Selected Topics in Advanced Optics

Week 2 – part 1

Olivier J.F. Martin
Nanophotonics and Metrology Laboratory

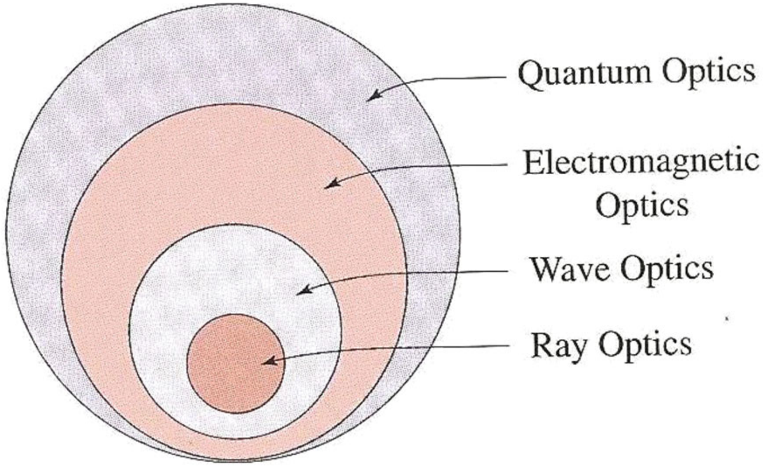
EPFL

Electromagnetic optics – Saleh & Teich Chapter 5

A more advanced reference:

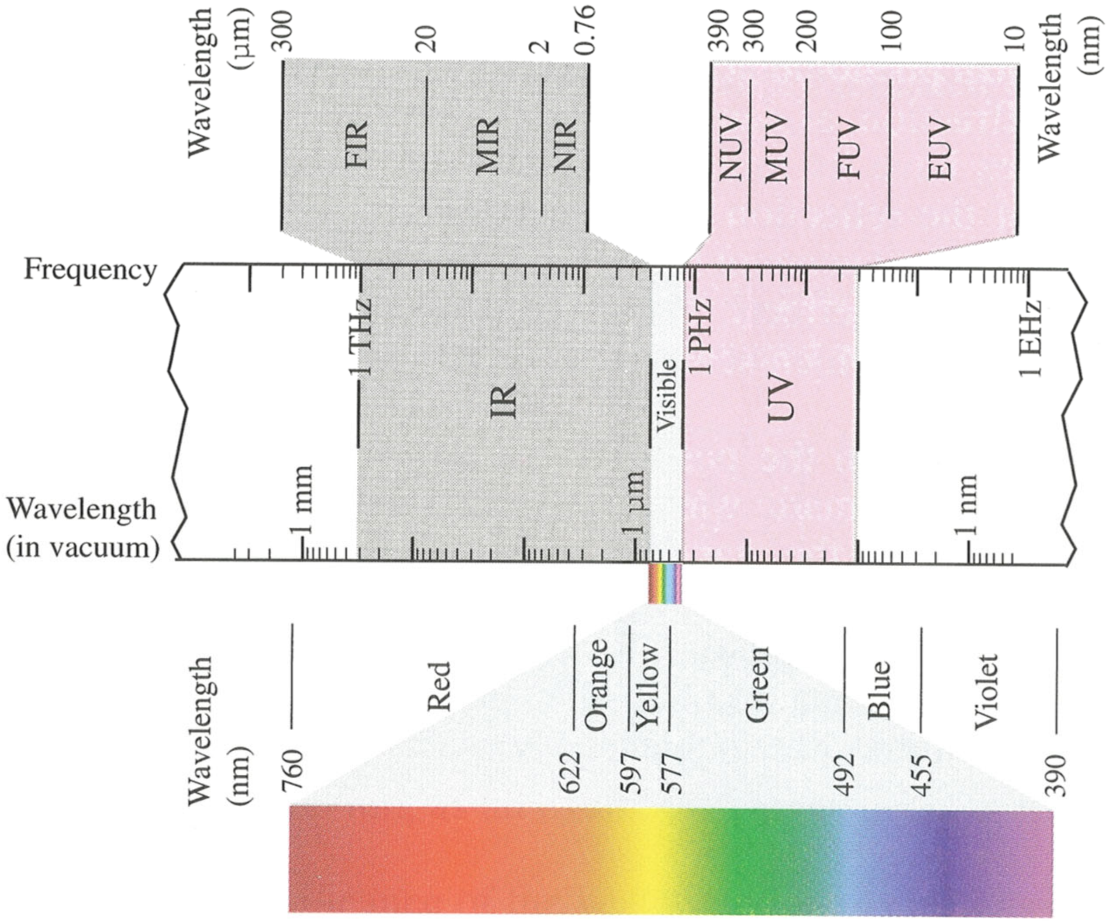
J.D. Jackson, Classical electrodynamics, 3rd Ed. (John Wiley, New York, 1999)

James Clerk Maxwell
(1831-1879)



The electromagnetic spectrum

- Maxwell's equations are valid over the entire electromagnetic spectrum:



Wavelength/frequency units

- Over the years, each field of physics/engineering/chemistry has developed its favourite units for the wavelength/energy of radiation:

	eV	nm	cm^{-1}	THz
Conversion Factors*	1	1240	8065.6	241.8
Ultraviolet (UV)	124 - 3.26	10 - 380	1000000 - 26300	30000 - 789
Visible (Vis)				
Violet	3.26 - 2.85	380 - 435	26300 - 23000	789 - 689
Blue	2.85 - 2.48	435 - 500	23000 - 20000	689 - 600
Cyan	2.48 - 2.38	500 - 520	20000 - 19200	600 - 577
Green	2.38 - 2.19	520 - 565	19200 - 17700	577 - 531
Yellow	2.19 - 2.10	565 - 590	17700 - 17000	531 - 508
Orange	2.10 - 1.98	590 - 625	17000 - 16000	508 - 480
Red	1.98 - 1.65	625 - 750	16000 - 13300	480 - 400
Near Infrared (NIR)	1.65 - 0.496	750 - 2500	13300 - 4000	400 - 120
Mid-Infrared (MIR)	496 - 124 meV	2.5 - 10 μm	4000 - 1000	120 - 30
Far Infrared (FIR)	124 - 12.4 meV	10 - 100 μm	1000 - 100	30 - 3.0
Terahertz Regime	41.4 - 1.24 meV	30 μm - 1 mm	334 - 10	10 - 0.3

A useful tool for unit conversions:
halas.rice.edu/conversions

Maxwell's equations (in MKSA units)

- Ingredients:

Electric field:	$\mathbf{E}(\mathbf{r},t)$	$[E]=V/m$
Magnetic field:	$\mathbf{H}(\mathbf{r},t)$	$[H]=A/m$
Displacement:	$\mathbf{D}(\mathbf{r},t)$	$[D]=C/m^2$
Magnetic flux density:	$\mathbf{B}(\mathbf{r},t)$	$[B]=Vs/m^2=T$

- Maxwell's equations:

$$\boxed{\begin{aligned}
&\nabla\times\mathbf{E}(\mathbf{r},t)=-\frac{\partial\mathbf{B}(\mathbf{r},t)}{\partial t}\\
&\nabla\times\mathbf{H}(\mathbf{r},t)=\frac{\partial\mathbf{D}(\mathbf{r},t)}{\partial t}+\mathbf{J}(\mathbf{r},t)\\
&\nabla\cdot\mathbf{D}(\mathbf{r},t)=\rho(\mathbf{r},t)\\
&\nabla\cdot\mathbf{B}(\mathbf{r},t)=0
\end{aligned}}$$

$\mathbf{J}(\mathbf{r},t)$ current density

$\rho(\mathbf{r},t)$ charge density

Maxwell's equations without sources

- This is the form generally used in optics

$$\boxed{\begin{aligned} &\nabla\times\mathbf{E}(\mathbf{r},t) = -\frac{\partial\mathbf{B}(\mathbf{r},t)}{\partial t} \qquad && \nabla\cdot\mathbf{D}(\mathbf{r},t) = 0 \\ &\nabla\times\mathbf{H}(\mathbf{r},t) = \frac{\partial\mathbf{D}(\mathbf{r},t)}{\partial t} \qquad && \nabla\cdot\mathbf{B}(\mathbf{r},t) = 0 \end{aligned}}$$

- The electric and magnetic properties of the medium are described by the constitutive relations:

$$\boxed{\begin{aligned} \mathbf{D} &= \varepsilon_0\mathbf{E} + \mathbf{P} \\ \mathbf{B} &= \mu_0\mathbf{H} + \mu_0\mathbf{M} \end{aligned}}$$

$\mathbf{P}$: polarization density

$\mathbf{M}$: magnetization density

- $\mathbf{P}$ and $\mathbf{M}$ depend on the applied fields $\boldsymbol{E}$ and $\boldsymbol{H}$. This dependence describes the response of the medium

Maxwell’s equations in vacuum

- In vacuum, the constitutive relations are

$$\mathbf{P} = \mathbf{0} \qquad \boxed{\mathbf{D} = \varepsilon_0 \mathbf{E}} \qquad \varepsilon_0 = \frac{10^7}{4\pi c_0^2} = 8.8541878 \cdot 10^{-12}\, F/m \ \text{(MKSA)}$$

$$\mathbf{M} = \mathbf{0} \qquad \boxed{\mathbf{B} = \mu_0 \mathbf{H}} \qquad \mu_0 = 4\pi \cdot 10^{-7}\, H/m$$

- And Maxwell's equations become:

$$\boxed{\begin{array}{ll} \nabla \times \mathbf{E} = -\mu_0 \dfrac{\partial \mathbf{H}}{\partial t} & \nabla \cdot \mathbf{E} = 0 \\ \nabla \times \mathbf{H} = \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} & \nabla \cdot \mathbf{H} = 0 \end{array}}$$

- From which one deduces the wave equation <u>for each field component</u>:

$$\boxed{\nabla^2 u - \frac{1}{c_0^2} \frac{\partial^2 u}{\partial t^2} = 0} \qquad \text{with } c_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \simeq 3 \cdot 10^8\, m/s$$

Maxwell's equations – cgs (Gaussian)

- Encore utilisées dans certains livres d'optique – Ingredients:

$$\text{Electric field}: \quad \mathbf{E}(\mathbf{r},t) \qquad [E] = \text{statvolt/cm}$$

$$\text{Magnetic field}: \quad \mathbf{H}(\mathbf{r},t) \qquad [H] = \text{oersted (Oe)}$$

$$\text{Displacement}: \quad \mathbf{D}(\mathbf{r},t) \qquad [D] = \text{statvolt/cm}$$

$$\text{Magnetic flux density}: \quad \mathbf{B}(\mathbf{r},t) \qquad [B] = \text{gauss}$$

- Maxwell's equations:

$$1\,\text{statvolt} = 299.792458\ \text{V}$$

$$\boxed{\begin{aligned}
\nabla \times \mathbf{E}(\mathbf{r},t) &= -\frac{1}{c}\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t} \\
\nabla \times \mathbf{H}(\mathbf{r},t) &= \frac{1}{c}\frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t} + \frac{4\pi}{c}\mathbf{J}(\mathbf{r},t) \\
\nabla \cdot \mathbf{D}(\mathbf{r},t) &= 4\pi\rho(\mathbf{r},t) \\
\nabla \cdot \mathbf{B}(\mathbf{r},t) &= 0
\end{aligned}}$$

$$\varepsilon_0 = 1$$

$$\mu_0 = 1$$

Selected Topics in Advanced Optics

Week 2 – part 2

Olivier J.F. Martin
Nanophotonics and Metrology Laboratory

EPFL

Maxwell's equations → boundary conditions

- In vacuum, the constitutive relations are

$$\mathbf{P}=\mathbf{0} \qquad \boxed{\mathbf{D}=\varepsilon_0\mathbf{E}} \qquad \varepsilon_0=\frac{10^7}{4\pi c_0^2}=8.8541878\cdot 10^{-12}\,F/m \text{ (MKSA)}$$

$$\mathbf{M}=\mathbf{0} \qquad \boxed{\mathbf{B}=\mu_0\mathbf{H}} \qquad \mu_0=4\pi\cdot 10^{-7}\,H/m$$

- And Maxwell's equations become:

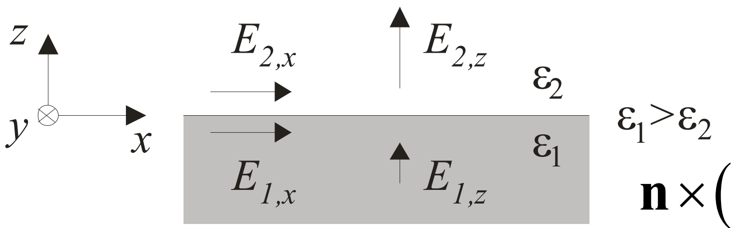
$$\boxed{\begin{array}{ll} \nabla\times\mathbf{E}=-\mu_0\dfrac{\partial\mathbf{H}}{\partial t} & \nabla\cdot\mathbf{E}=0 \\ \nabla\times\mathbf{H}=\varepsilon_0\dfrac{\partial\mathbf{E}}{\partial t} & \nabla\cdot\mathbf{H}=0 \end{array}}$$

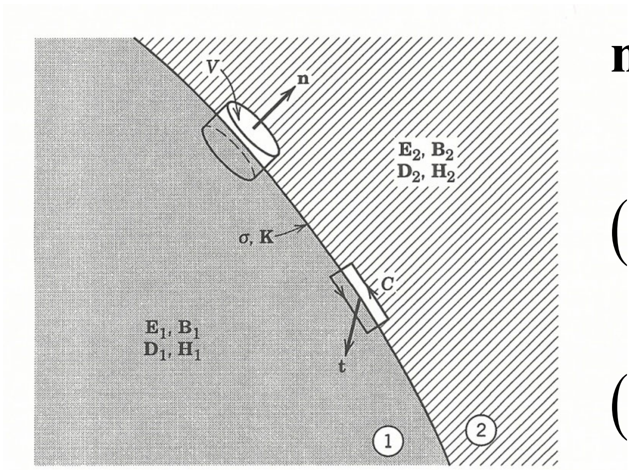
- From which one deduces the wave equation <u>for each field component</u>:

$$\boxed{\nabla^2 u-\frac{1}{c_0^2}\frac{\partial^2 u}{\partial t^2}=0} \qquad \text{with } c_0=\frac{1}{\sqrt{\varepsilon_0\mu_0}}\simeq 3\cdot 10^8\,m/s$$

Boundary conditions

- The form of Maxwell's equations contains also the boundary conditions for the electromagnetic fields at the interface between two media





- <u>Without charges or currents</u>:

$$\mathbf{n} \times \left(\mathbf{E}_2 - \mathbf{E}_1 \right) = 0$$

parallel component of $\mathbf{E}$ continuous

$$\mathbf{n} \times \left(\mathbf{H}_2 - \mathbf{H}_1 \right) = 0$$

parallel component of $\mathbf{H}$ continuous

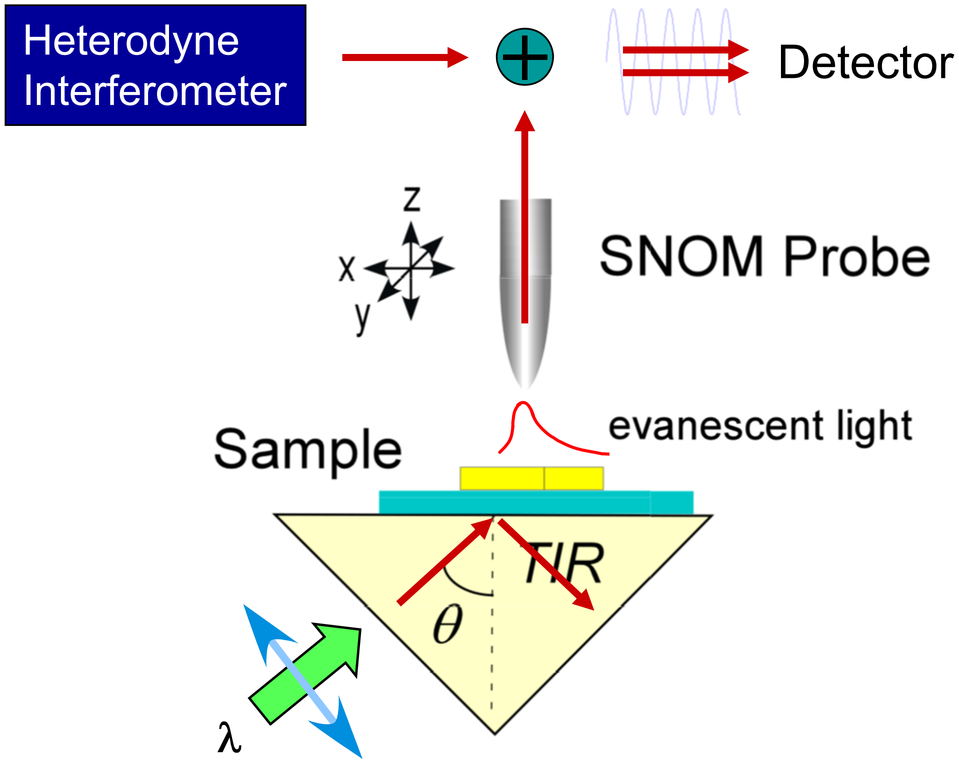
$$\left(\mathbf{B}_2 - \mathbf{B}_1 \right) \cdot \mathbf{n} = 0$$

normal component of $\mathbf{B}$ continuous

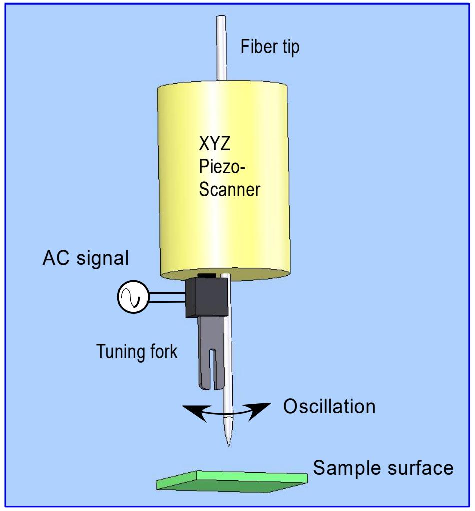
$$\left(\mathbf{D}_2 - \mathbf{D}_1 \right) \cdot \mathbf{n} = 0$$

normal component of $\mathbf{D}$ continuous

Photon scanning near-field optical microscope (PSTM)



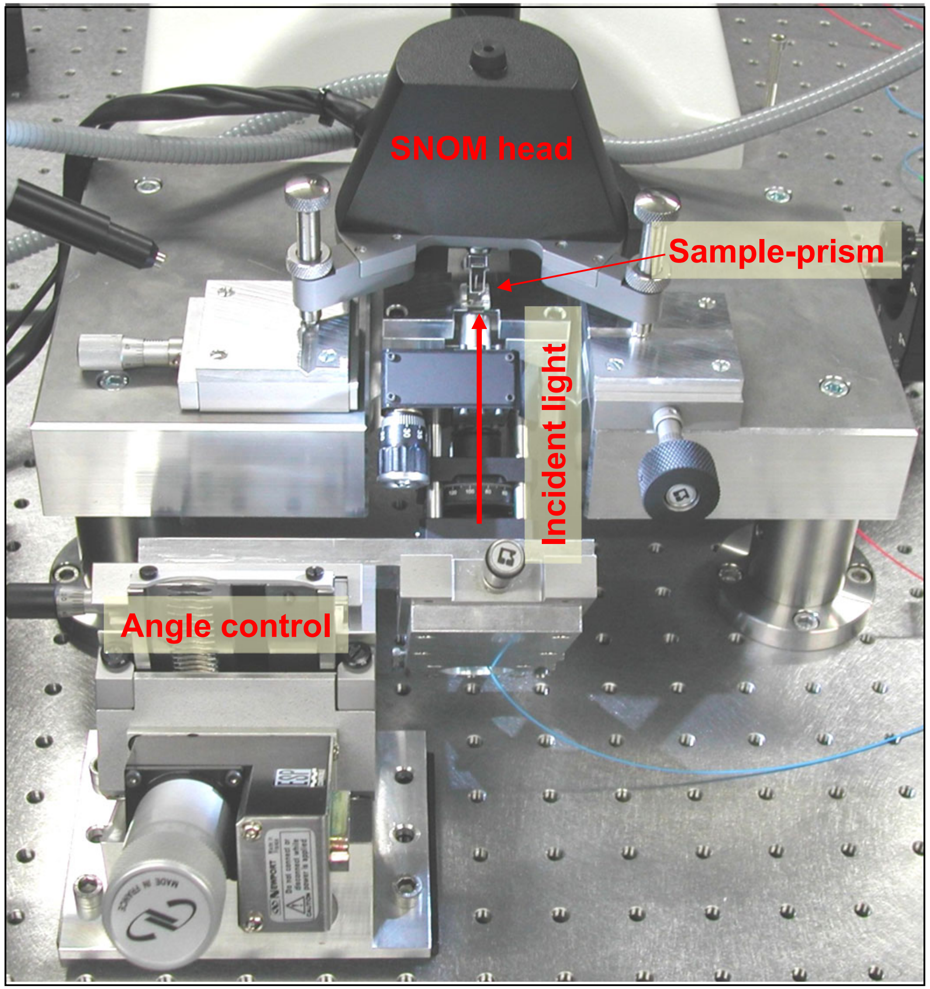
- Shear force AFM:



resolution: x,y ~ 1 nm
z ~ 5 nm

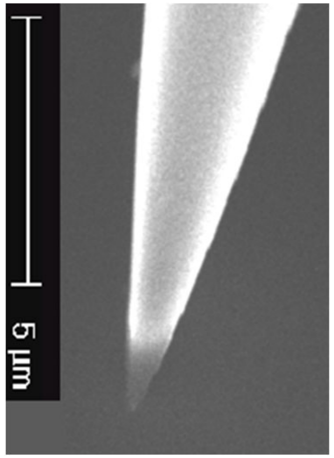
- Coherent PSTM combined with heterodyne interferometer
  → measurement of the amplitude and phase of the field !

Photon scanning near-field optical microscope (PSTM)

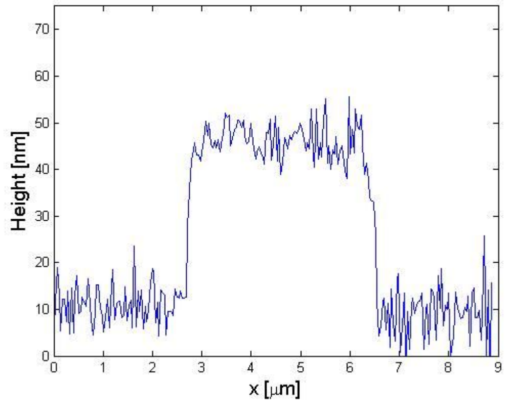


Tuning fork

Fiber probe (ρ ~ 50 nm)



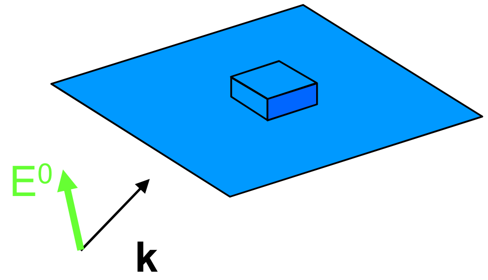
Gold waveguide topography

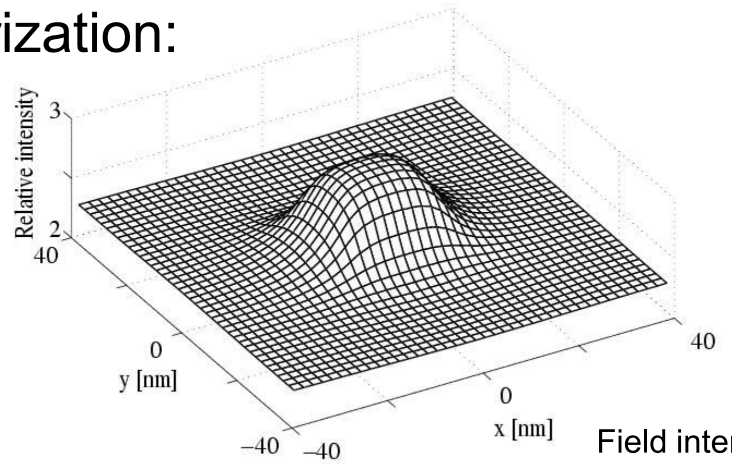


Olivier J.F. Martin

Light confinement in near-field optics

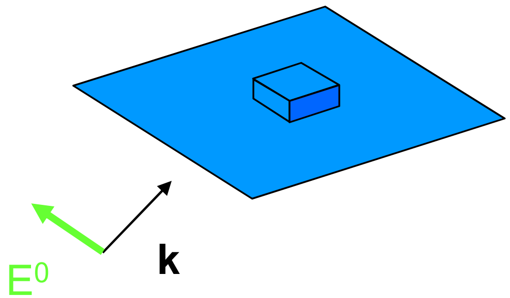
• p-polarization:

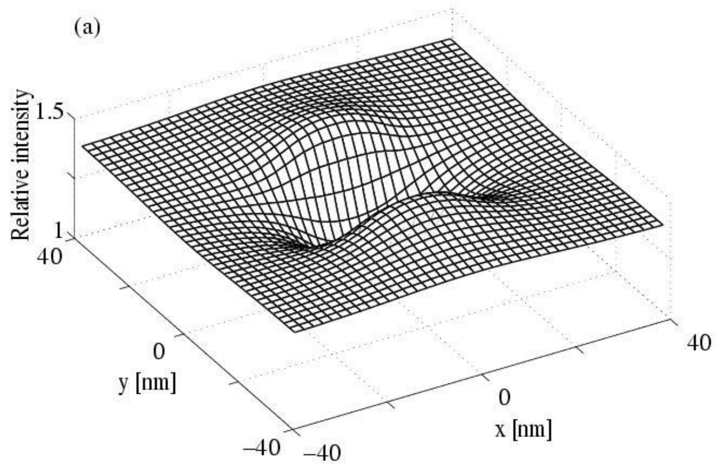




Field intensity
5 nm above object

• s-polarization:



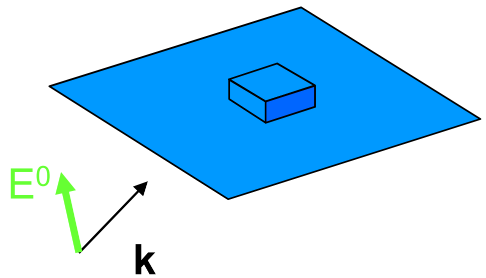


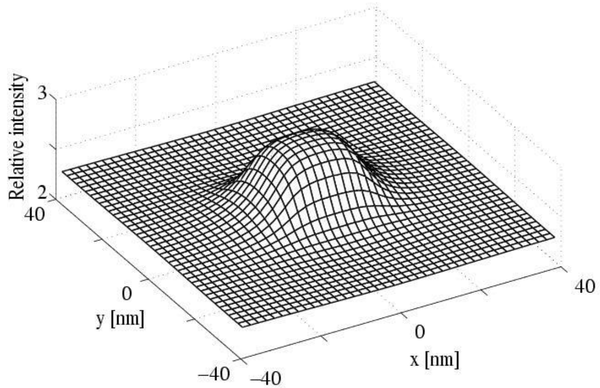
Depolarization fields

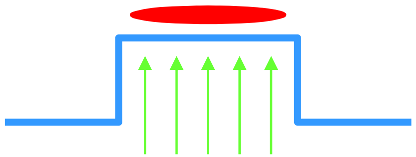
- In matter: $E = E^0 + E^d$

E^d is such that E fullfills the EM boundary conditions

- p-polarization:

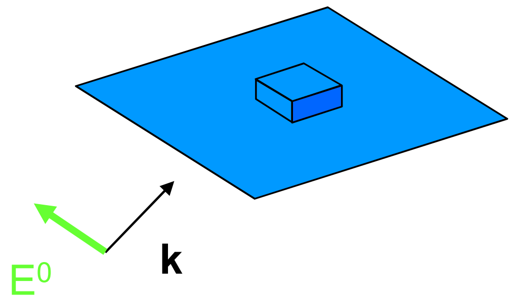


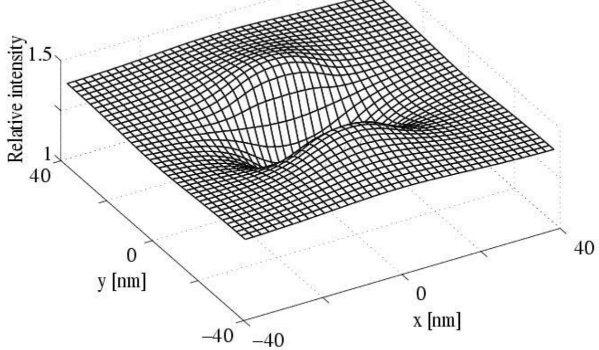




- s-polarization:

(a)





Experimental proof

VOLUME 77, NUMBER 27 PHYSICAL REVIEW LETTERS 30 DECEMBER 1996

Observation of Light Confinement Effects with a Near-Field Optical Microscope

Jean-Claude Weeber, Eric Bourillot, Alain Dereux, and Jean-Pierre Goudonnet
Laboratoire de Physique, Optique Submicronique, Université de Bourgogne, F-21004 Dijon, France

Yong Chen
Laboratoire de Microstructures et Microélectronique, CNRS, 196 Avenue H. Ravera, F-92225 Bagneux, France

Christian Girard
Laboratoire de Physique Moléculaire UA CNRS 772, Université de Franche-Comté, F-25030 Besançon, France
(Received 4 September 1996)

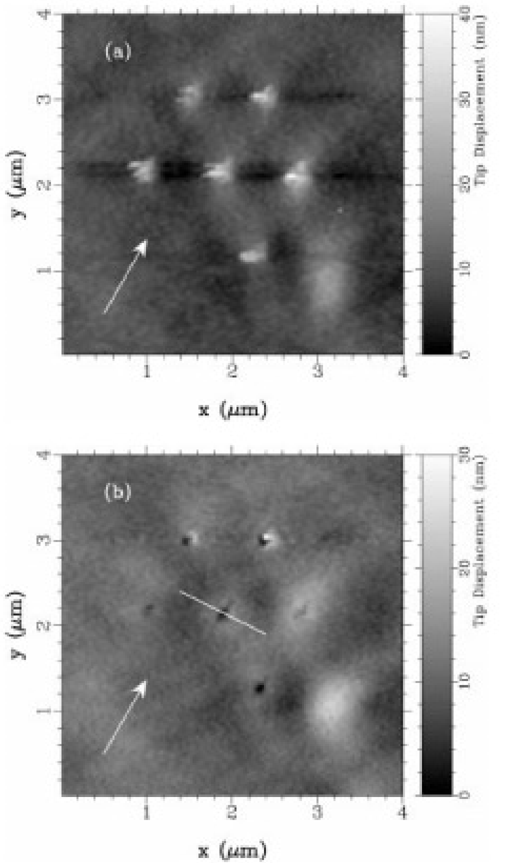


FIG. 2. Near-field optical images of the sample presented in Fig. 1 for two incident polarizations: TM (a) and TE (b). The arrows indicate the direction of the projection of the incident wave vector parallel to the surface of the substrate. The cut line in Fig. 3 follows the white line in (b).

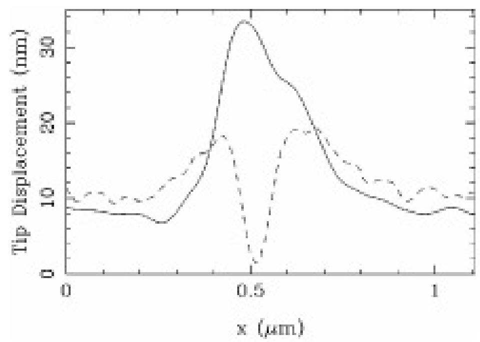
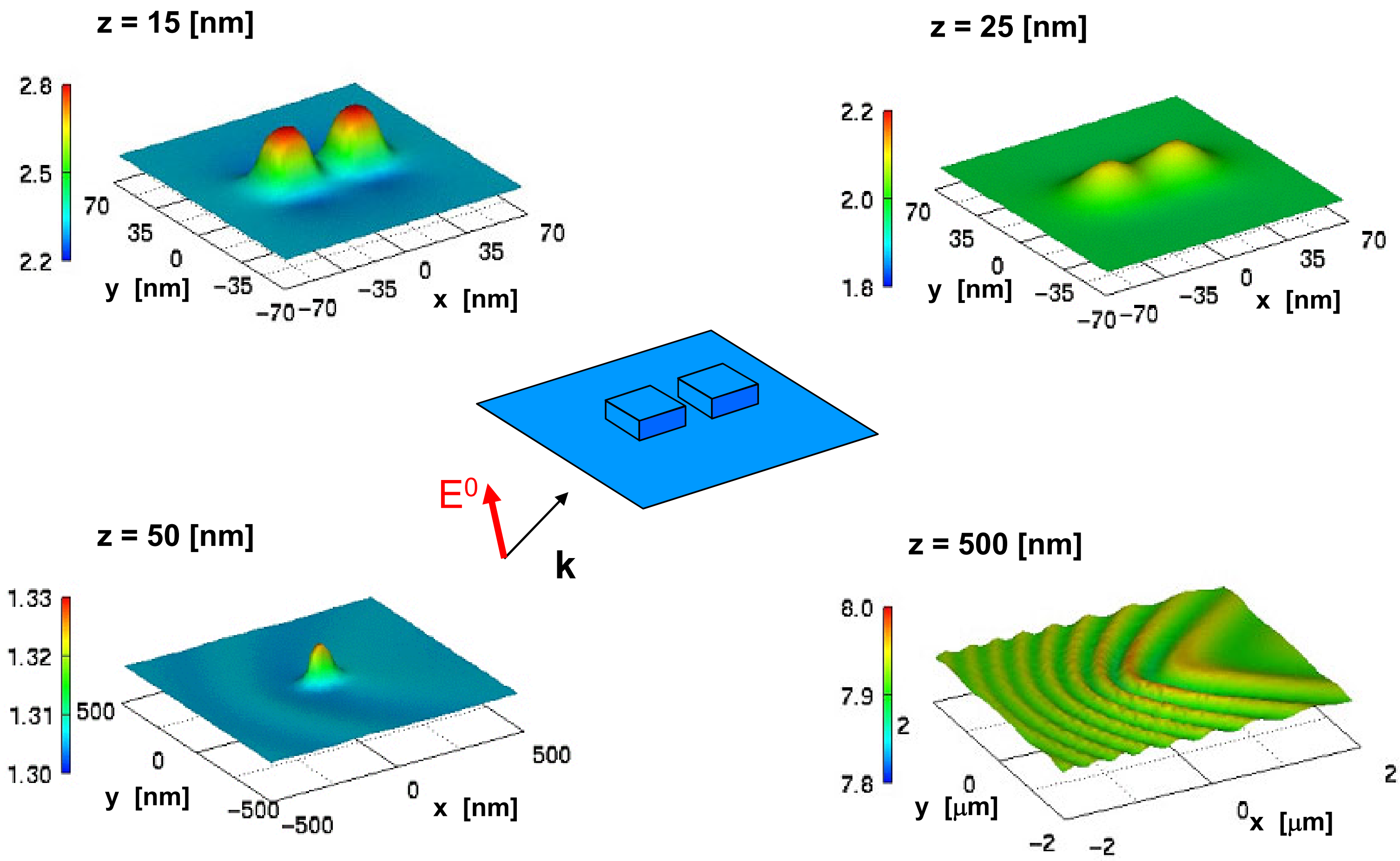


FIG. 3. Cuts of the intensity profiles above the central pad of the microstructure of Fig. 1 for the two incident polarizations TM (solid line) and TE (dashed line). The followed cut line appears in white on Fig. 2(b).

Light confinement in the near-field



Selected Topics in Advanced Optics

Week 2 – part 3

Olivier J.F. Martin
Nanophotonics and Metrology Laboratory

EPFL

Monochromatic electromagnetic waves

- All components of the electric and magnetic fields are harmonic functions of time with the same frequency ν:

$$\boxed{\begin{aligned}\mathbf{E}(\mathbf{r},t) &= \mathrm{Re}\{\mathbf{E}(\mathbf{r})\exp(j\omega t)\}\\ \mathbf{H}(\mathbf{r},t) &= \mathrm{Re}\{\mathbf{H}(\mathbf{r})\exp(j\omega t)\}\end{aligned}}$$

- For monochromatic waves, all time derivatives in Maxwell's equations become multiplication with $j\omega$ (for the complex fields):

$$\boxed{\begin{aligned}\nabla\times\mathbf{E}(\mathbf{r},t) &= -j\omega\mathbf{B}(\mathbf{r},t) & \qquad \nabla\cdot\mathbf{D}(\mathbf{r},t) &= 0\\ \nabla\times\mathbf{H}(\mathbf{r},t) &= j\omega\mathbf{D}(\mathbf{r},t) & \qquad \nabla\cdot\mathbf{B}(\mathbf{r},t) &= 0\end{aligned}}$$

Monochromatic electromagnetic plane waves

- For monochromatic plane waves each component of the electromagnetic field is a harmonic function in time and space:

$$\boxed{\begin{aligned}\mathbf{E}(\mathbf{r},t) &= \mathrm{Re}\left\{\mathbf{E}_0 \exp(-j\mathbf{k}\cdot\mathbf{r})\exp(j\omega t)\right\}\\ \mathbf{H}(\mathbf{r},t) &= \mathrm{Re}\left\{\mathbf{H}_0 \exp(-j\mathbf{k}\cdot\mathbf{r})\exp(j\omega t)\right\}\end{aligned}}$$

- Each temporal derivative in Maxwell's equations becomes a multiplication with $j\omega$ and each spatial derivative in Maxwell's equations becomes a vector multiplication with $-j\mathbf{k}\cdot$:

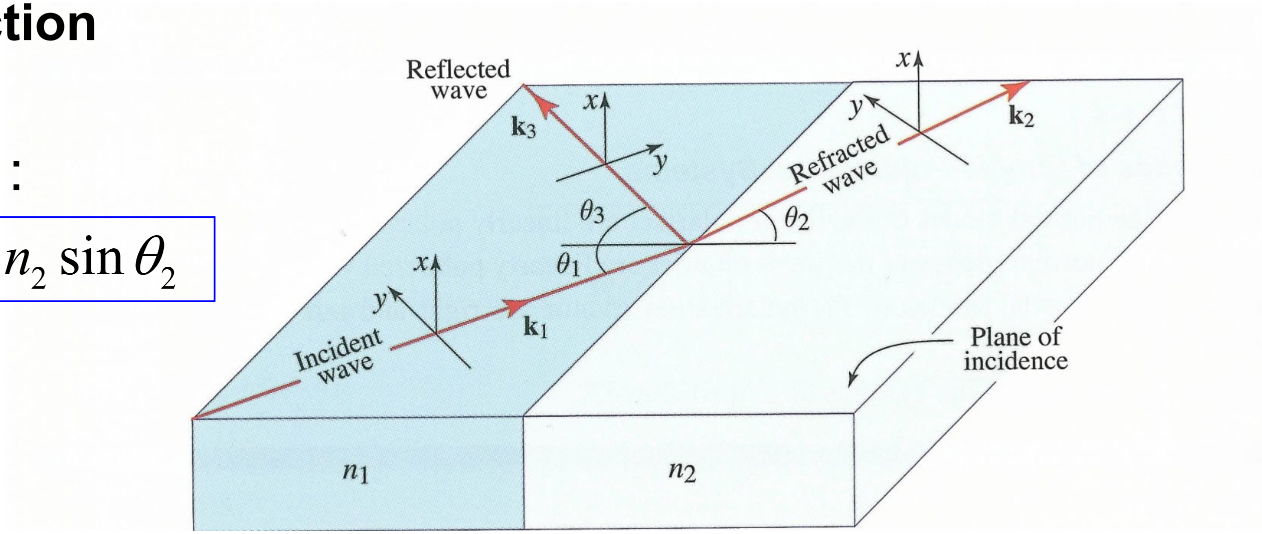
$$\boxed{\begin{aligned}\mathbf{k}\times\mathbf{E}(\mathbf{r},t) &= \omega\mathbf{B}(\mathbf{r},t) & \qquad \mathbf{k}\cdot\mathbf{D}(\mathbf{r},t) &= 0\\ -\mathbf{k}\times\mathbf{H}(\mathbf{r},t) &= \omega\mathbf{D}(\mathbf{r},t) & \qquad \mathbf{k}\cdot\mathbf{B}(\mathbf{r},t) &= 0\end{aligned}}$$

Maxwell's equations for a plane wave

Reflection and refraction

Snell law :

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$



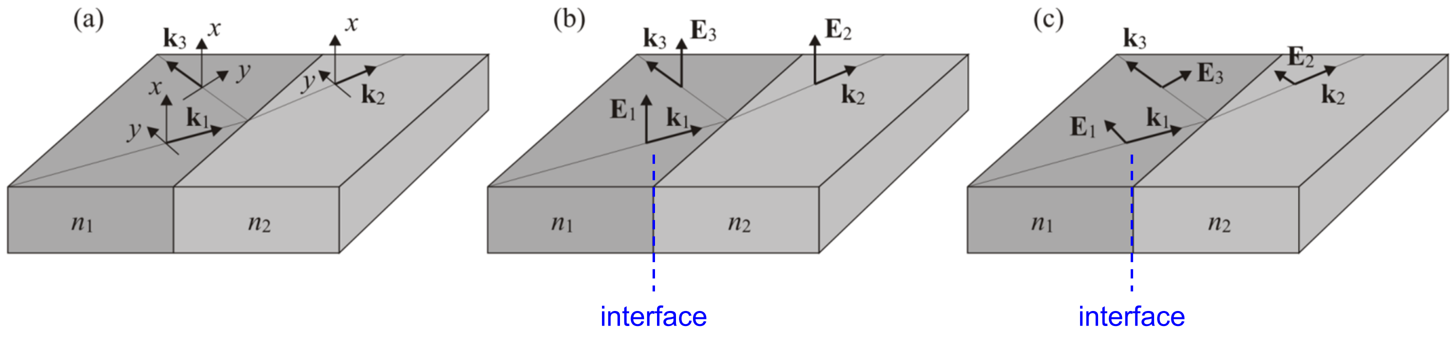
- Each field can be represented by its Jones vector (in a different plane):

$$\mathbf{J}_1 = \begin{bmatrix} A_{1x} \\ A_{1y} \end{bmatrix} \quad \mathbf{J}_2 = \begin{bmatrix} A_{2x} \\ A_{2y} \end{bmatrix} \quad \mathbf{J}_3 = \begin{bmatrix} A_{3x} \\ A_{3y} \end{bmatrix}$$

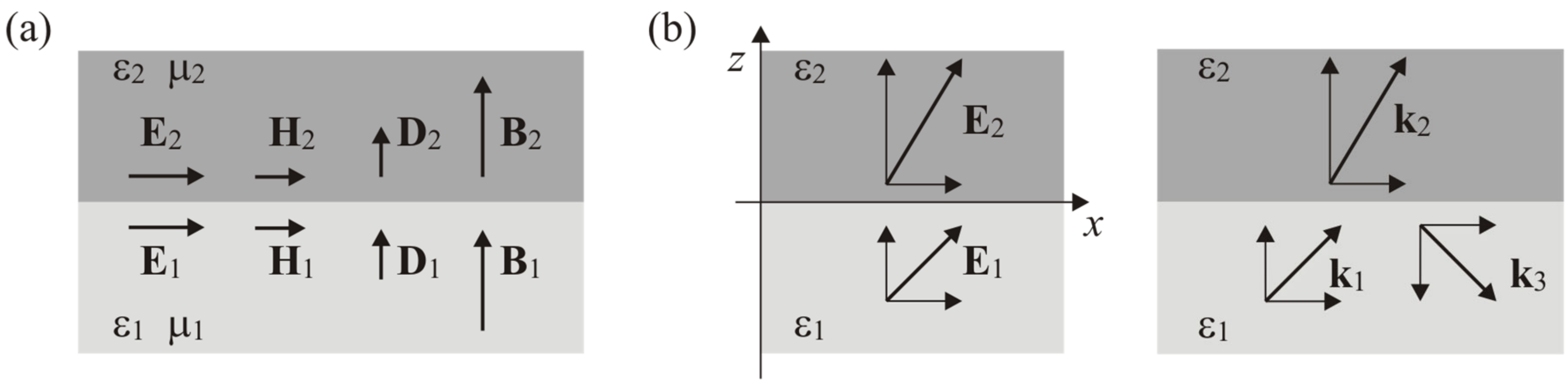
- x-polarized mode: transverse electric (TE), or orthogonal, or s-polarized (s=senkrecht)
- y-polarized mode: transverse magnetic (TM), or parallel, or p-polarized (p=parallel)

Reflection and refraction

- The different polarizations:



- Conservation of field components and momentum



Olivier J.F. Martin

Reflection and refraction

- The two polarizations are independent → Jones matrices are diagonal:

$$\mathbf{t} = \begin{bmatrix} t_x & 0 \\ 0 & t_y \end{bmatrix} \qquad \mathbf{r} = \begin{bmatrix} r_x & 0 \\ 0 & r_y \end{bmatrix}$$

$$E_{2x} = t_x E_{1x} \qquad E_{2y} = t_y E_{1y}$$

$$E_{3x} = r_x E_{1x} \qquad E_{3y} = r_y E_{1y}$$

- Reflection and transmission coefficients (general case):

$$\boxed{\begin{aligned} r_x &= \frac{\eta_2 \sec\theta_2 - \eta_1 \sec\theta_1}{\eta_2 \sec\theta_2 + \eta_1 \sec\theta_1} \qquad t_x = 1 + r_x \\ r_y &= \frac{\eta_2 \cos\theta_2 - \eta_1 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1} \qquad t_y = \left(1 + r_y\right)\frac{\cos\theta_1}{\cos\theta_2} \end{aligned}}$$

(TE) $\eta = \sqrt{\mu / \varepsilon}$

(TM) $\sec = \dfrac{1}{\cos}$

Reflection and refraction

- For nonlossy, nonmagnetic materials (Fresnel coefficients):

$$\boxed{\begin{array}{ll} r_x = \dfrac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} & t_x = 1 + r_x \\ r_y = \dfrac{n_1 \sec\theta_1 - n_2 \sec\theta_2}{n_1 \sec\theta_1 + n_2 \sec\theta_2} & t_y = \left(1 + r_y\right)\dfrac{\cos\theta_1}{\cos\theta_2} \end{array}}$$

(TE)

(TM)

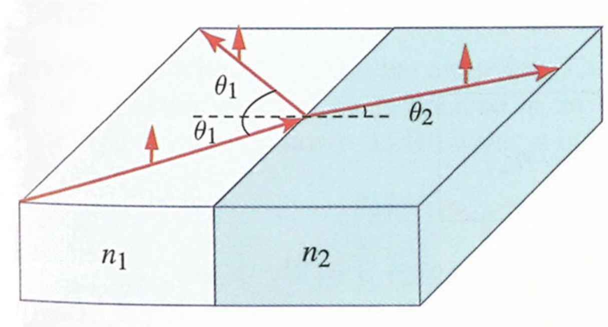
- Determination of the refraction angle (Snell's law gives only the sinus):

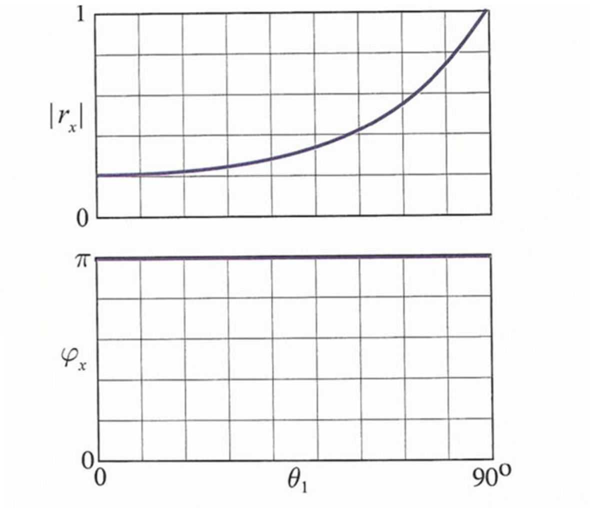
$$\cos\theta_2 = \pm\sqrt{1 - \sin^2\theta_2} = \pm\sqrt{1 - \left(n_1 / n_2\right)^2 \sin^2\theta_1}$$

- The Fresnel coefficients are generally complex (amplitude and phase) !

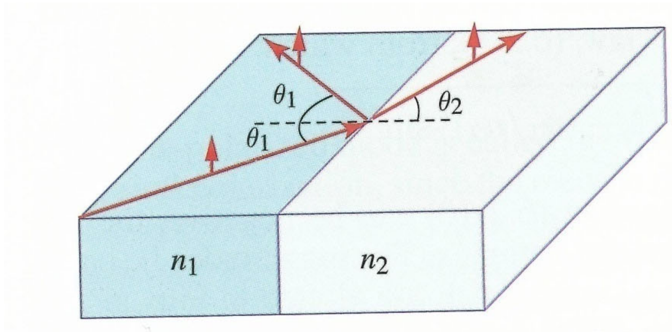
TE or s-polarization

- External reflection ($n_1 < n_2$):

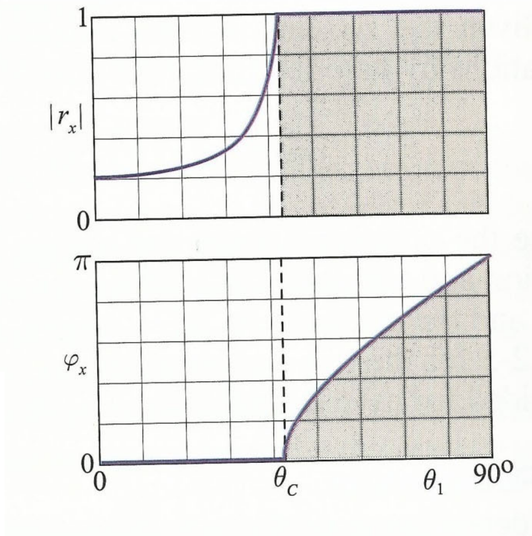




- Internal reflection ($n_1 > n_2$):



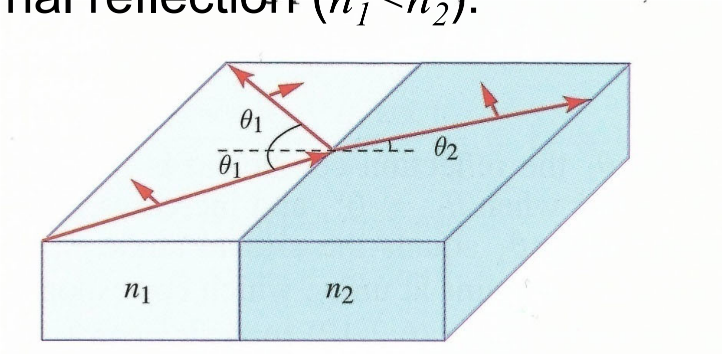
$$\theta_c = \sin^{-1}\left(n_2 / n_1\right)$$

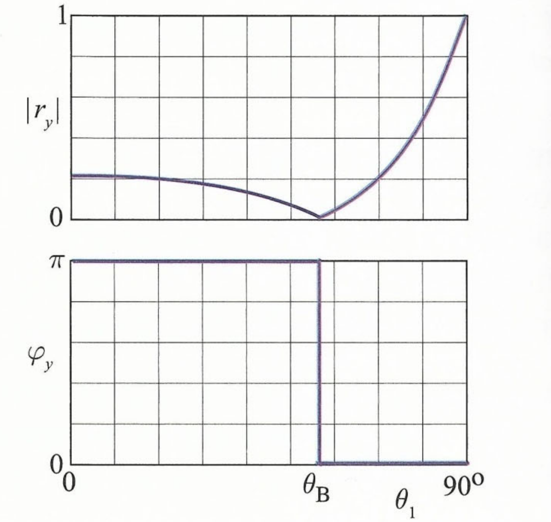


Olivier J.F. Martin

TM or p-polarization

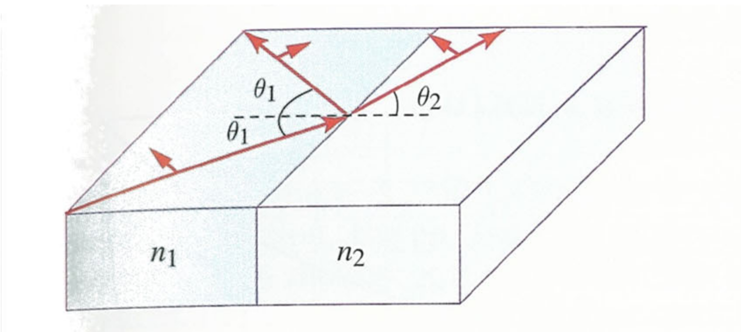
- External reflection ($n_1<n_2$):

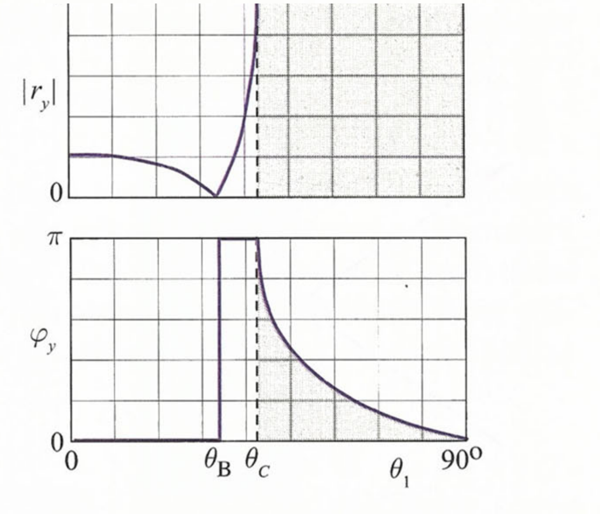




$$\theta_B = \tan^{-1}\left(n_2 / n_1\right)$$

- Internal reflection ($n_1>n_2$):





$$\theta_c = \sin^{-1}\left(n_2 / n_1\right)$$

Optical tunnel effect

- Under total internal reflection, an evanescent field is created in the second medium. This field can become propagating if a third medium is brought close to the first interface.