Selected Topics in Advanced Optics

Week 1 – part 2

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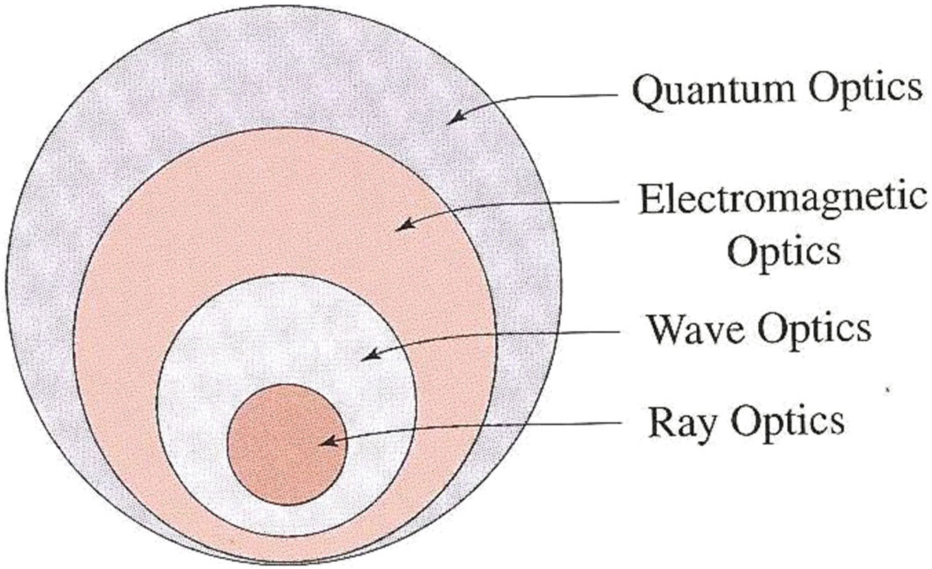
Module 1: Summary of fundamental optics

- Ray optics – Saleh & Teich Chapter 1
- Wave optics – Saleh & Teich Chapter 2
- Maxwell's optics – Saleh & Teich Chapter 5
- Polarization optics – Saleh & Teich Chapter 6

Different perspectives on optics:



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The postulates of ray optics

- Light travels in the form of rays
- A medium is characterized by a quantity $n \geq 1$ (in general…) called the refractive index
- The refractive index is defined by the ratio of the speed of light in vacuum c_0 over the speed of light c in the medium:

$$\boxed{n = c_0 / c}$$

- The time taken by light to travel a distance d is $d / c = nd / c_0$
- nd is the optical pathlength
- In an inhomogoneous medium $n = n(\mathbf{r})$ is a function of the position and the pathlength is defined as an integral:

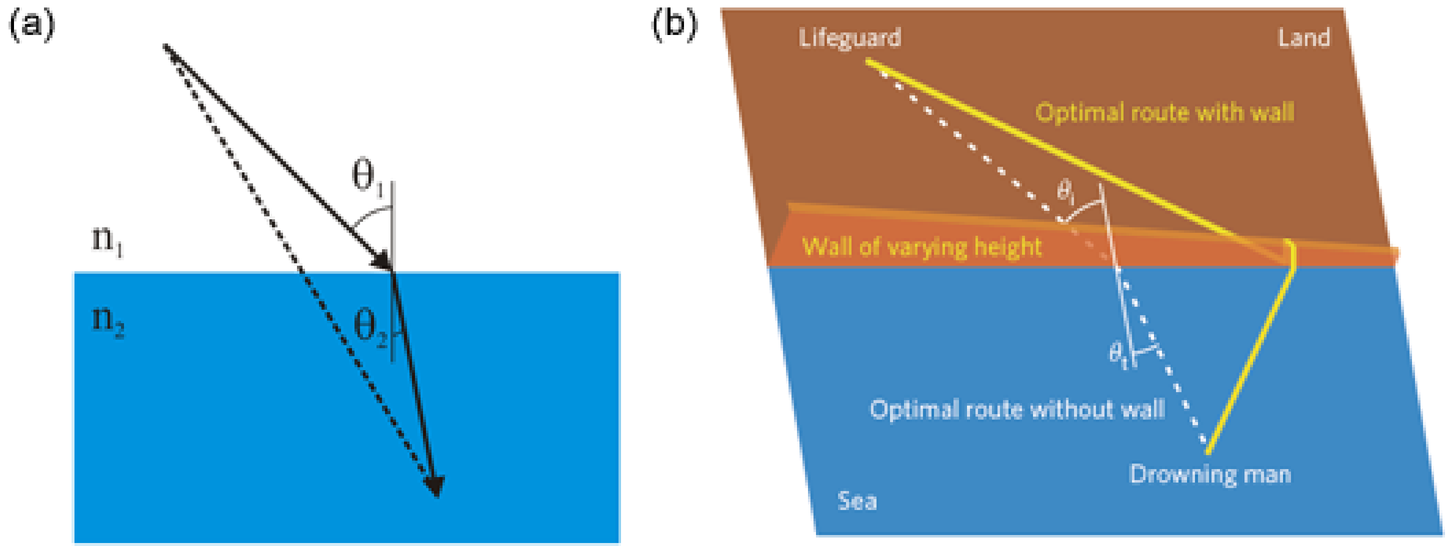
$$\text{pathlength} = \int_A^B n(\mathbf{r})\, ds$$

The postulates of ray optics

- Fermat principle: Optical rays travelling between two points follow a path such that the time of travel (or the optical pathlength) is an extremum (in most cases a minimum):

$$\delta \int_A^B n(\mathbf{r})\, ds = 0$$

- Refraction:

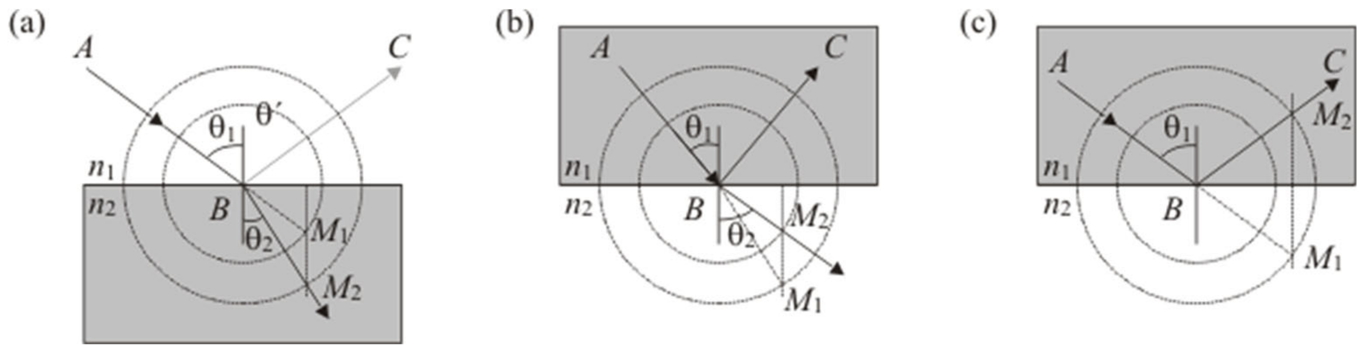


N. Yu, Nature Materials vol. 13, p. 139 (2014)

Refraction – Geometrical construction

- Snell law:

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$



- Total internal reflection (no refraction) for:

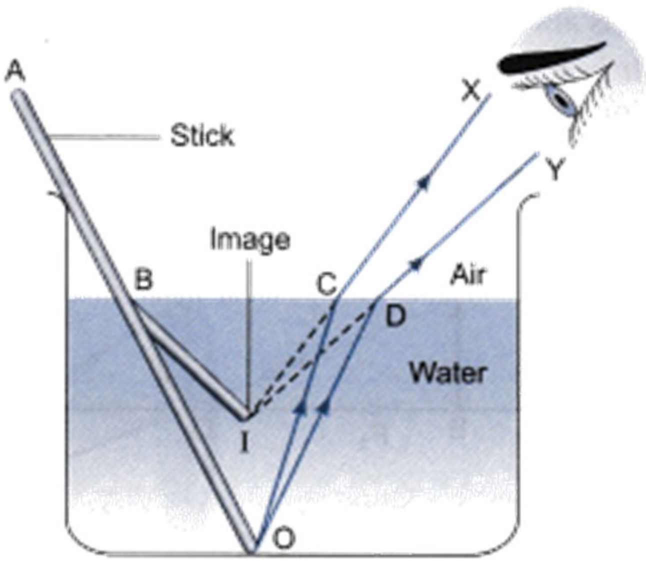
$$\boxed{\theta_c = \arcsin(n_2 / n_1)}$$

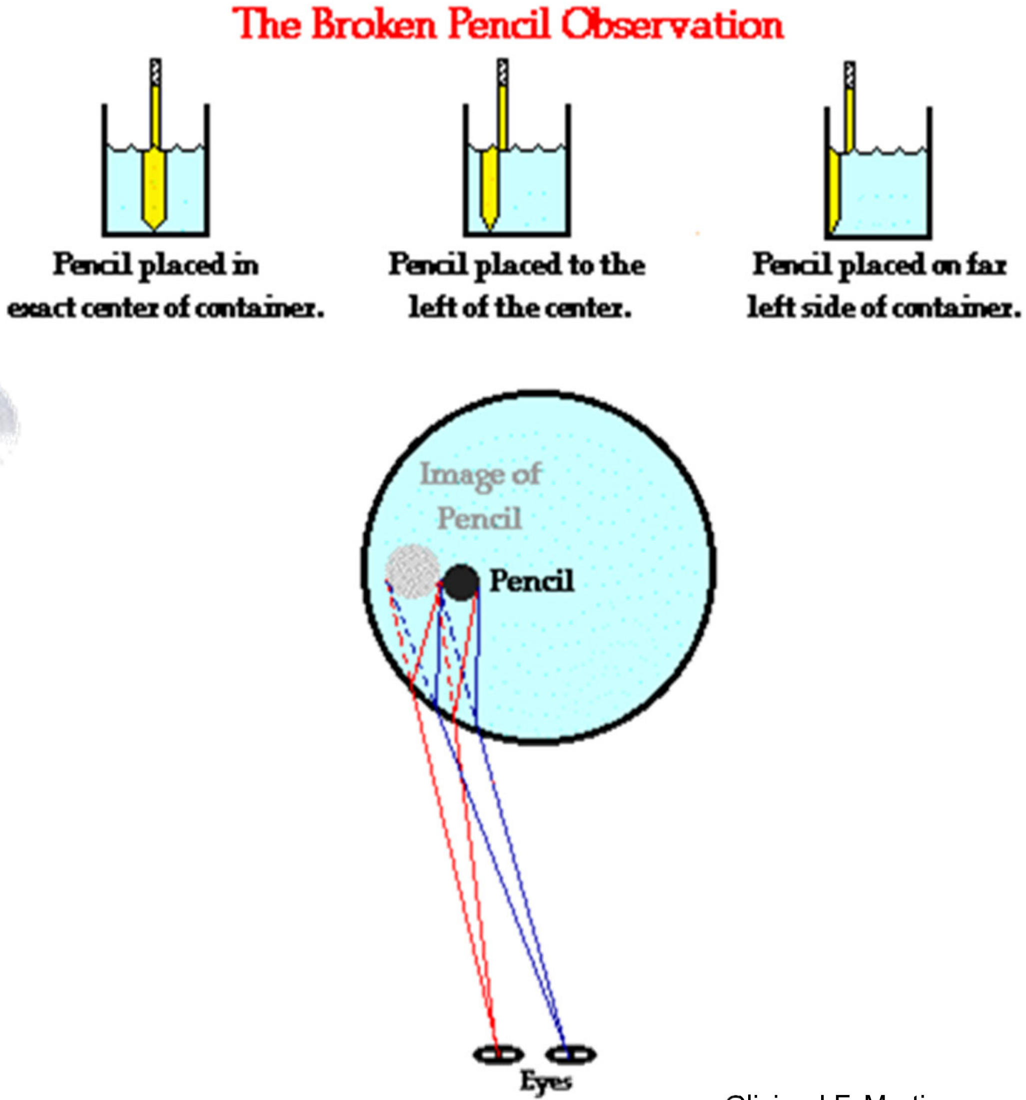
- Requires $n_1 > n_2$ (e.g. waveguides)

Refraction – Snell law

- Our eye assumes that light transmission is along straight lines → the object must be broken under the water

physicsclassroom.com





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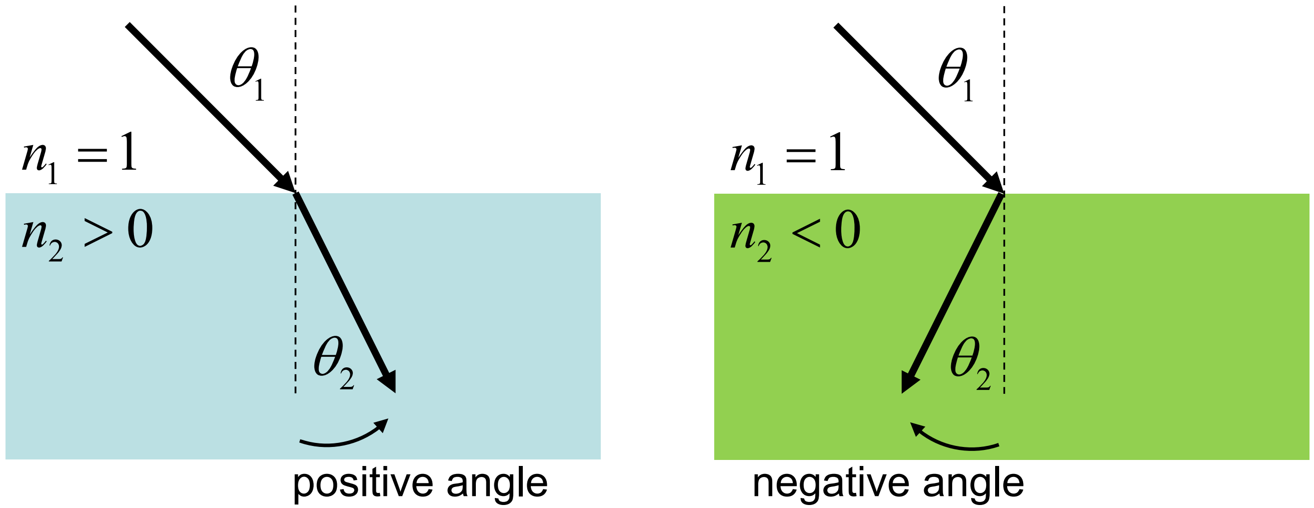
Negative refraction

- In principle, the refractive index is positive… there are however exotic situations where it can be negative
- In that case, the refracted angle becomes also negative!

$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$



Perfect lens

- A source point becomes a point image (which is impossible with conventional lenses)

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Negative Refraction Makes a Perfect Lens

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(Received 25 April 2000)

With a conventional lens sharpness of the image is always limited by the wavelength of light. An unconventional alternative to a lens, a slab of negative refractive index material, has the power to focus all Fourier components of a 2D image, even those that do not propagate in a radiative manner. Such "superlenses" can be realized in the microwave band with current technology. Our simulations show that a version of the lens operating at the frequency of visible light can be realized in the form of a thin slab of silver. This optical version resolves objects only a few nanometers across.

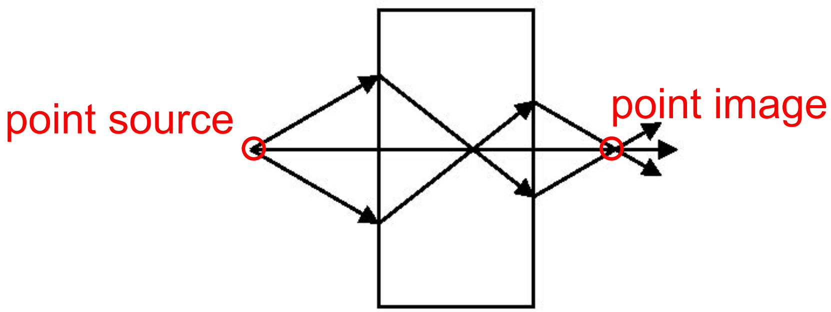
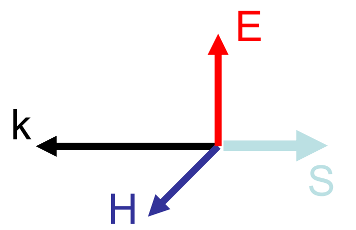
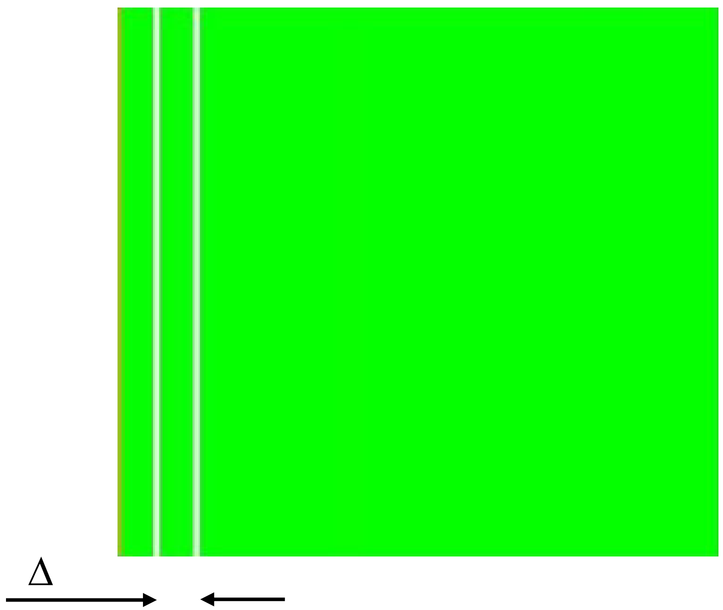


FIG. 1. A negative refractive index medium bends light to a negative angle with the surface normal. Light formerly diverging from a point source is set in reverse and converges back to a point. Released from the medium the light reaches a focus for a second time.

Negative phase velocity

- $c = c_0 / n$ what happens if $n < 0$?

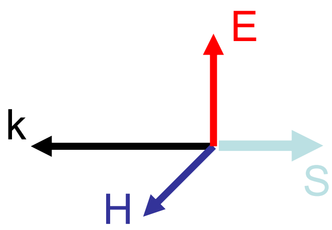


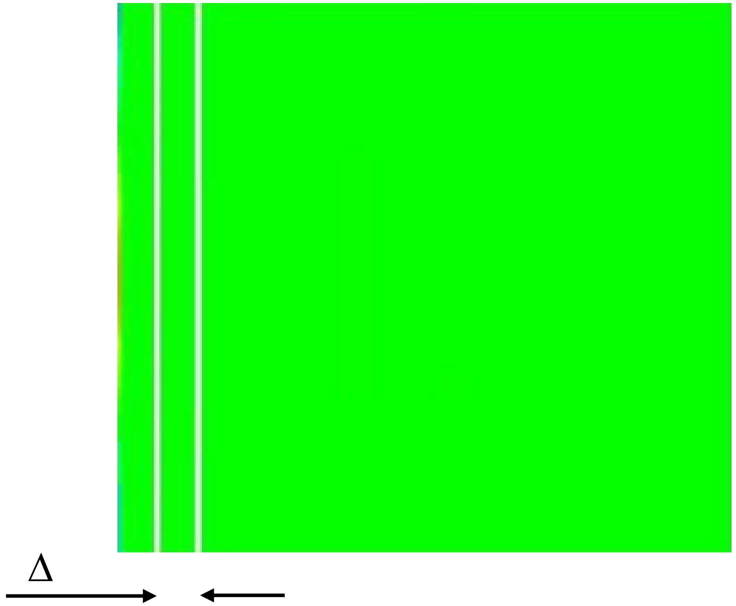


- Phase accumulated over a distance Δ: $\Delta\phi = n\dfrac{\omega}{c}\Delta < 0$

Negative refraction

- $c = c_0 / n$ what happens if $n < 0$?





- Phase accumulated over a distance Δ: $\Delta\phi = n\dfrac{\omega}{c}\Delta < 0$

Selected Topics in Advanced Optics

Week 1 – part 3

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Wave optics

- Scalar theory for the optical wave (one field component): $u(\mathbf{r},t)$
- Wave equation:

$$\boxed{\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0}$$

- Speed of light: $c = \dfrac{c_0}{n}$ $\qquad c_0 = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}} \sim 3.0 \cdot 10^8 m/s$ (MKSA!)

$$c_0 = 3.0 \cdot 10^8 \mathrm{m/s} = 30 \mathrm{cm/ns} = 0.3 \mathrm{mm/ps} = 0.3 \mu\mathrm{m/fs}$$

Wave equation in different coordinate systems

- Cartesian $u(x, y, z)$

$$\nabla^2 = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

- Spherical $u(r, \theta, \phi)$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}$$

- Cylindrical $u(\rho, \phi, z)$

$$\nabla^2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial u}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 u}{\partial\phi^2} + \frac{\partial^2 u}{\partial z^2}$$

For a very good reference: G. B. Arfken and H.J. Weber, Mathematical Methods for Physicists, 5th Ed. (Academic Press, New York, 2001).

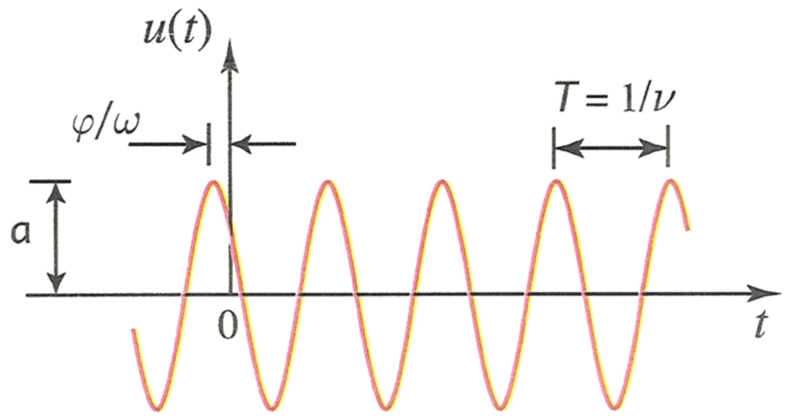
Wave optics

- The wave equation is linear (linear optics, as opposed to nonlinear optics):
  - The optical properties of materials do not depend on light intensity
  - The principle of superposition is valid $u(\mathbf{r},t) = u_1(\mathbf{r},t) + u_2(\mathbf{r},t)$
  - The frequency of light does not change when it passes through a medium
  - Two beams of light cannot interact (light cannot be used to control light)

Monochromatic harmonic waves

- Scalar wave (real function, representing one field component)
- Harmonic time dependence: $u(\mathbf{r},t) = a(\mathbf{r})\cos[\omega t + \varphi(\mathbf{r})]$

$a(\mathbf{r})$	: amplitude	[?]
$\varphi(\mathbf{r})$	: phase	[rad]
$\omega = 2\pi\nu$	: angular frequency	[rad / s]
ν	: frequency	[Hz]
$T = 1/\nu$	: period	[s]



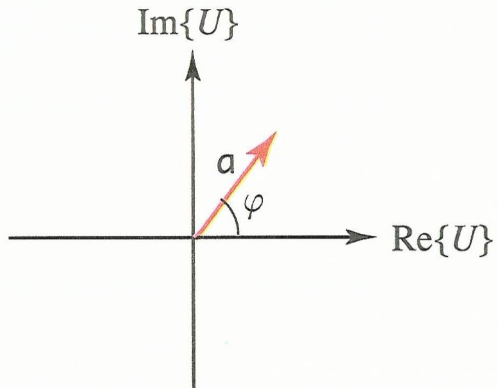
Complex representation

- To take advantage of complex calculus one introduces the complex wave function $U(\mathbf{r},t)$

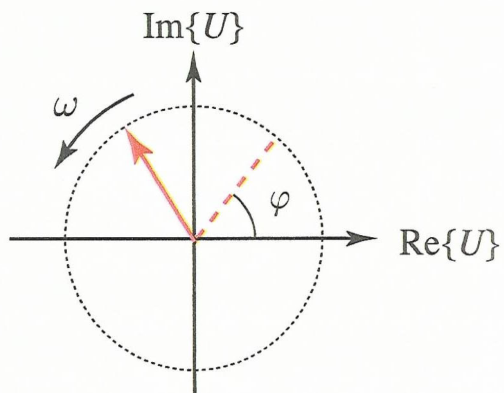
$$u(\mathbf{r},t) = \mathrm{Re}\{U(\mathbf{r},t)\} = \frac{1}{2}\{U(\mathbf{r},t) + U^*(\mathbf{r},t)\}$$

$$U(\mathbf{r},t) = a(\mathbf{r})\exp(j\varphi(\mathbf{r}))\exp(j\omega t) = \underbrace{U(\mathbf{r})}_{\text{complex amplitude}}\exp(j\omega t)$$

complex amplitude:



phasor:



Complex representation (cont.)

- The complex wave function also fulfills the wave equation:

$$U(\mathbf{r},t) = U(\mathbf{r})\exp(j\omega t) \qquad \nabla^2 U - \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2} = 0$$

- Leading to Helmholtz equation: $\boxed{\nabla^2 U + k^2 U = 0}$

- With the wave number $k = \dfrac{\omega}{c} = \dfrac{2\pi\nu}{c}$

Monochromatic plane wave

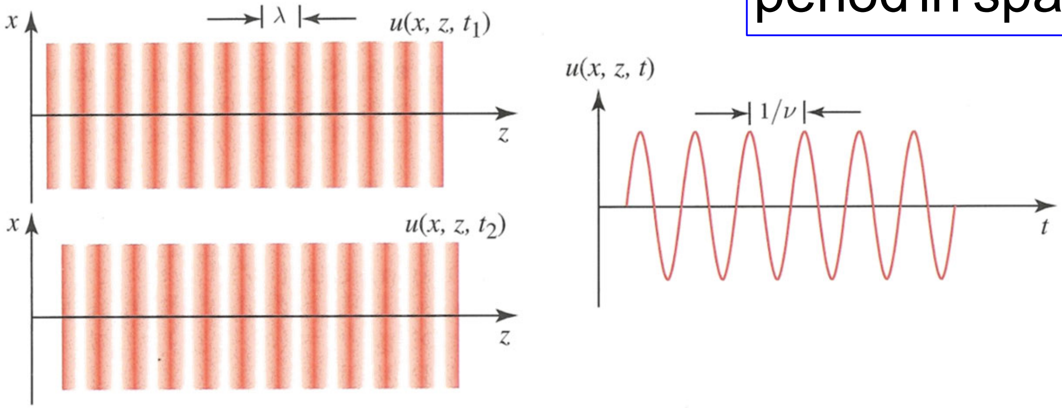
- From the generic form: $U(\mathbf{r},t) = U(\mathbf{r})\exp(j\omega t)$
- We assume also a "harmonic" form in space:

$$\boxed{U(\mathbf{r},t) = A\exp\left(-j\mathbf{k}\cdot\mathbf{r} + j\omega t\right) = A\exp\left[-j\left(k_x x + k_y y + k_z z\right) + j\omega t\right]}$$

- Wave vector: $\mathbf{k} = \left(k_x, k_y, k_z\right)$ $\boxed{k_x^2 + k_y^2 + k_z^2 = k^2 = \dfrac{\omega^2}{c^2}}$

- Plane wave travelling in the direction z:

$$\boxed{\begin{aligned} &\text{period in time}: T = 1/\nu = 2\pi/\omega \\ &\text{period in space}: \lambda = 2\pi/k \end{aligned}}$$



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Monochromatic plane wave (cont.)

- The wave fronts are planes with a distance between consecutive planes:

$$\lambda = \frac{2\pi}{k} = \frac{c}{\nu} = \frac{c_0}{n\nu} \quad \text{wavelength}$$

- Through a medium n: $\boxed{c = c_0 / n \quad \lambda = \lambda_0 / n \quad k = nk_0 \quad \omega = const.}$

  All the wave parameters vary when the wave changes from one medium to another one, except the frequency (= energy)

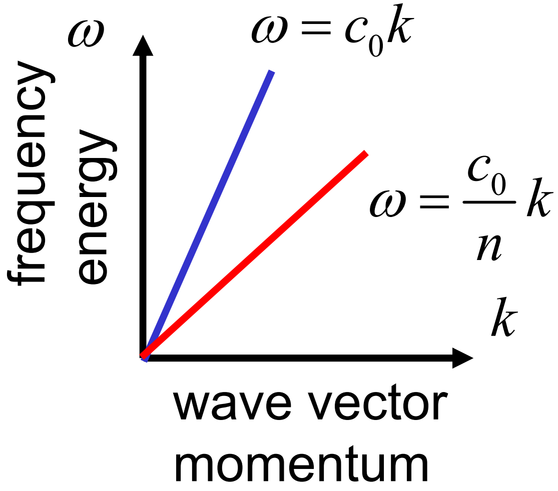
- A plane wave has constant intensity in the entire space and hence carries an infinite energy, which is not physical!

Dispersion diagram

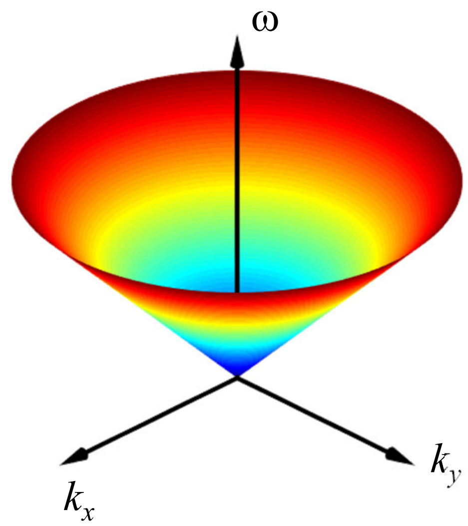
- For any wave, there is a relation between momentum (k) and energy (ω)
- This relation can be represented in a dispersion diagram, which depends on the medium in which the wave propagates

$$k_x^2 + k_y^2 = k^2 = \frac{\omega^2}{c^2}$$

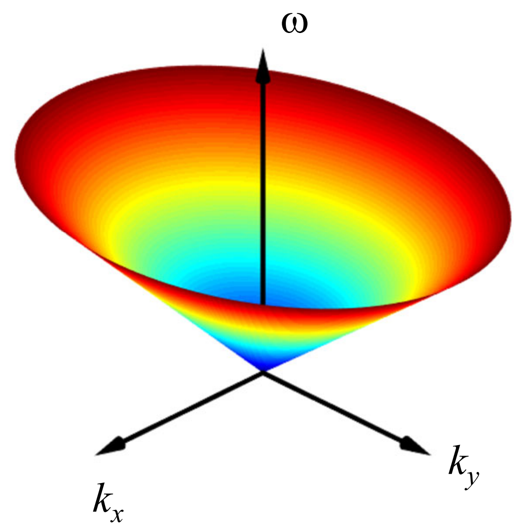
$$k = \frac{\omega}{c} = \frac{2\pi\nu}{c}$$



Isotropic medium:



Anisotropic medium:



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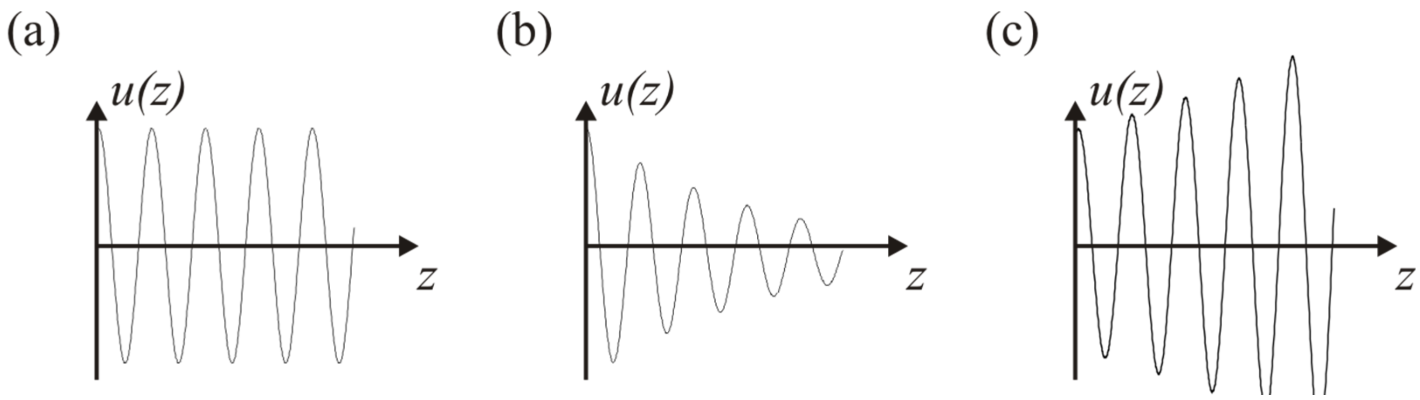
Direction of propagation

- Assuming a time dependence $\exp(+j\omega t)$
  - Forward propagating wave: $\exp(+j\omega t - jkr)$
  - Backward propagating wave: $\exp(+j\omega t + jkr)$
- Assuming a time dependence $\exp(-j\omega t)$
  - Forward propagating wave: $\exp(-j\omega t + jkr)$
  - Backward propagating wave: $\exp(-j\omega t - jkr)$
- The choice of the sign for the temporal dependence is arbitrary… but can be very confusing!
- Usually, engineering books (like Saleh & Teich) use $\exp(+j\omega t)$;
  many physics textbooks use $\exp(-j\omega t)$. In this lecture we will use both!

Evanescent wave

- Wave propagating along direction z $(\mathbf{k} = (k_x, k_y, k_z) = (0, 0, k))$
- Complex wave vector $k = k' + jk''$

$$U(\mathbf{r}, t) = Ae^{-jkz+j\omega t} = Ae^{k''z}e^{-jk'z+j\omega t}$$



$(k'' = 0)$ $(k'' < 0)$ $(k'' > 0)$

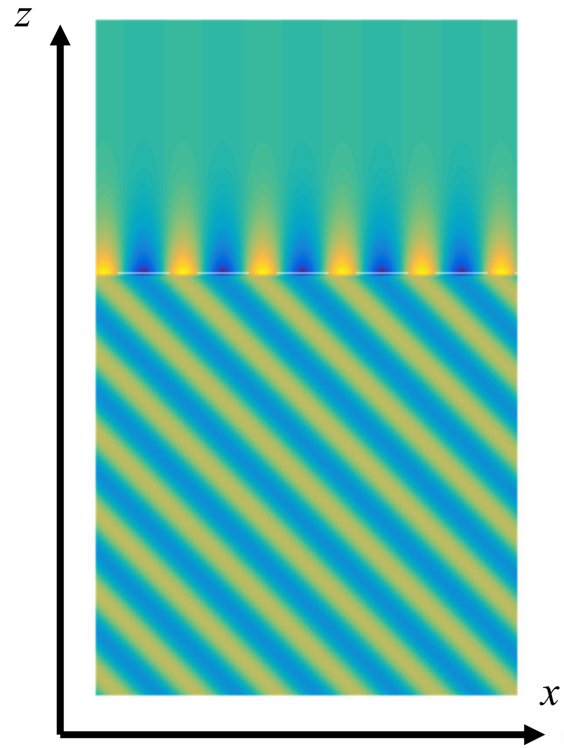
- Careful: the sign condition on k'' depends on the choice for the temporal dependence!

Evanescent wave in one direction

- A wave can propagate in different directions, let's assume

$$\mathbf{k} = (k_x, k_y, k_z) = (k_x, 0, k_z)$$

- This wave can propagate along the x-direction and be evanescent along the z-direction if k_x is real and k_z is purely imaginary
- This situation occurs under total internal reflection at an interface: the wave transmitted above the interface is evanescent in the direction normal to the interface (and propagates along the interface)



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Evanescent wave caused by a large wave vector component

- Let's consider again $\mathbf{k} = (k_x, k_y, k_z) = (k_x, 0, k_z)$

- We have $k_x^2 + k_y^2 + k_z^2 = k_x^2 + k_z^2 = \dfrac{\omega^2}{c^2}$

$$\boxed{k_z = \pm\sqrt{\frac{\omega^2}{c^2} - k_x^2}}$$

- When $k_x^2 \le \dfrac{\omega^2}{c^2}$, k_z is real and the wave propagates in z-direction
- When $k_x^2 > \dfrac{\omega^2}{c^2}$, k_z is imaginary and the wave is evanescent in z-direction

- The choice for the ± sign is complicated and usually determined by considerations on energy conservation and causality (the signal cannot arrive before it is emitted) we will discuss this in detail when we discuss negative refraction