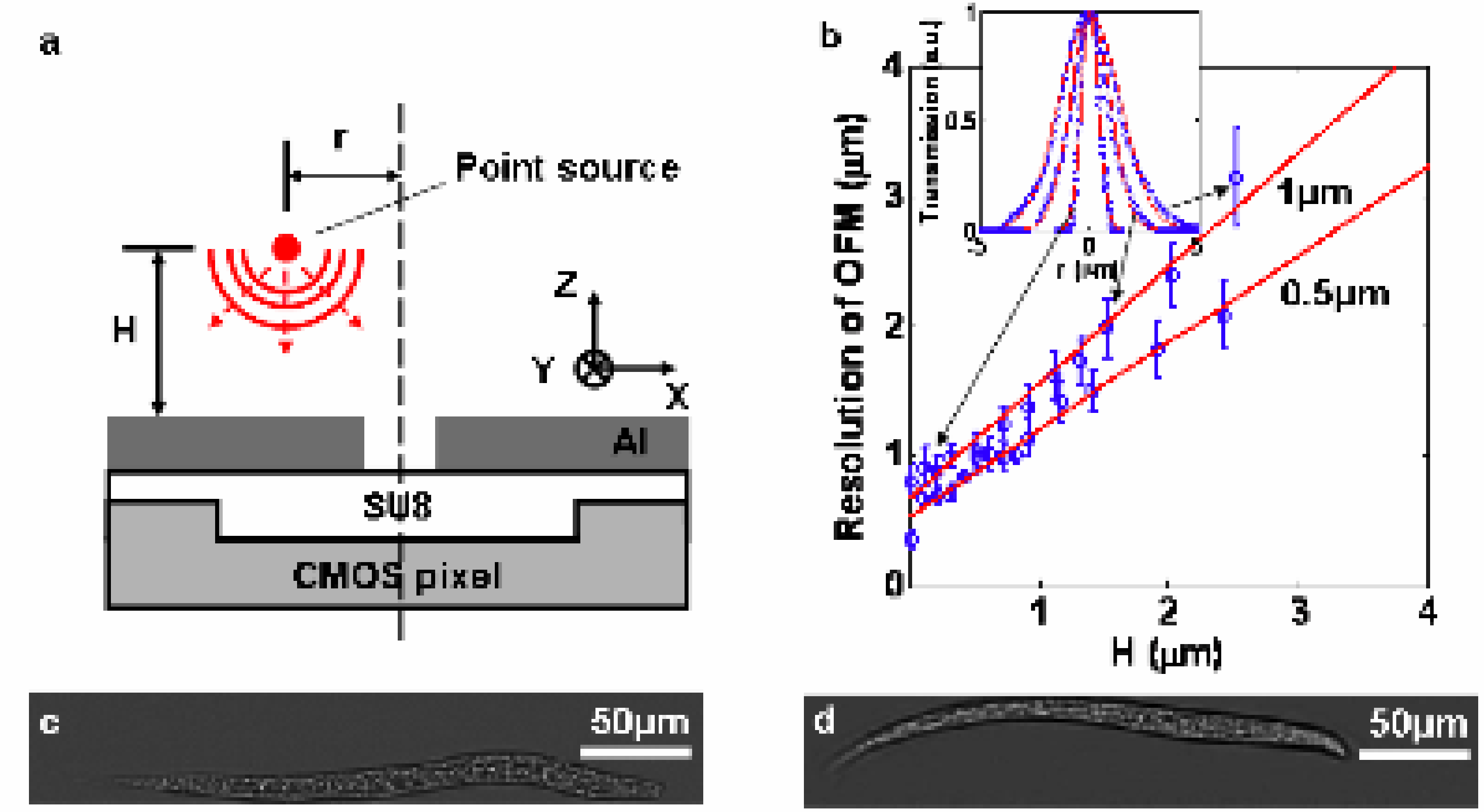
Nucleated blood cell image is from http://www.wadsworth.org/chemheme/heme/glass/slide_011_nrbc.htm

Integrated OFM



Lensless high-resolution on-chip optofluidic microscopes for Caenorhabditis elegans and cell imaging (pdf) Cui XQ, Lee LM, Heng X, Zhong WW, Sternberg PW, Psaltis D, Yang CH Proceedings of the National Academy of Sciences of the United States of America, Vol. 105, pp. 10670-10675, August 2008

Resolution


a
r
Point source
H
Z
Y
X
Al
SU8
CMOS pixel
b
Resolution of OFM (μm)
H (μm)
Transmission (a.u.)
r (μm)
1μm
0.5μm
c
50μm
d
50μm