

# **Selected Topics in Advanced Optics**

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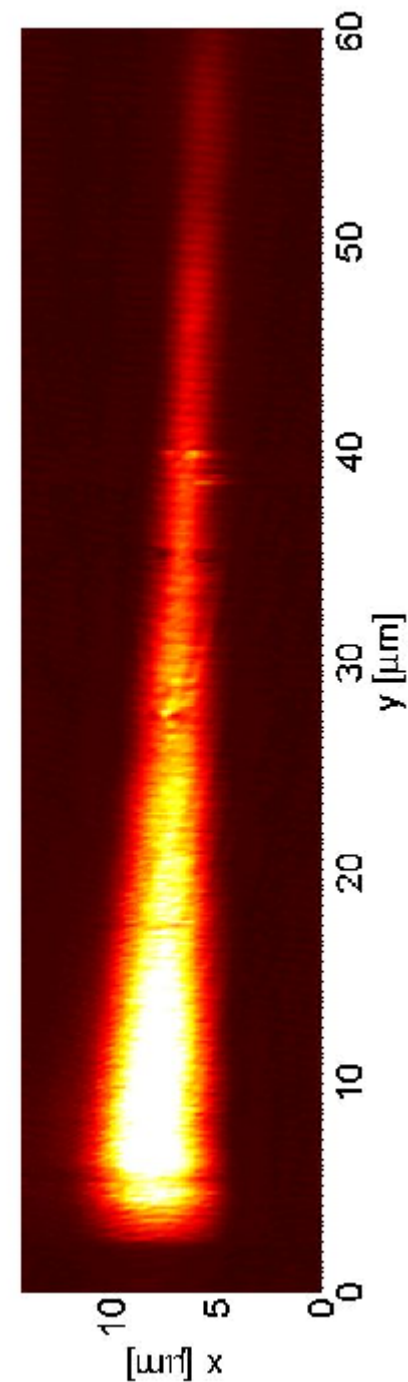
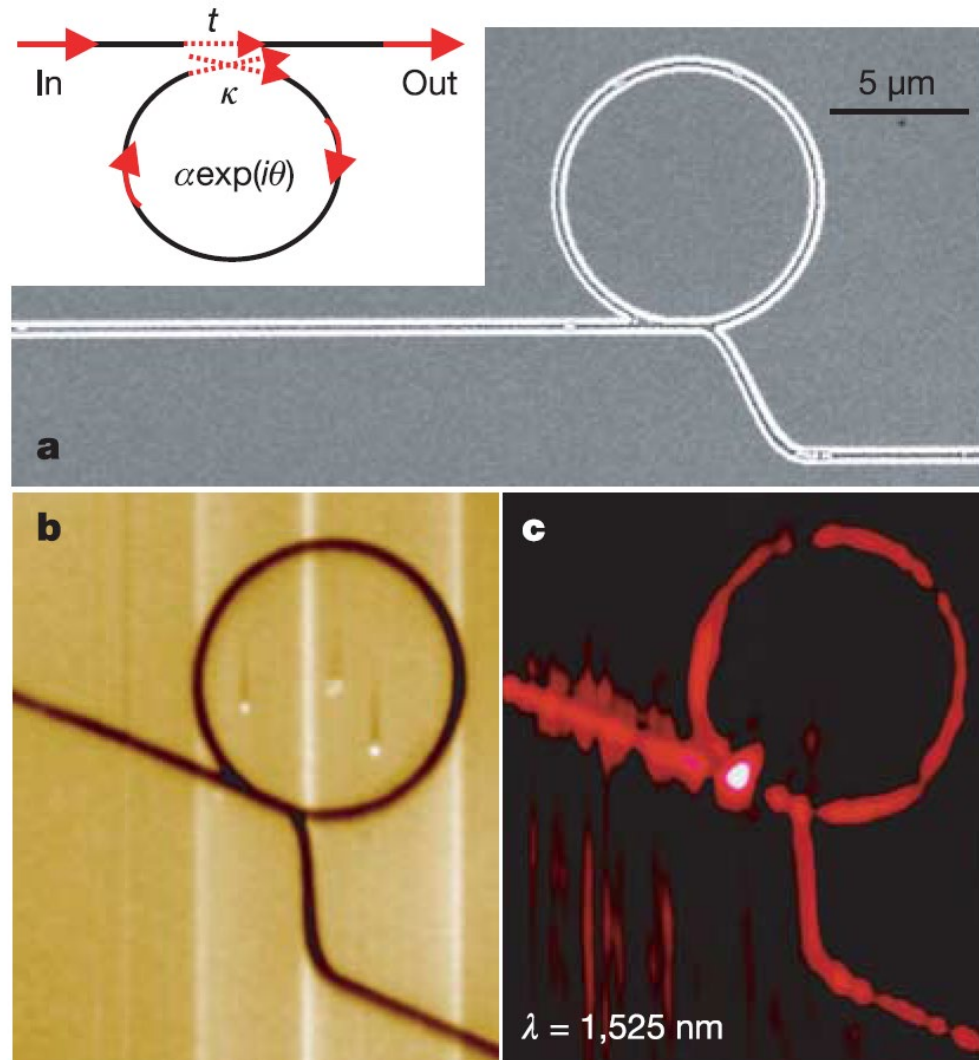
## **Week 6 – part 1**

Olivier J.F. Martin  
Nanophotonics and Metrology Laboratory



# Optics of metals and plasmonics

## Part II – Propagating plasmons

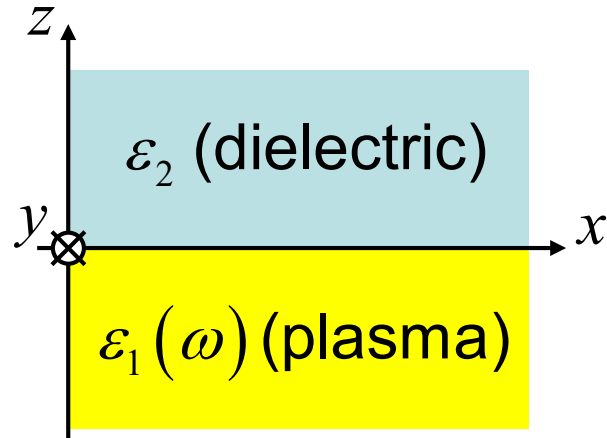


## Some references

- H. Raether, Surface Plasmons on Smooth and Rough Surfaces and on Gratings, Springer Tracts in Modern Physics **111** (Springer, New York, 1988).
- S.A. Maier, Plasmonics: Fundamentals and Applications (Springer, 2007).
- J. Homola, Surface plasmon resonance based sensors, Springer Series in Chemical Sensors and Biosensors **4** (Springer, Berlin, 2006).
- P. Berini, “Long-range surface plasmon polaritons”, *Adv. in Opt. and Photon.* **1**, 484-588 (2009).
- M.L. Brongersma and P.G. Kik (Eds.), Surface plasmon nanophotonics (Springer, 2007).

# SPP dispersion relation on a semi-infinite metal (Raether)

p-polarized wave:



$$\begin{cases} \mathbf{E}_2 = (E_{2x}, 0, E_{2z}) \exp i(k_{2x}x + k_{2z}z - \omega t) \\ \mathbf{H}_2 = (0, H_{2y}, 0) \exp i(k_{2x}x + k_{2z}z - \omega t) \end{cases}$$

$$\begin{cases} \mathbf{E}_1 = (E_{1x}, 0, E_{1z}) \exp i(k_{1x}x - k_{1z}z - \omega t) \\ \mathbf{H}_1 = (0, H_{1y}, 0) \exp i(k_{1x}x - k_{1z}z - \omega t) \end{cases}$$

Maxwell's equations :

- (1)  $\nabla \times \mathbf{E}_i = i\omega\mu_0 \mathbf{H}_i$
- (2)  $\nabla \times \mathbf{H}_i = -i\omega\epsilon_0 \epsilon_i \mathbf{E}_i$
- (3)  $\epsilon_0 \epsilon_i \nabla \cdot \mathbf{E}_i = 0$
- (4)  $\nabla \cdot \mathbf{H} = 0$

Continuity at the interface :

$$\begin{aligned} E_{1x} &= E_{2x} \\ H_{1y} &= H_{2y} \quad (5) \\ \epsilon_1 E_{1z} &= \epsilon_2 E_{2z} \\ k_{1x} &= k_{2x} = k_x \end{aligned}$$

## SPP dispersion relation on a semi-infinite metal

$$(2) \text{ leads to : } \frac{\partial H_{iy}}{\partial z} = i\omega\epsilon_0\epsilon_i E_{ix} \quad \begin{aligned} k_{1z}H_{1y} &= -\omega\epsilon_0\epsilon_1 E_{1x} \\ k_{2z}H_{2y} &= \omega\epsilon_0\epsilon_2 E_{2x} \end{aligned}$$

$$\text{using (5) : } \begin{cases} H_{1y} - H_{2y} = 0 \\ \frac{k_{1z}}{\epsilon_1} H_{1y} + \frac{k_{2z}}{\epsilon_2} H_{2y} = 0 \end{cases}$$

$$\text{solution if } \det = 0 : \det = \frac{k_{2z}}{\epsilon_2} + \frac{k_{1z}}{\epsilon_1}$$

$$\text{furthermore : } k_x^2 + k_{iz}^2 = \epsilon_i \frac{\omega^2}{c^2}$$

$$\text{it follows : } k_x = \frac{\omega}{c} \left( \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2} \right)^{1/2}$$

## SPP dispersion relation on a semi-infinite metal

Metal with complex permittivity :  $\varepsilon_1 = \varepsilon'_1 + i\varepsilon''_1$

covered with dielectric :  $\varepsilon_2$  real

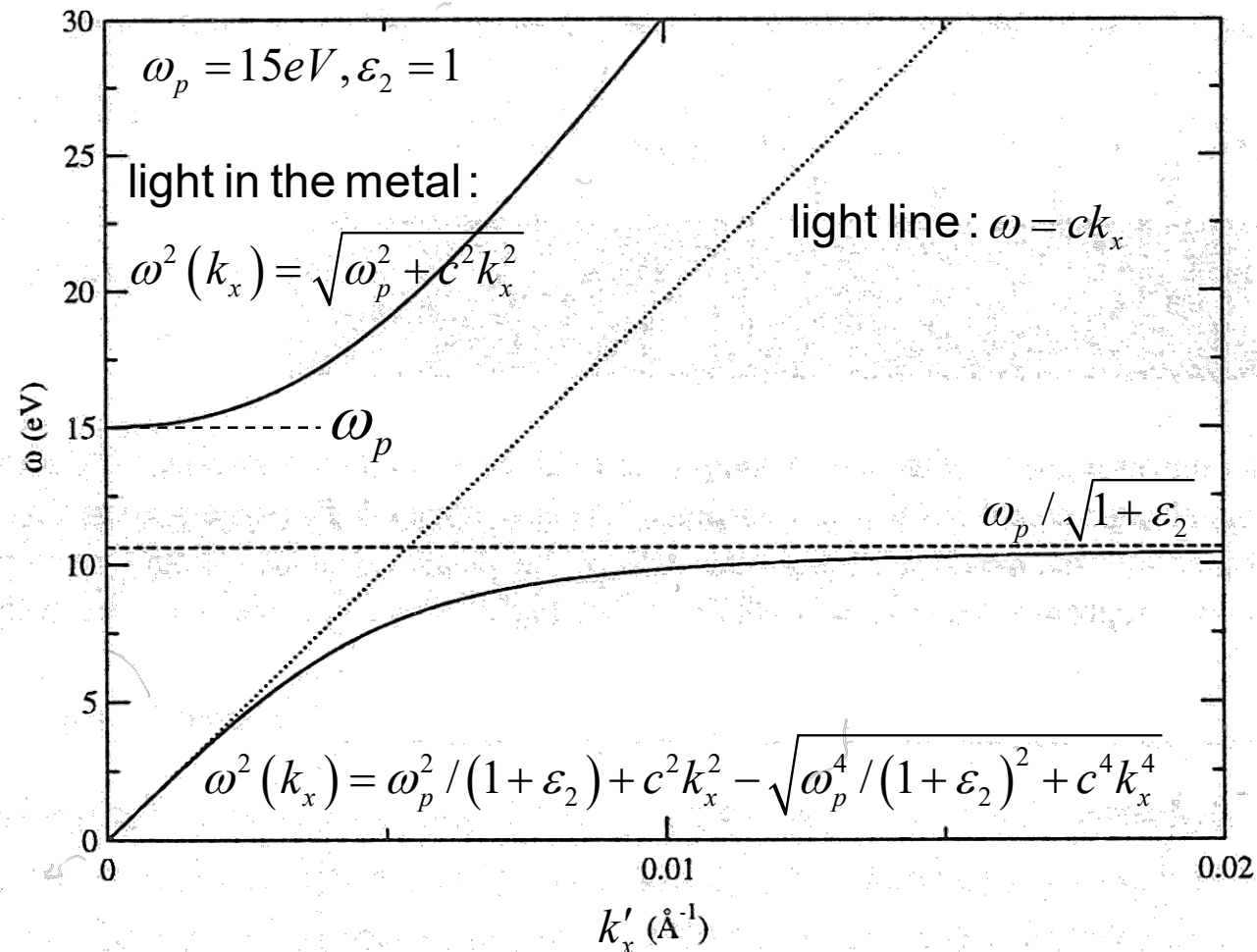
Complex propagation vector :  $k_x = k'_x + ik''_x$

$$k'_x = \frac{\omega}{c} \left( \frac{\varepsilon'_1 \varepsilon_2}{\varepsilon'_1 + \varepsilon_2} \right)^{1/2} \quad k''_x = \frac{\omega}{c} \left( \frac{\varepsilon'_1 \varepsilon_2}{\varepsilon'_1 + \varepsilon_2} \right)^{3/2} \frac{\varepsilon''_1}{2(\varepsilon'_1)^2}$$

# SPP dispersion relation on a semi-infinite Drude metal in air

Drude metal:  $\varepsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$

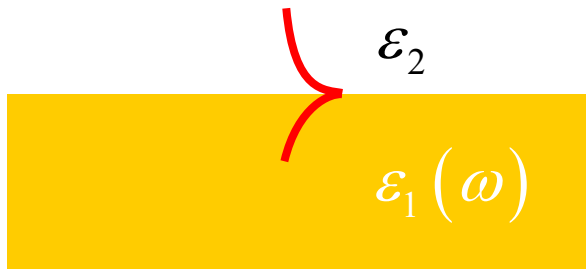
$$k'_x = \frac{\omega}{c} \left( \frac{\omega^2 - \omega_p^2}{2\omega^2 - \omega_p^2} \right)^{1/2} \quad (\omega \gg \gamma)$$



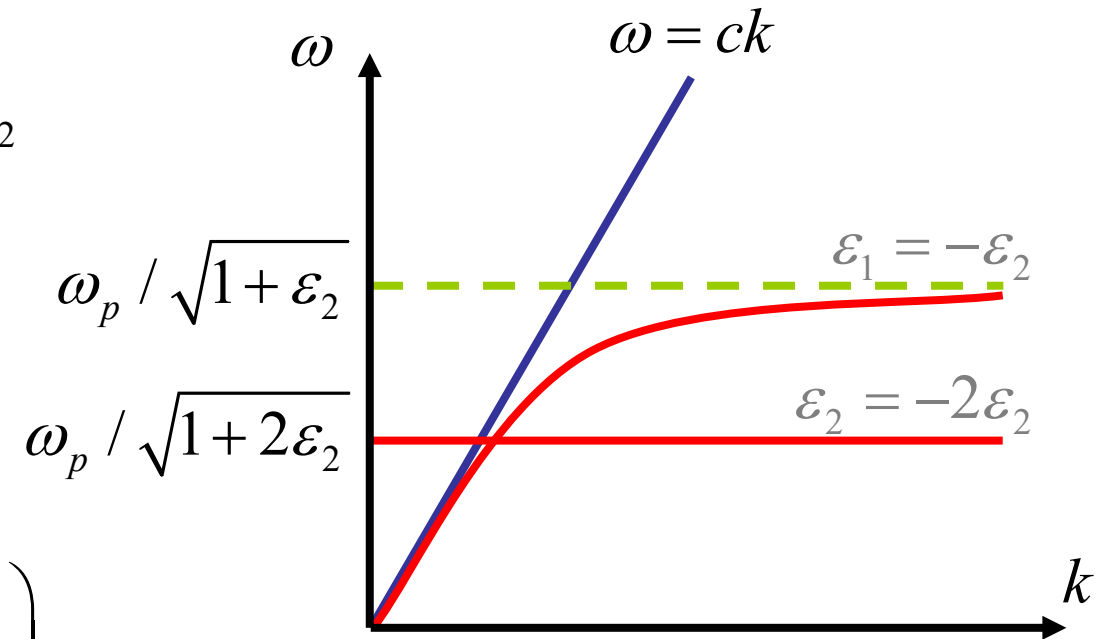
# Localized and propagating plasmons for a Drude metal

$$\epsilon_1(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$

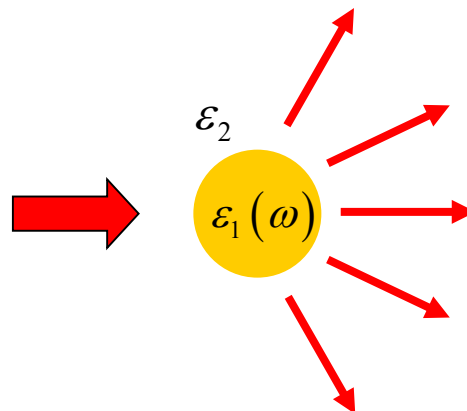
- The propagating or SPP mode is a dispersive mode: the frequency  $\omega$  and the momentum  $k$  change continuously
- The localized plasmon resonance does not disperse: its frequency  $\omega$  is constant, irrespective of the momentum  $k$
- Propagating mode (SPP) :



$$k'_x = \frac{\omega}{c} \left( \frac{\omega^2 - \omega_p^2}{2\omega^2 - \omega_p^2} \right)^{1/2}$$



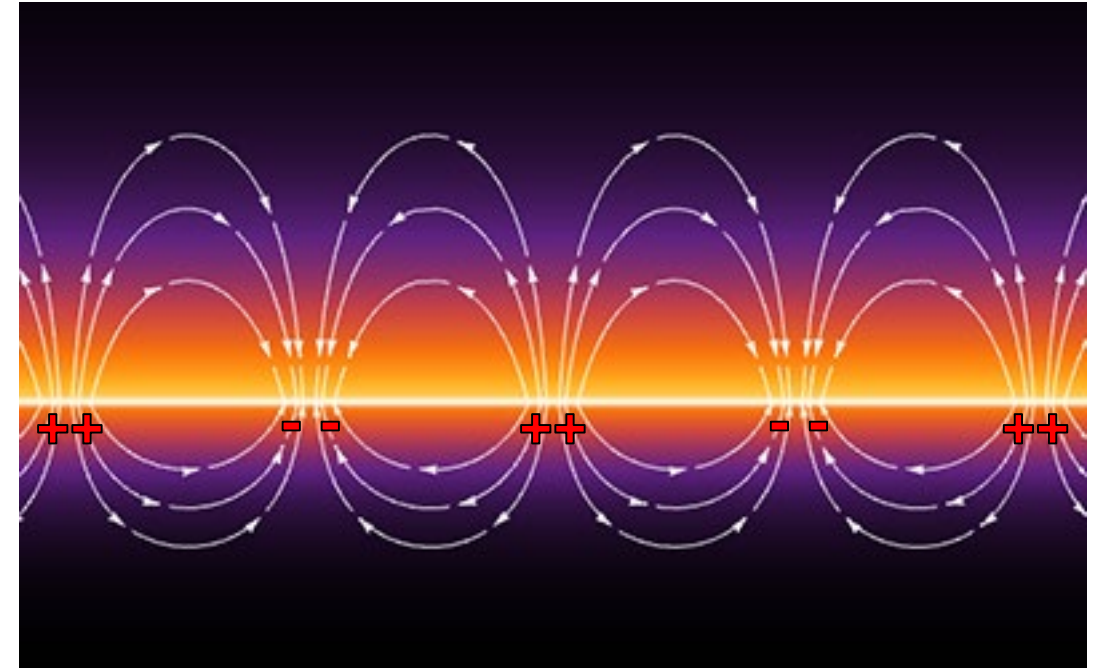
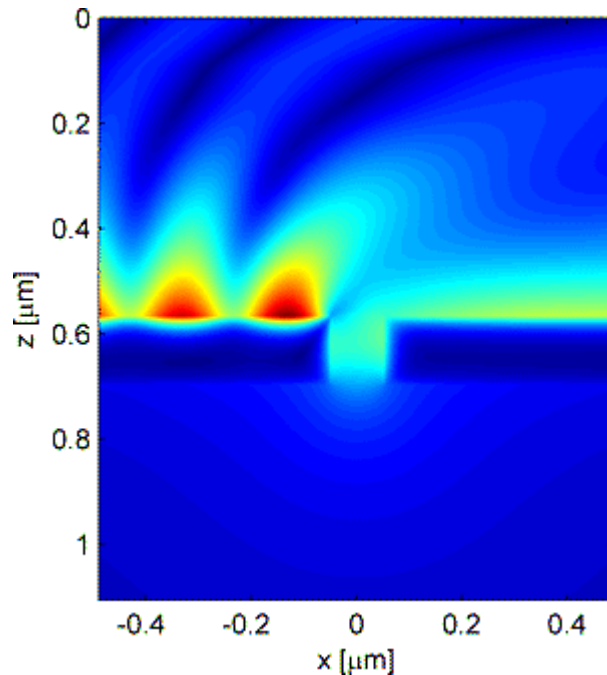
- Localized mode (LSPR):



$$\alpha(\omega) \approx \left( \frac{\epsilon_1(\omega) - \epsilon_2}{\epsilon_1(\omega) + 2\epsilon_2} \right)$$

## SPP dispersion relation on a semi-infinite metal

- There is localization of the charges along the metal surface
- The field distribution decays very rapidly away from the interface



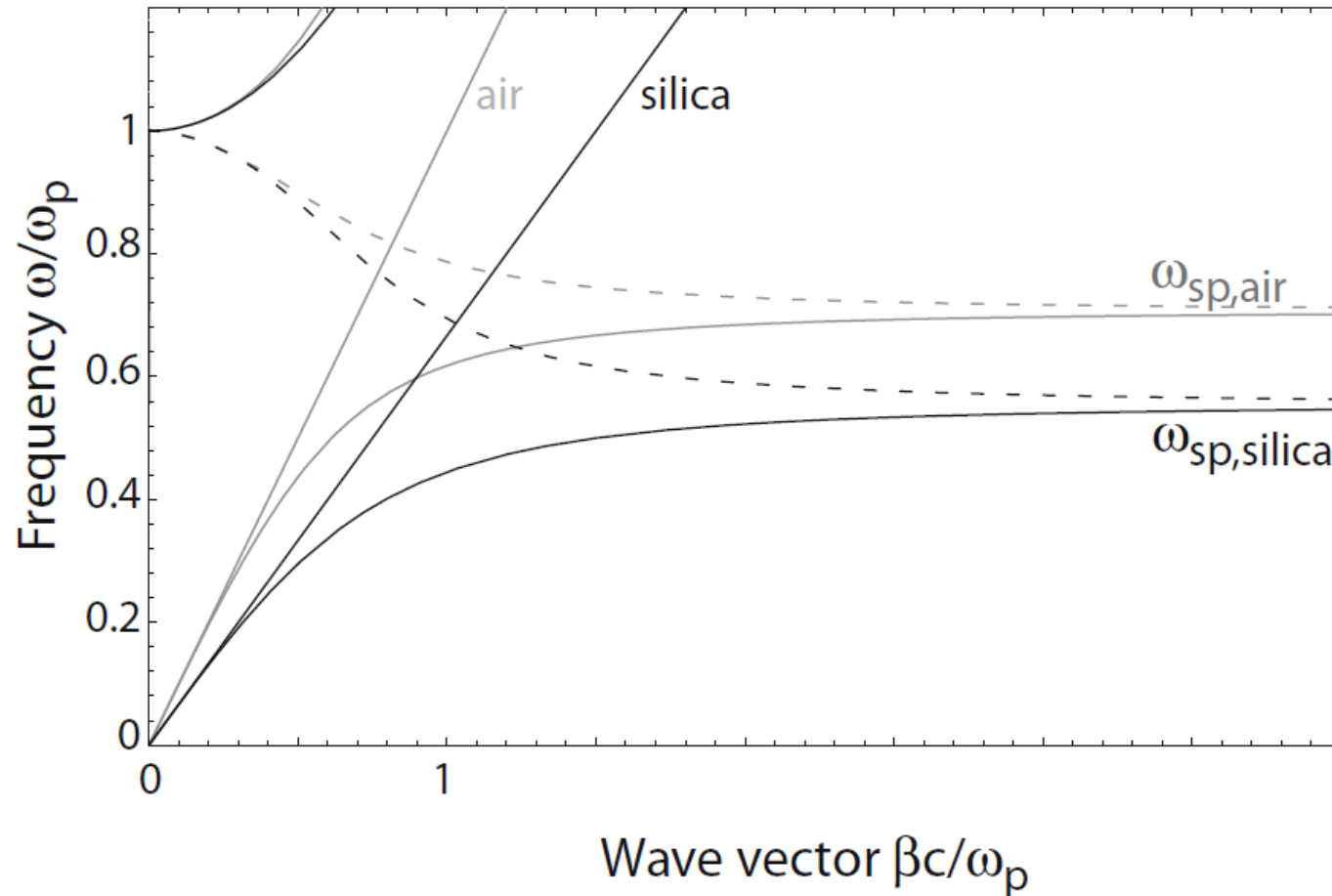
*D.R. Smith, Duke University*

SPP coupling to a groove in a metallic film

*Oulu University, Finland*

## SPP dispersion relation on a semi-infinite metal

- The dispersion relation changes with the dielectric function of the covering material on the top of the metal film



S.A. Maier, Plasmonics: Fundamentals and Applications (Springer, 2007).

# **Selected Topics in Advanced Optics**

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## **Week 6 – part 2**

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## SPP dispersion relation on a thin film

Derivation based on the reflectivity (p-polarized wave):

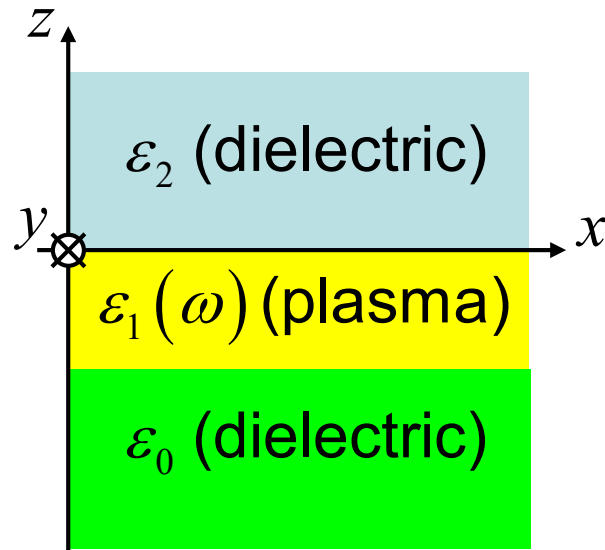
For each interface :

$$r_{ij} = \left( \frac{k_{iz}}{\varepsilon_i} - \frac{k_{jz}}{\varepsilon_j} \right) / \left( \frac{k_{iz}}{\varepsilon_i} + \frac{k_{jz}}{\varepsilon_j} \right) \quad t_{ij} = 1 + r_{ij}$$

For the total system :

$$r_{012} = \frac{E_r}{E_0} = \frac{r_{01} + r_{12} \exp(2i\alpha)}{1 + r_{01}r_{12} \exp(2i\alpha)} \quad \alpha = k_{1z}d$$

$(2i\alpha < 0, \text{ since } \varepsilon_1 < 0)$

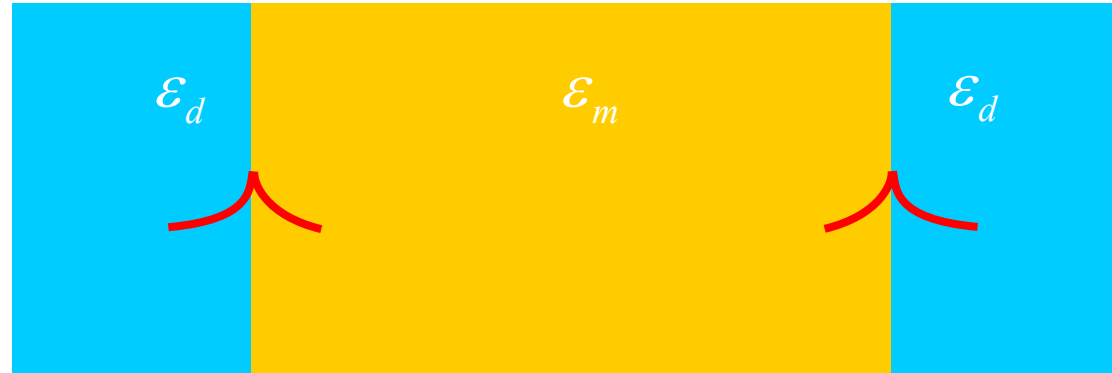


The zero of the denominator of  $r_{012}$  give the dispersion relation of the system:

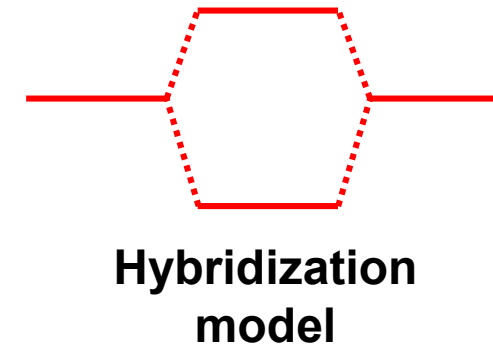
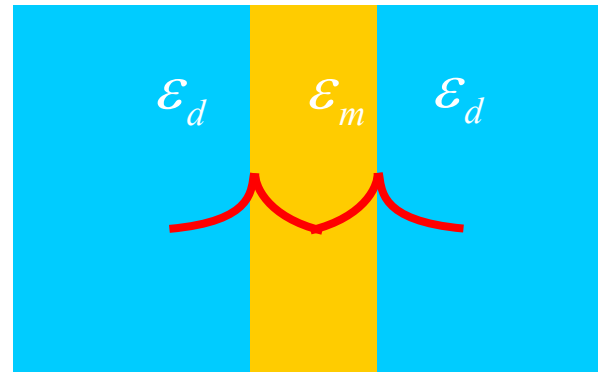
$$1 + r_{01}r_{12} \exp(2i\alpha) = 0$$

## SPP dispersion relation on a thin film – Intuitive approach

- Consider two surface plasmons on two interfaces:



- When the metal becomes thin, the two modes will couple:



- This will produce two different modes: a long-range and a short range mode

## Long-range and short-range SPP (propagation normal to the plane of the figure)

- Propagation along  $z$
- The long-range mode has symmetric dominant electric field  $E_y$
- The short-range mode has antisymmetric  $E_y$
- Other components:
  - $H_x$  same as  $E_y$
  - $E_z$  opposite as  $E_y$

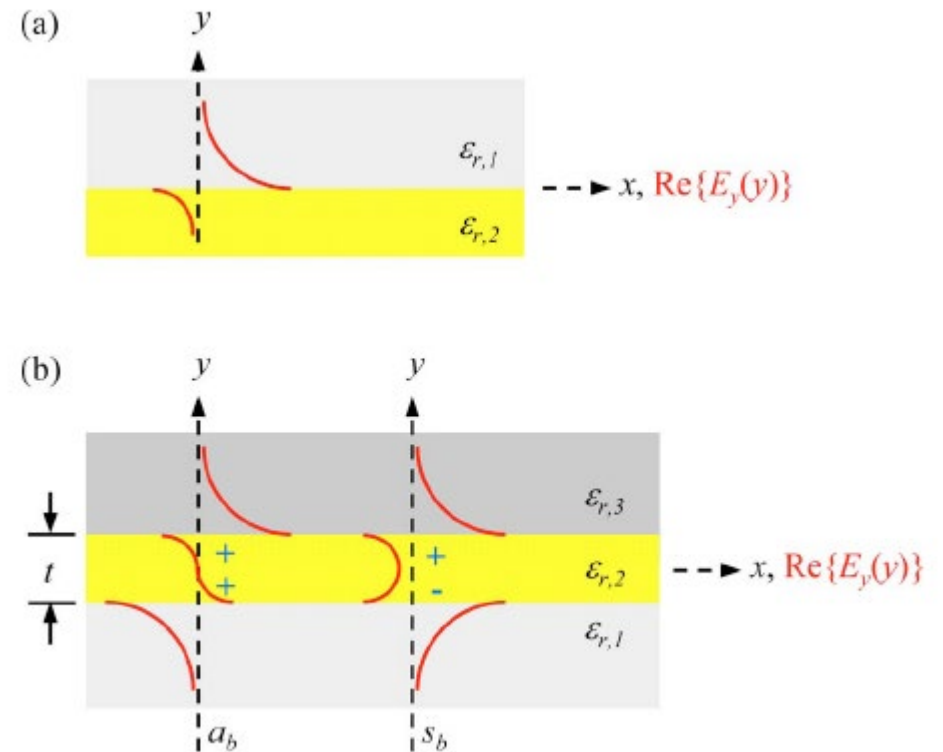


Table 1. Model Properties of SPP Waveguides

| Mode, Structure                        | $n_{\text{eff}} - n_1$ | MPA<br>(dB/mm) | $L_e$<br>( $\mu\text{m}$ ) | $\delta_w$<br>( $\mu\text{m}$ ) |
|----------------------------------------|------------------------|----------------|----------------------------|---------------------------------|
| SPP, single interface                  | 0.013                  | 44             | 98                         | 1.27                            |
| LRSP (S <sub>b</sub> mode), metal slab | 0.0023                 | 1.2            | 3776                       | 6.12                            |

## Long-range vs. short range SPP (propagation in the plane of the figure)

- They have opposite charge distribution:

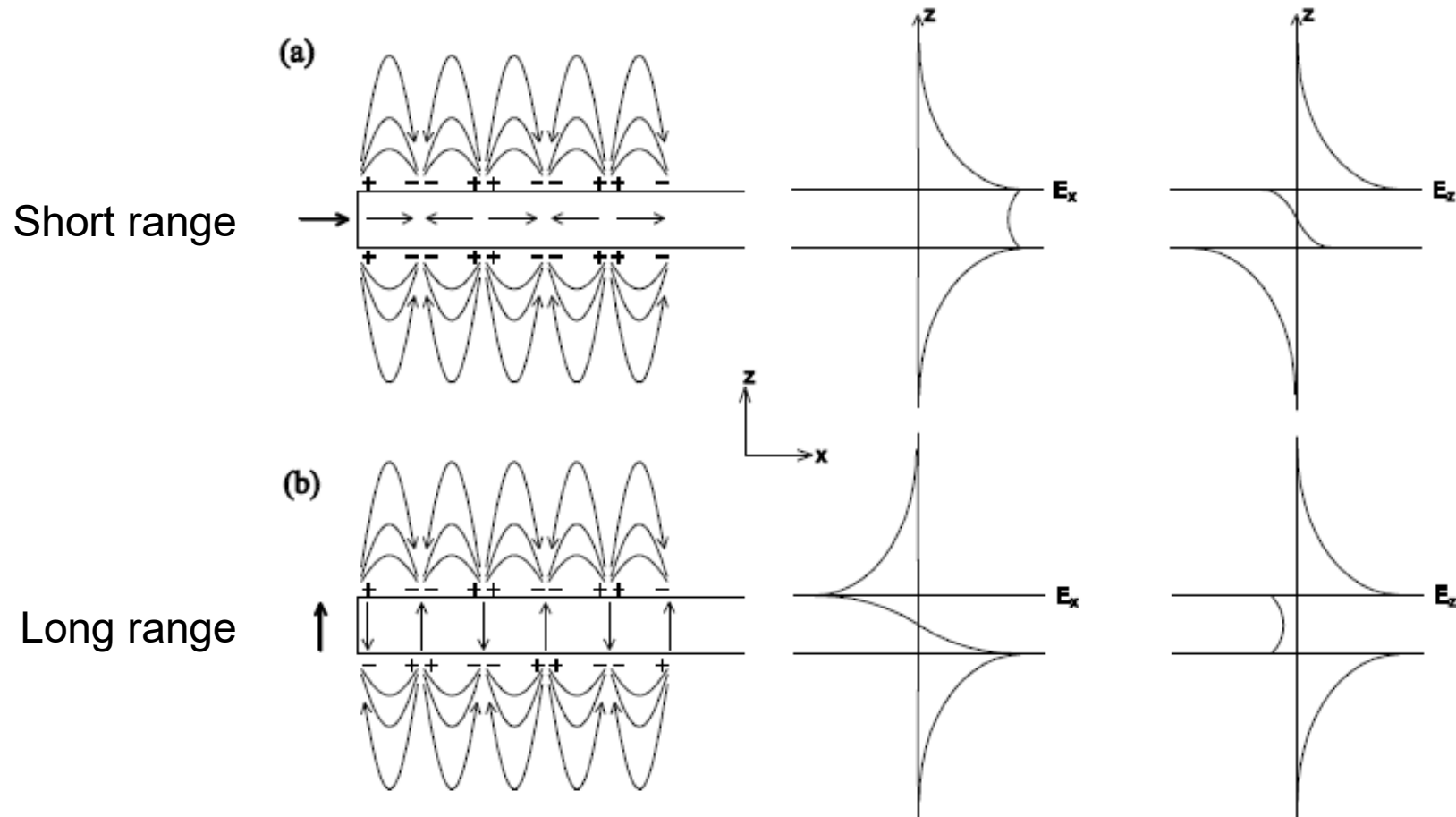
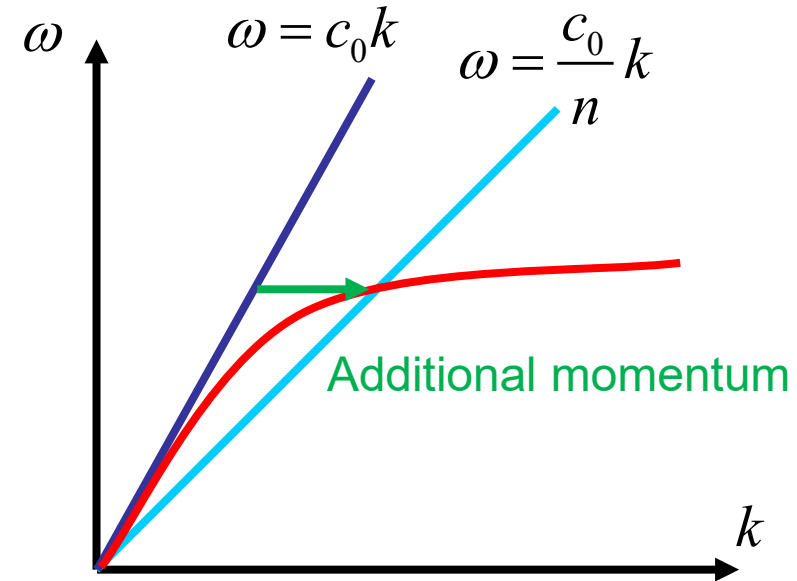


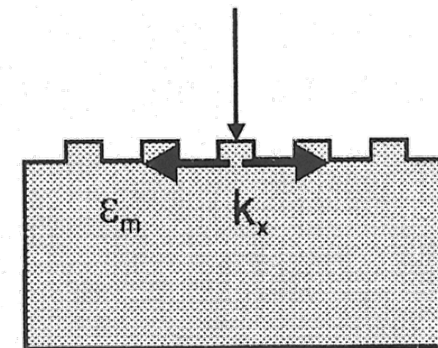
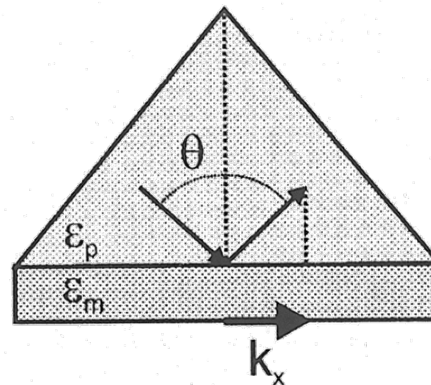
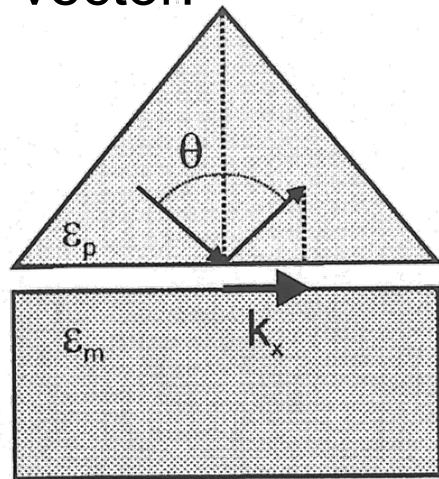
Fig. 3. Effect of horizontal and vertical dipole sources on an infinitely long metallic nanoparticle and the resulting electric field distributions. (a) A horizontal dipole excites the even propagating mode, while (b) a vertical dipole excites the odd mode.

## Excitation of SPP

- Direct excitation is not possible (from air),  $k$  is too short !

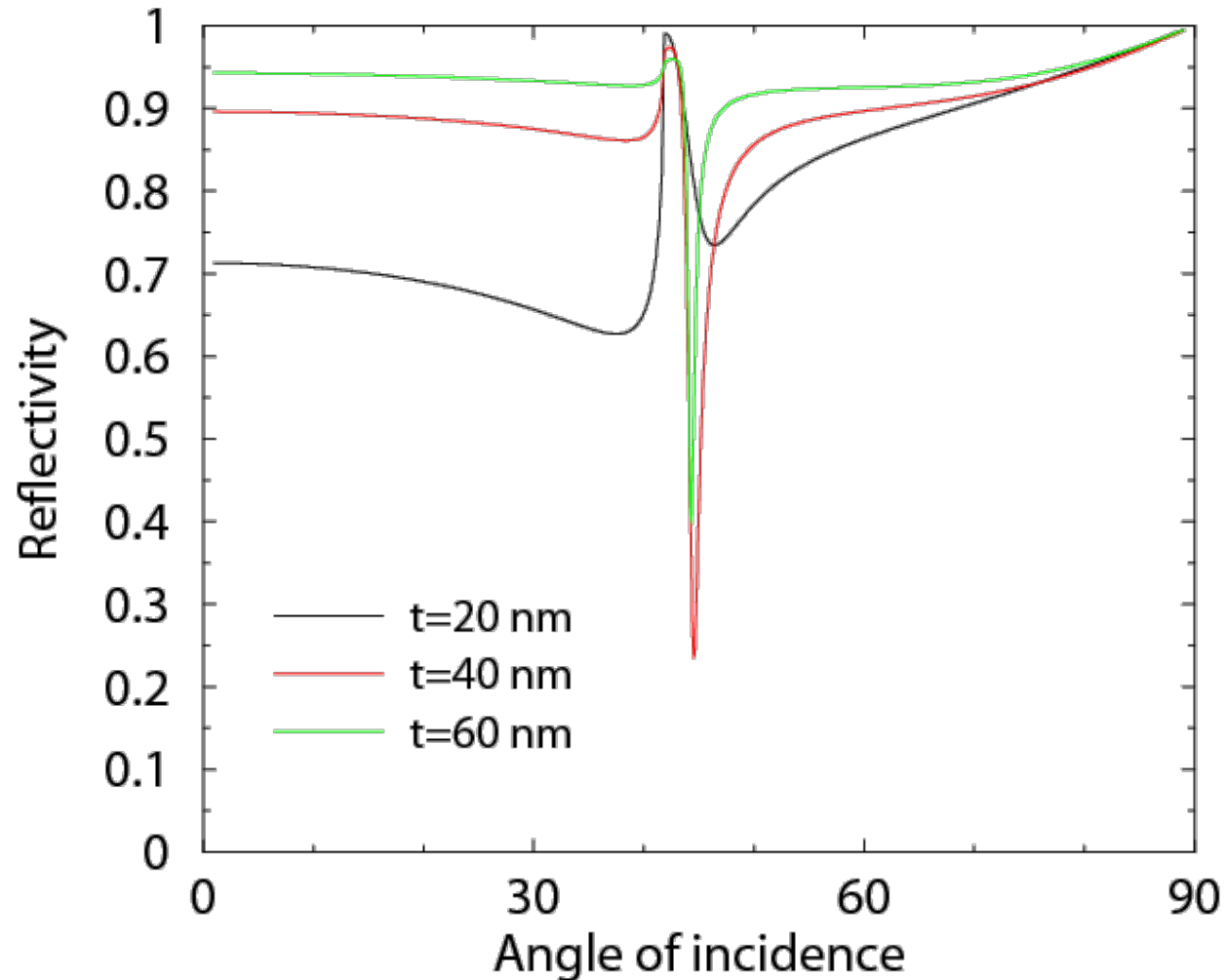
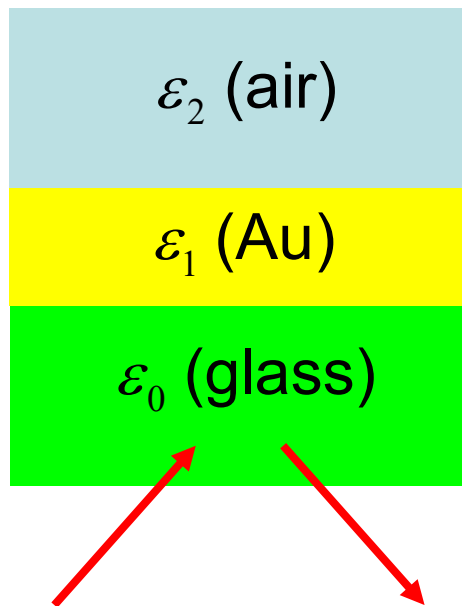


- There are different ways to provide additional momentum to match the SPP propagation vector:



## SPP excitation on a thin film

- Au film on a glass substrate, different thicknesses: when the SPP is excited, most of the incident energy goes into that mode and very little light is reflected



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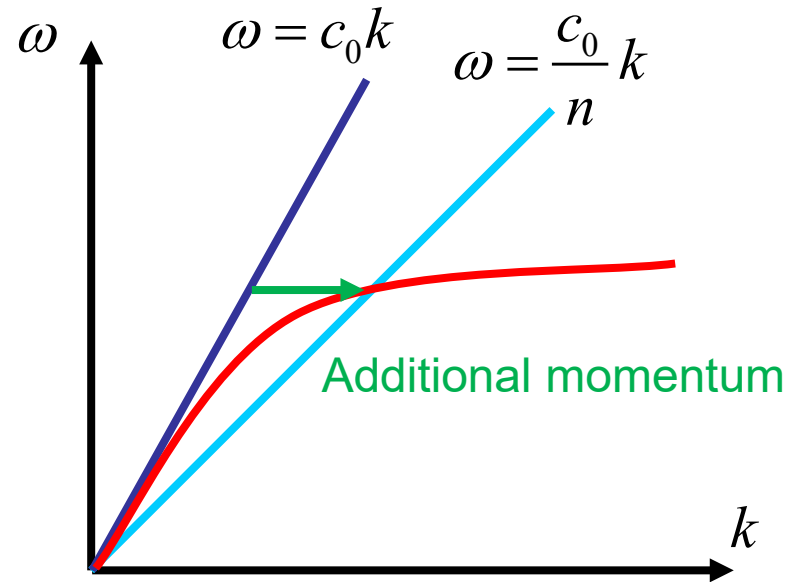
## **Week 6 – part 3**

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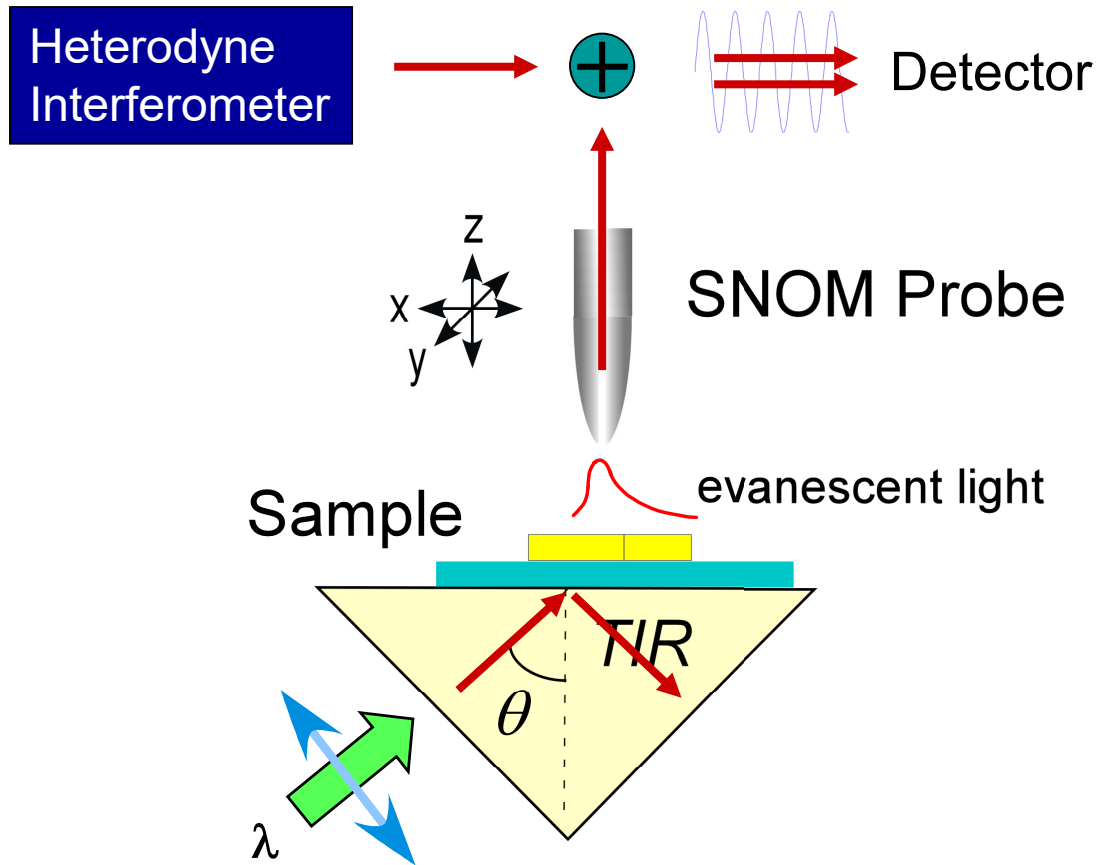
# Direct SPP measurement using heterodyne photon scanning tunneling microscopy (PSTM)

- The SPP has a larger momentum  $k$  than a wave propagating in free space
- Its effective wavelength  $\lambda$  should be smaller ( $\lambda = 2\pi / k$ )

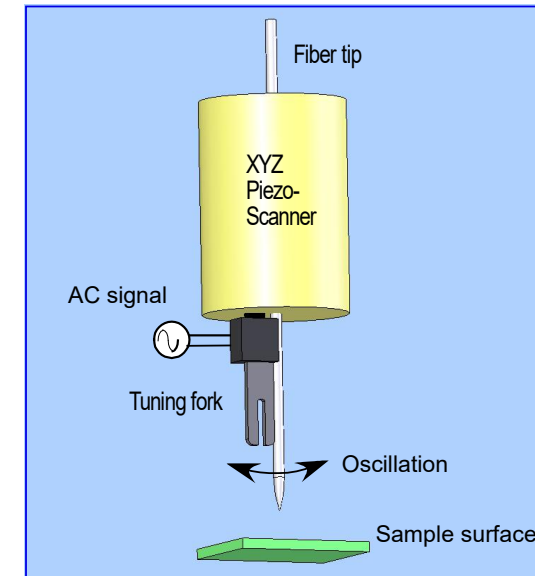


- Can we measure this experimentally?

# Direct SPP measurement using heterodyne photon scanning tunneling microscopy (PSTM)



- Shear force AFM:

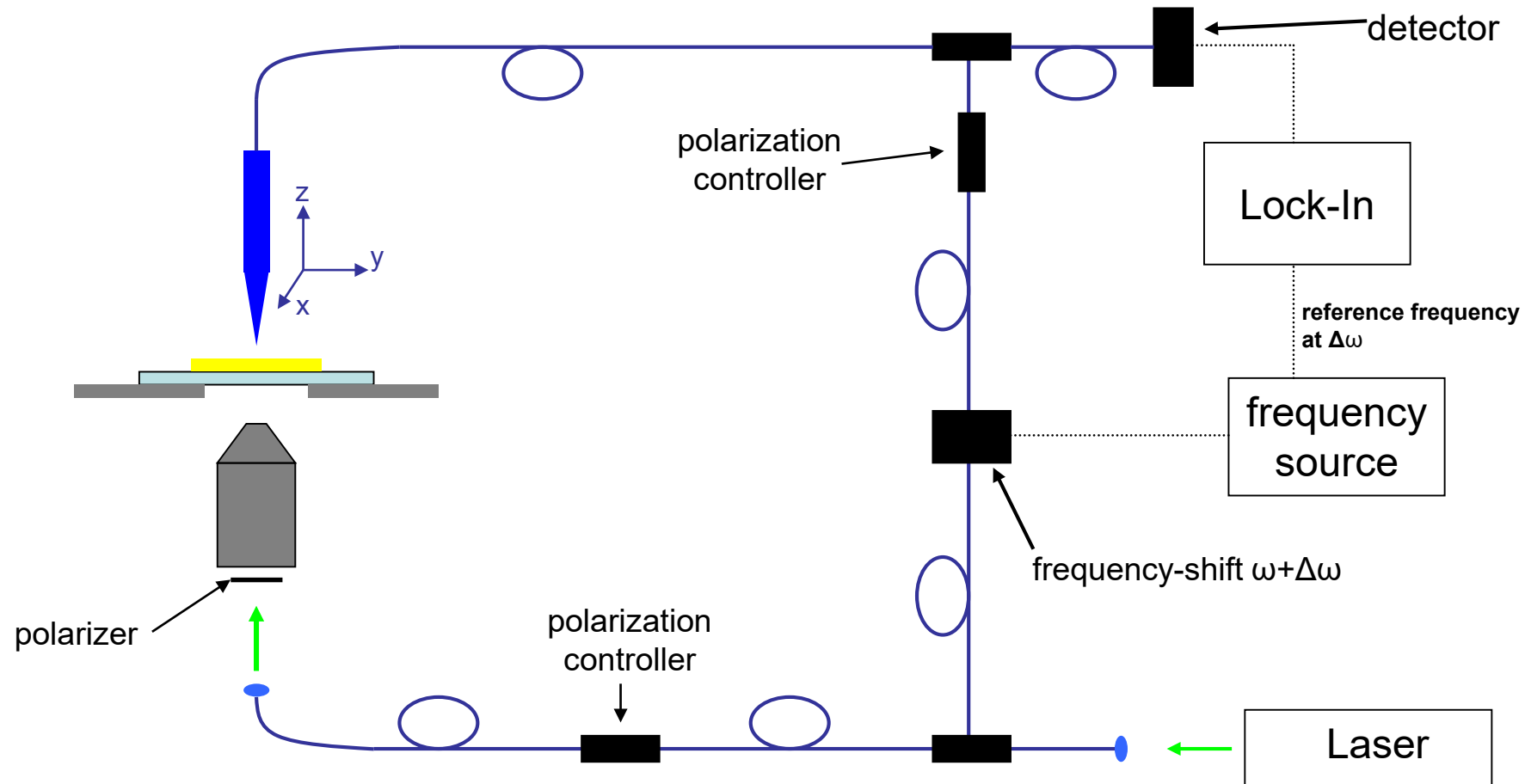


resolution:  $x, y \sim 1 \text{ nm}$   
 $z \sim 5 \text{ nm}$

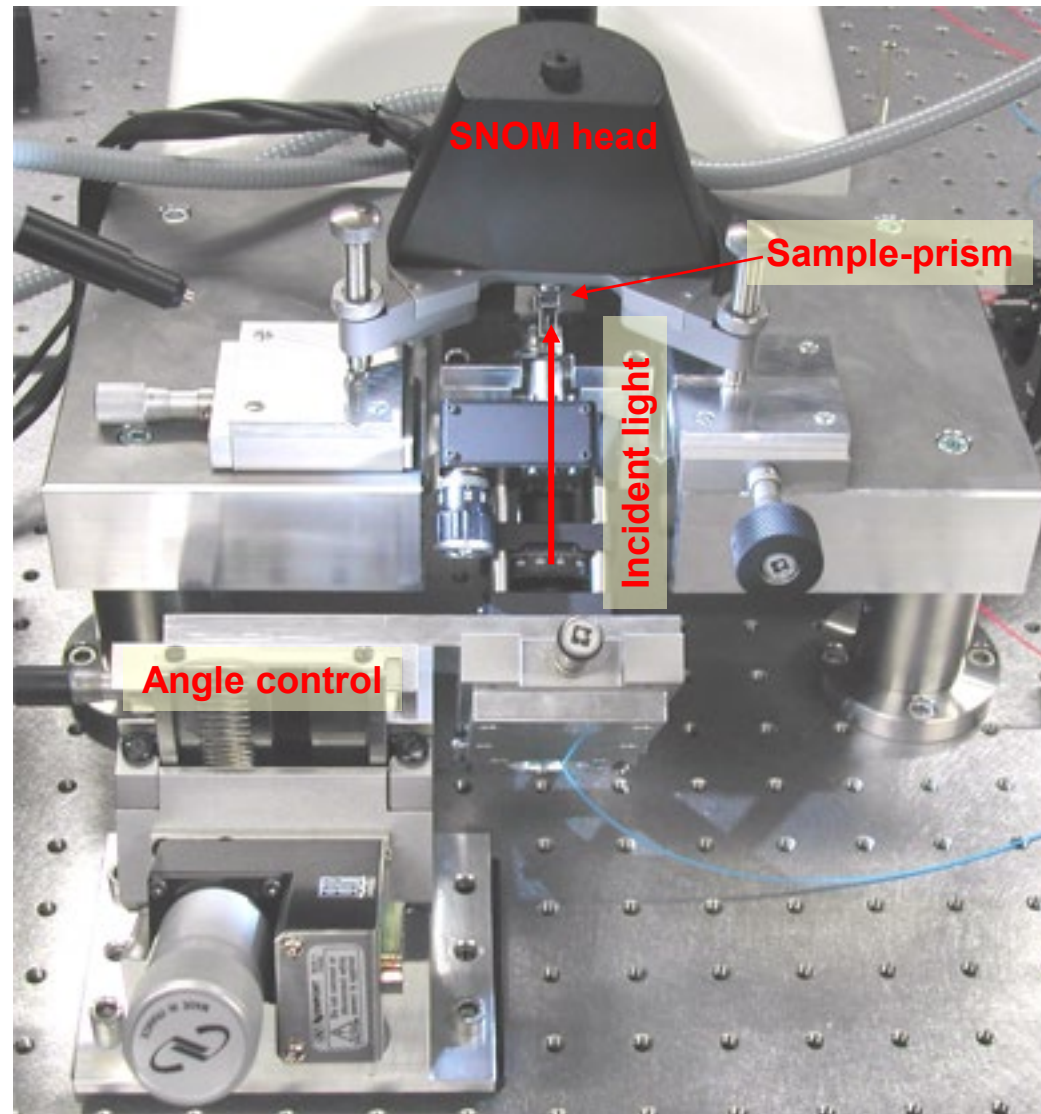
- Coherent PSTM combined with heterodyne interferometer  
→ measurement of the amplitude and phase of the field !

# Heterodyne photon scanning tunneling microscopy (PSTM)

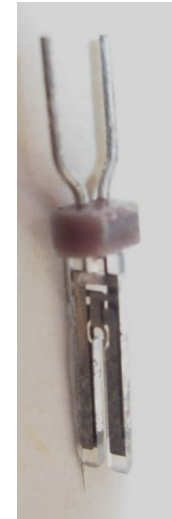
- In addition to the phase measurement, this interferometric technique provides very high sensitivity (shot-noise limited)



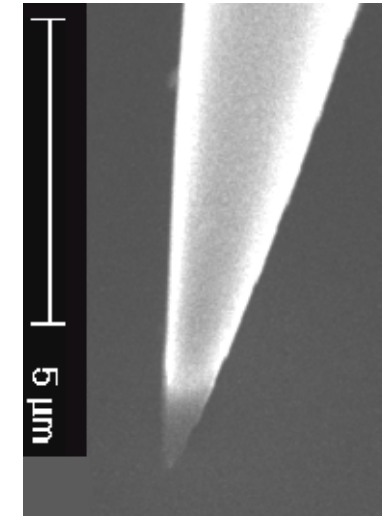
# Heterodyne photon scanning tunneling microscopy (PSTM)



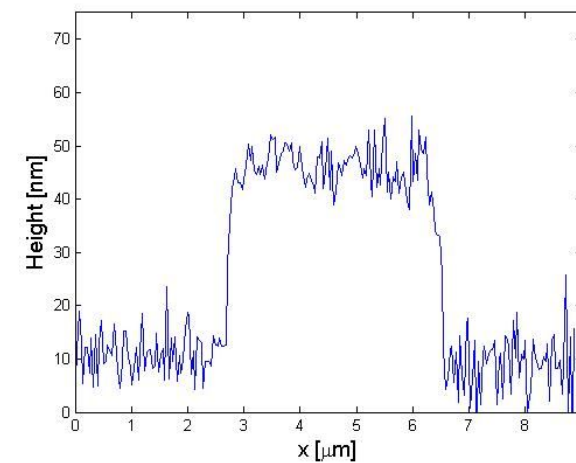
Tuning fork



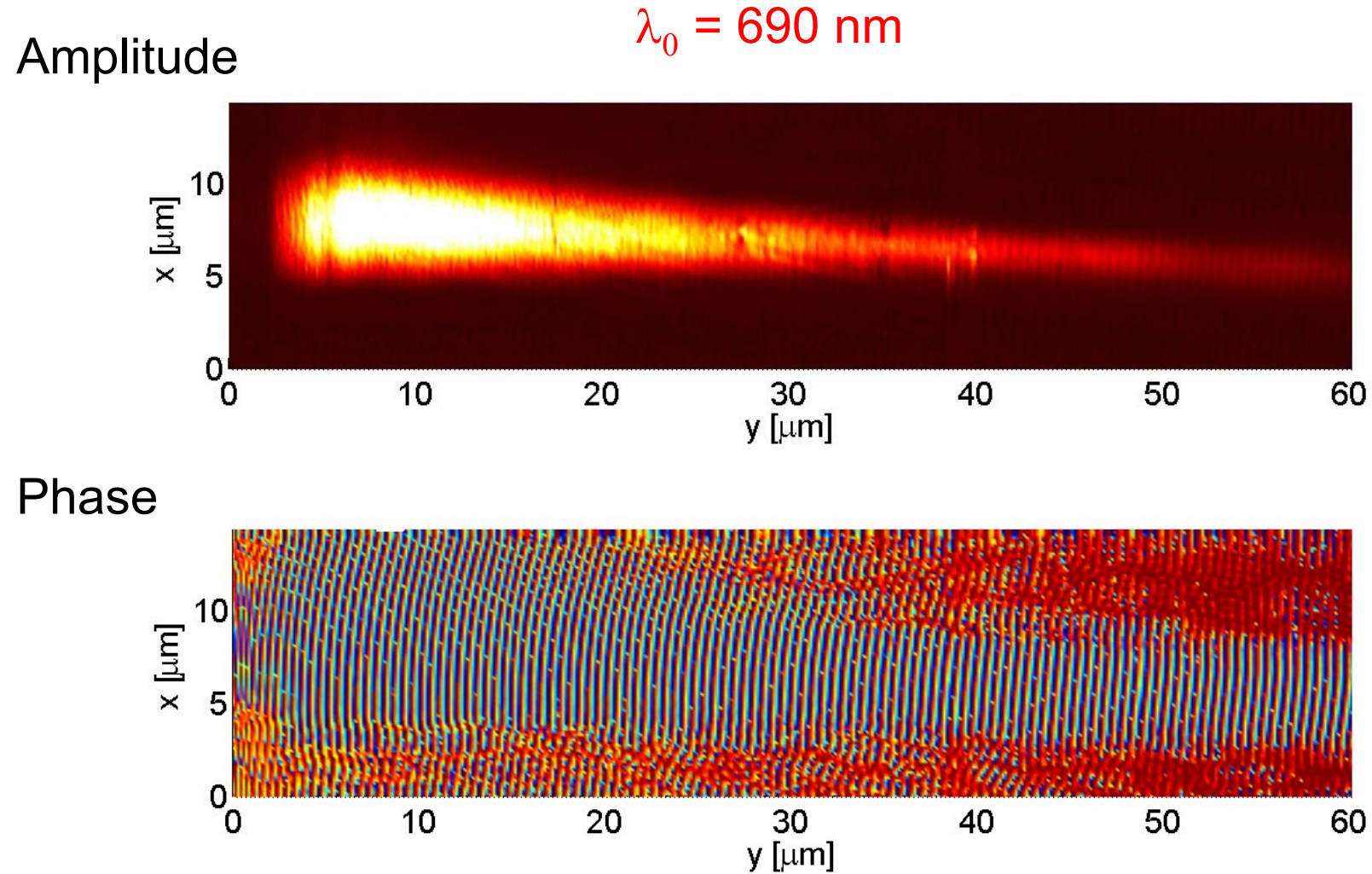
Fiber probe ( $\rho \sim 50 \text{ nm}$ )



Gold waveguide topography

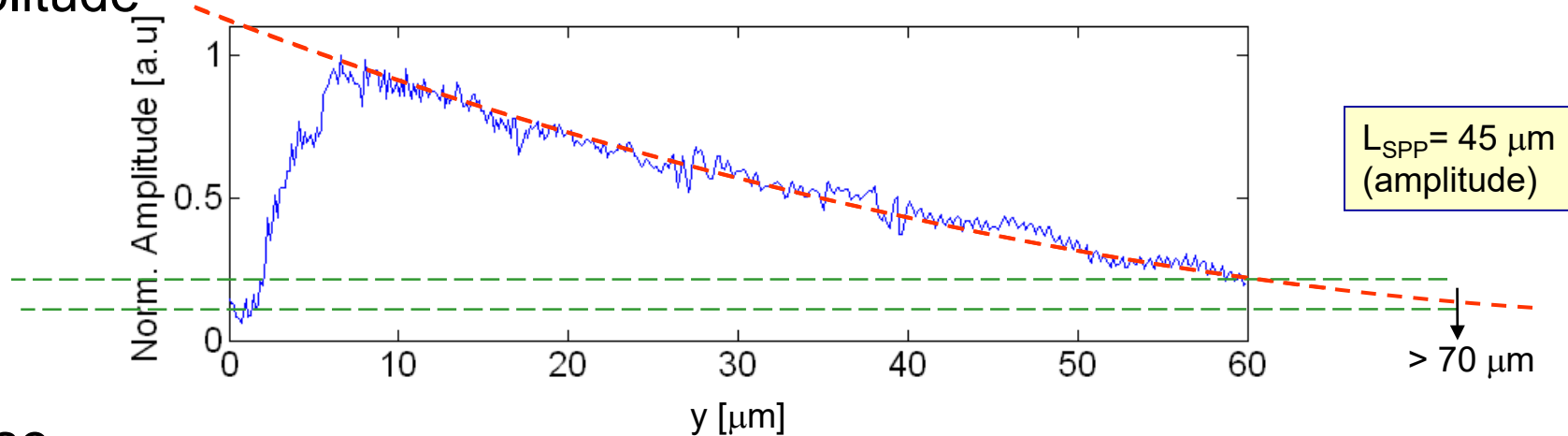


# Direct SPP measurement using heterodyne photon scanning tunneling microscopy (PSTM)

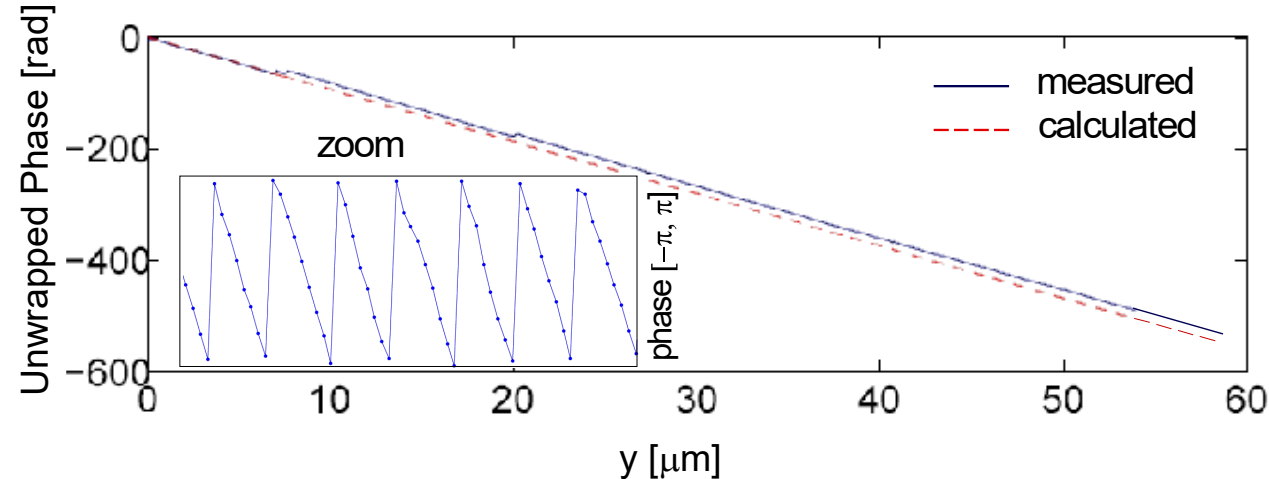


# Direct SPP measurement using heterodyne photon scanning tunneling microscopy (PSTM)

Amplitude



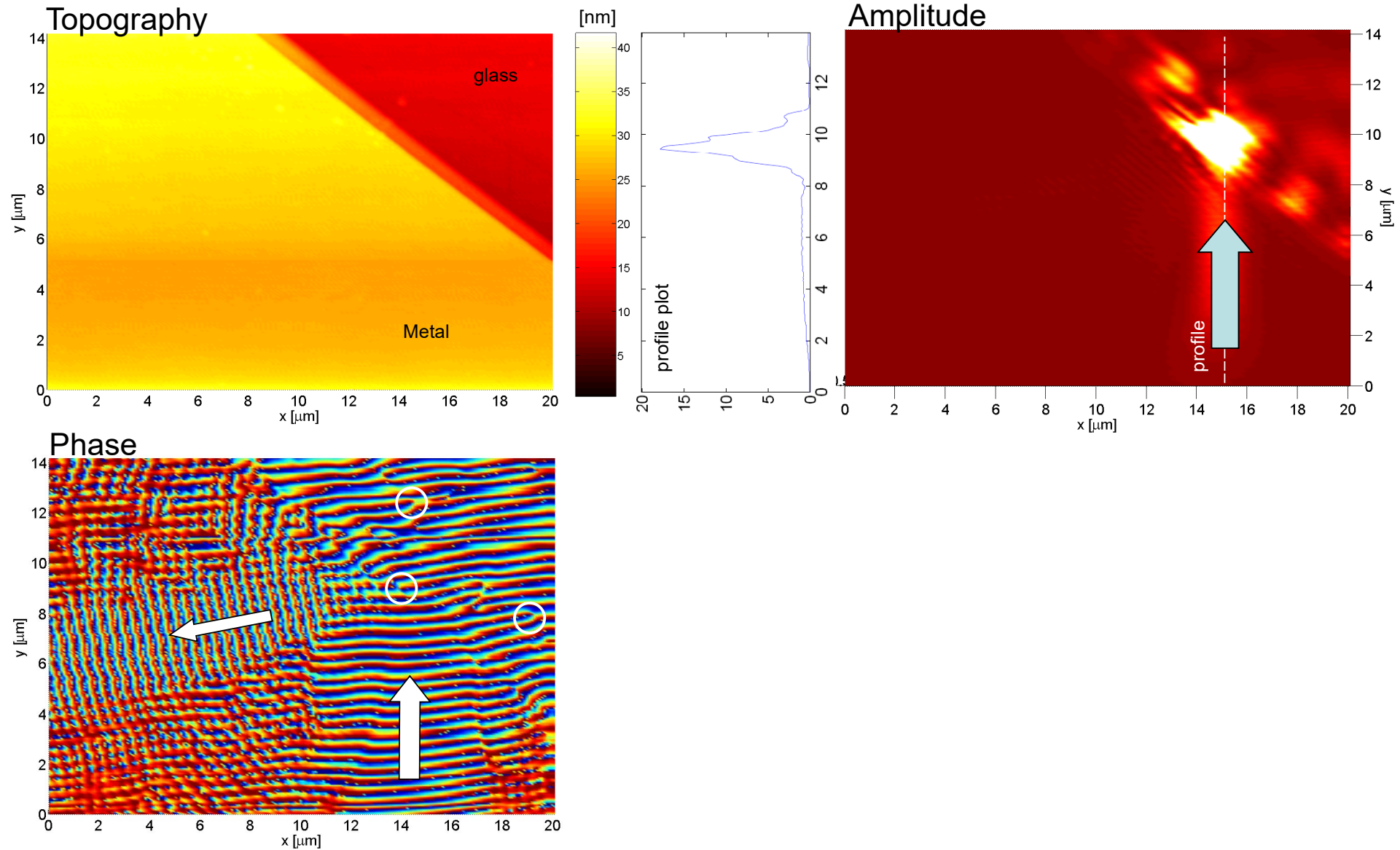
Phase



$$\lambda_{\text{SPP}} = \frac{1}{2\pi} \left( -\frac{\Delta\phi}{\delta y} \right)^{-1} = 670 \text{ nm}$$

- The illumination wavelength was  $\lambda_0 = 690 \text{ nm}$ , hence the effective wavelength is smaller, which is a characteristic of a surface wave!

# SPP reflection/refraction



## Link with localized plasmon resonance

- If one takes a metal particle and stretches it, one should slowly reach the condition for a propagating plasmon

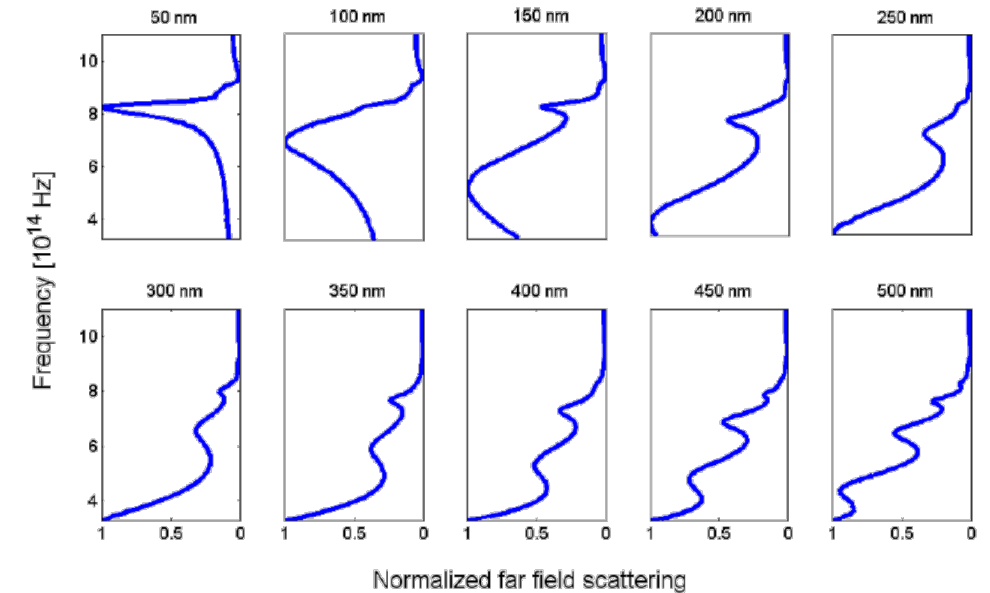


Fig. 6. Normalized scattering spectra for particles of  $L = 50\text{-}500\text{nm}$ , excited by a  $\theta = 0^\circ$  dipole source. For successively larger lengths  $L$ , higher order modes are excited.

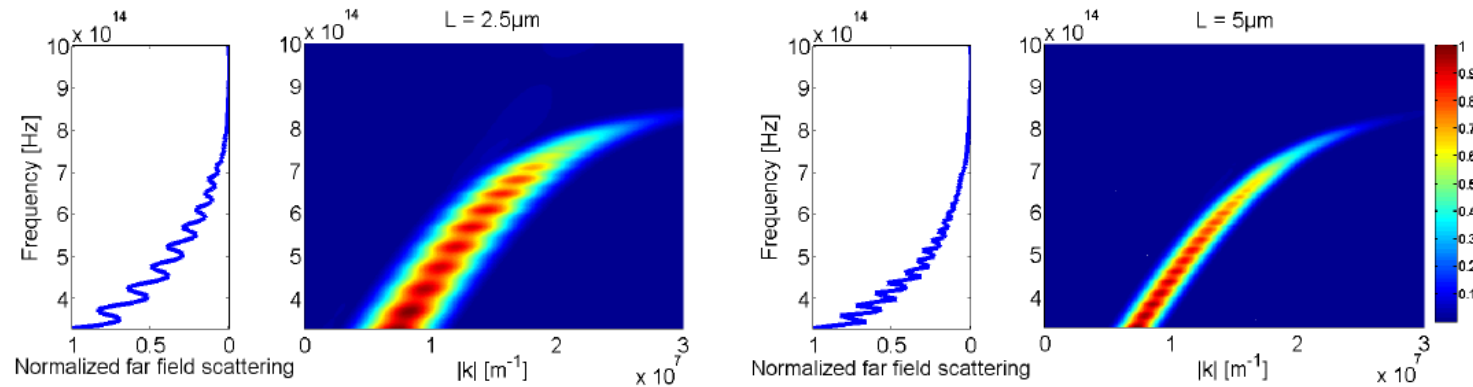


Fig. 7. Fourier analysis and scattering spectra results for  $L = 2.5$  and  $5\mu\text{m}$ . We excite the even propagating surface plasmon mode with a horizontally oriented dipole source. As can be seen, for a finite particle length, this mode is nothing more than the superposition of many higher order modes.

# SPP tunneling

- Long-range SPP can tunnel over large interruptions

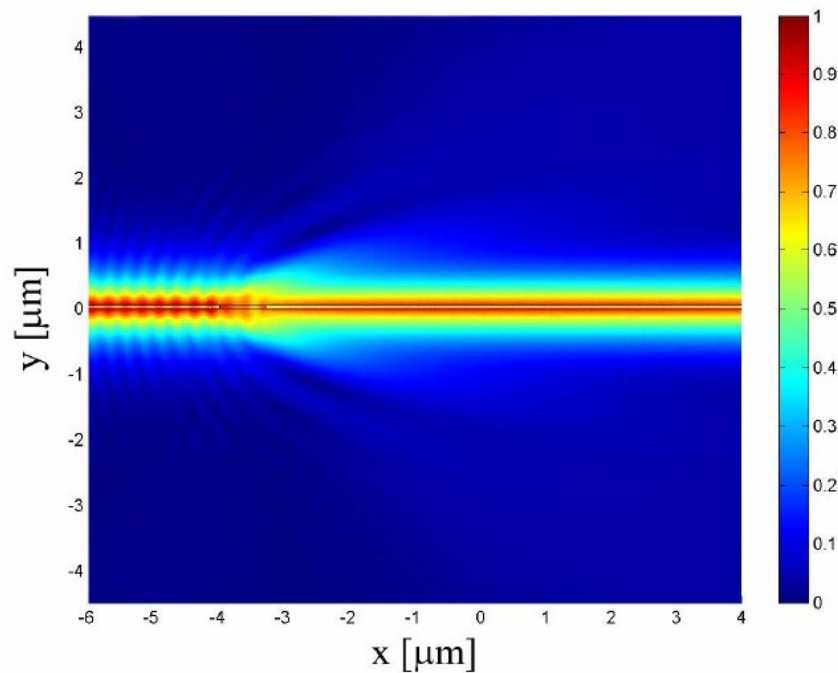
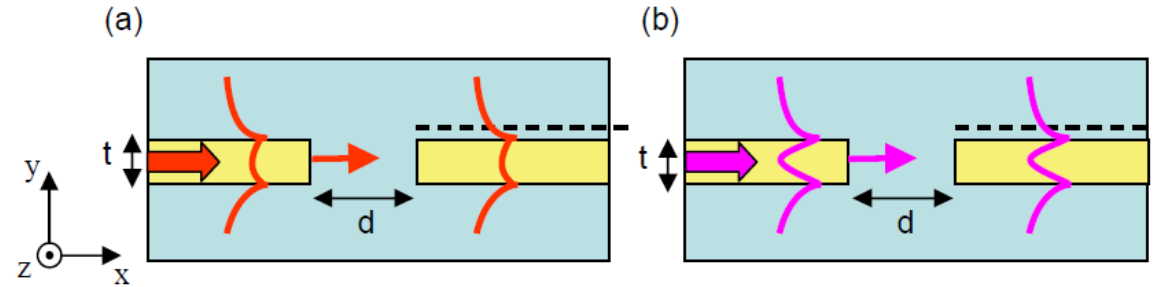


Fig. 8. Amplitude of magnetic field when a LR SPP mode is propagating on a  $t=40$  nm thick gold film with a  $d=700$  nm interruption, see Fig. 7(a). The movie shows the field distribution for interruptions between  $d=50$  nm and  $d=4.7$   $\mu\text{m}$ .

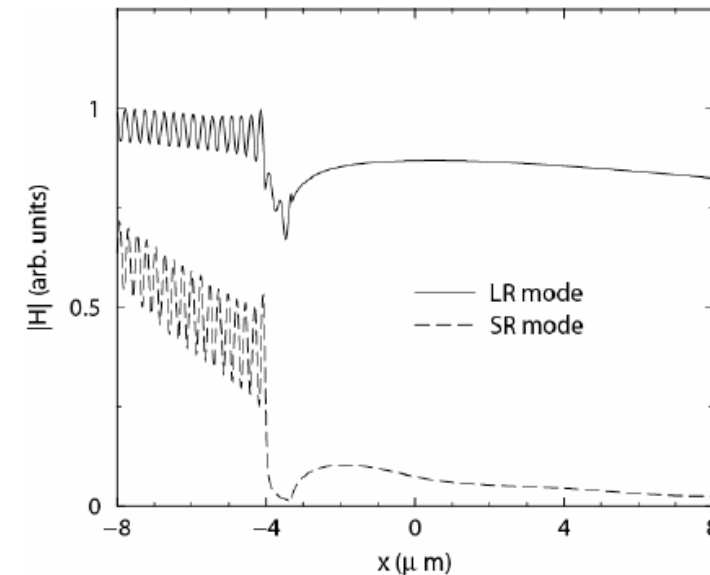


Fig. 9. Amplitude of the magnetic field 5 nm above the  $t=40$  nm thick metallic film (see the dashed lines in Fig. 7) for long-range (LR) and short-range (SR) excitations. The film has a  $d=700$  nm long interruption.

S. Sidorenko, Optics Express vol. 15, p. 6380 (2007)

# **Selected Topics in Advanced Optics**

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## **Week 6 – part 4**

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Nanophotonics and Metrology Laboratory

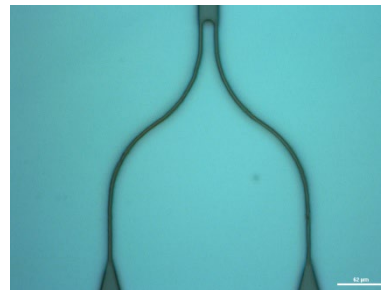


# Plasmonic waveguides

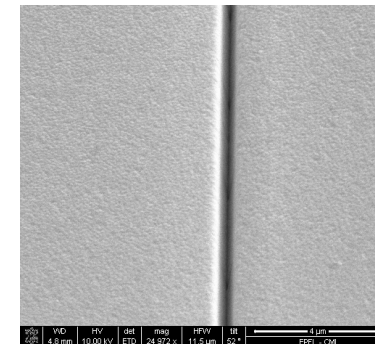
- Strip:



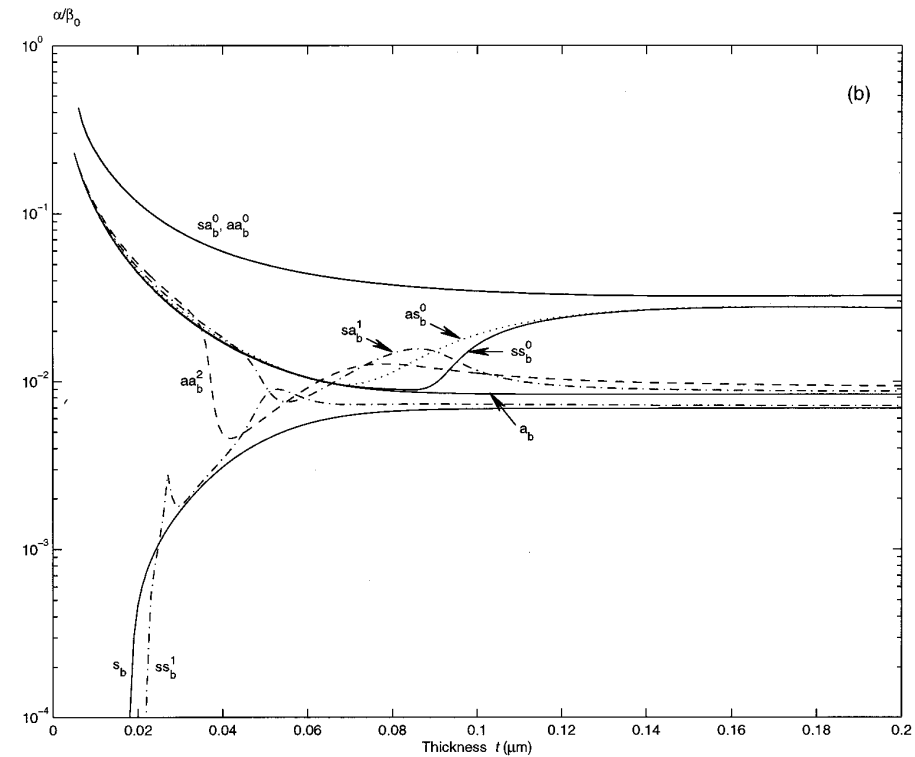
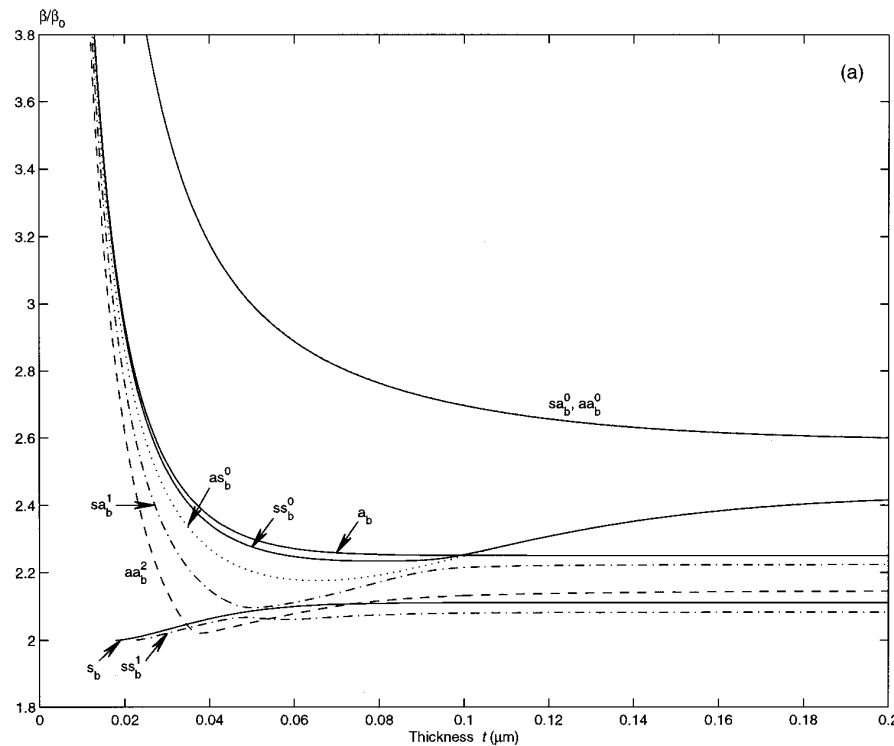
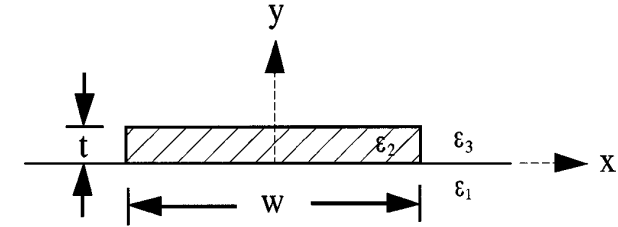
- Dielectric loaded:



- Groove or channel:



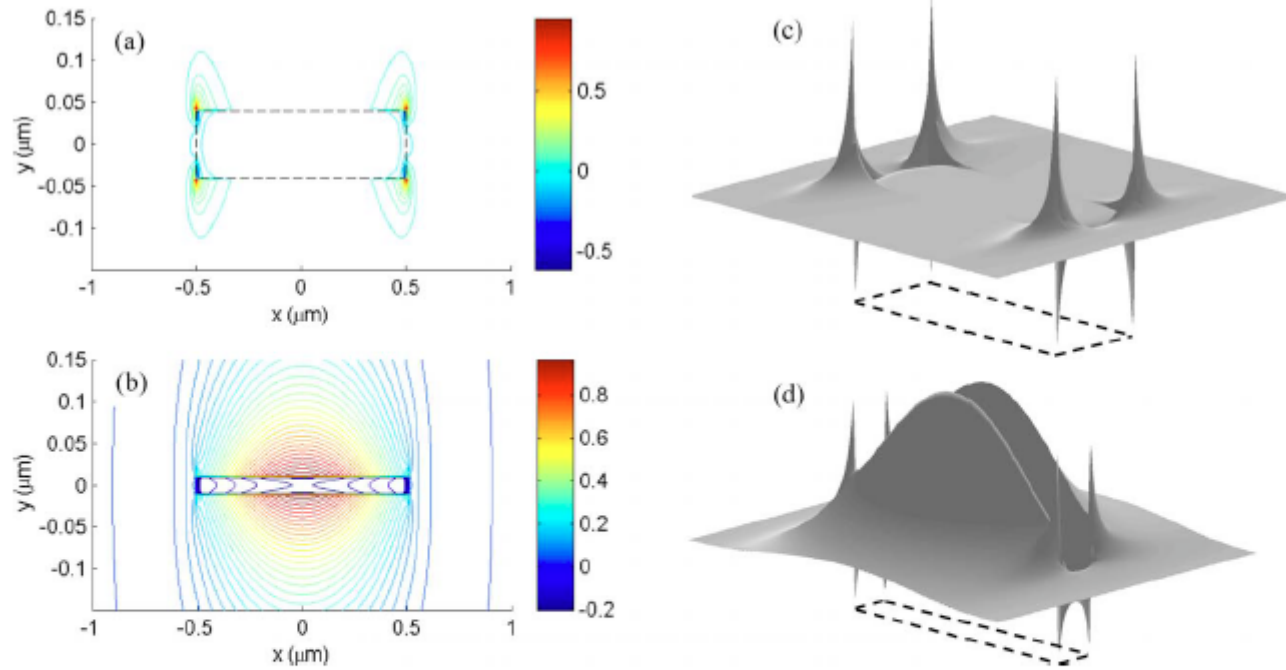
# Asymmetric plasmonic strip waveguide



P. Berini et al., "Plasmon-polariton waves guided by thin lossy metal films of finite width: Bound modes of asymmetric structures", *Phys. Rev. B*, vol. 63, p. 125417 (2001).

# Symmetric plasmonic strip waveguide

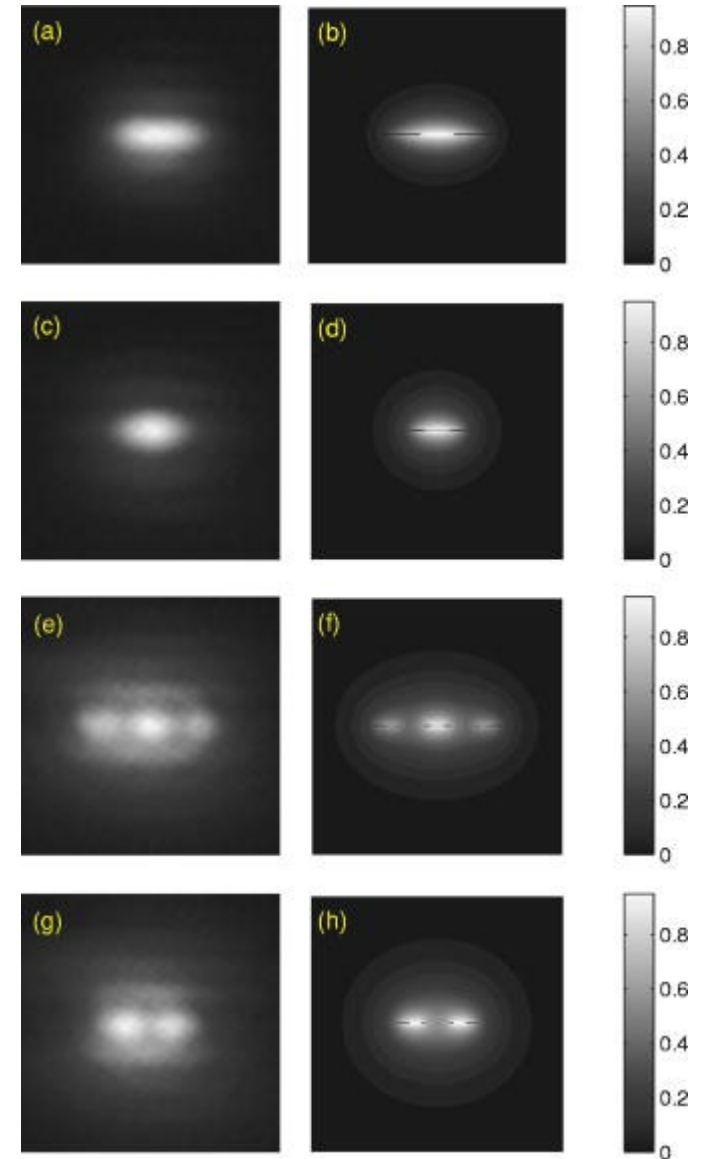
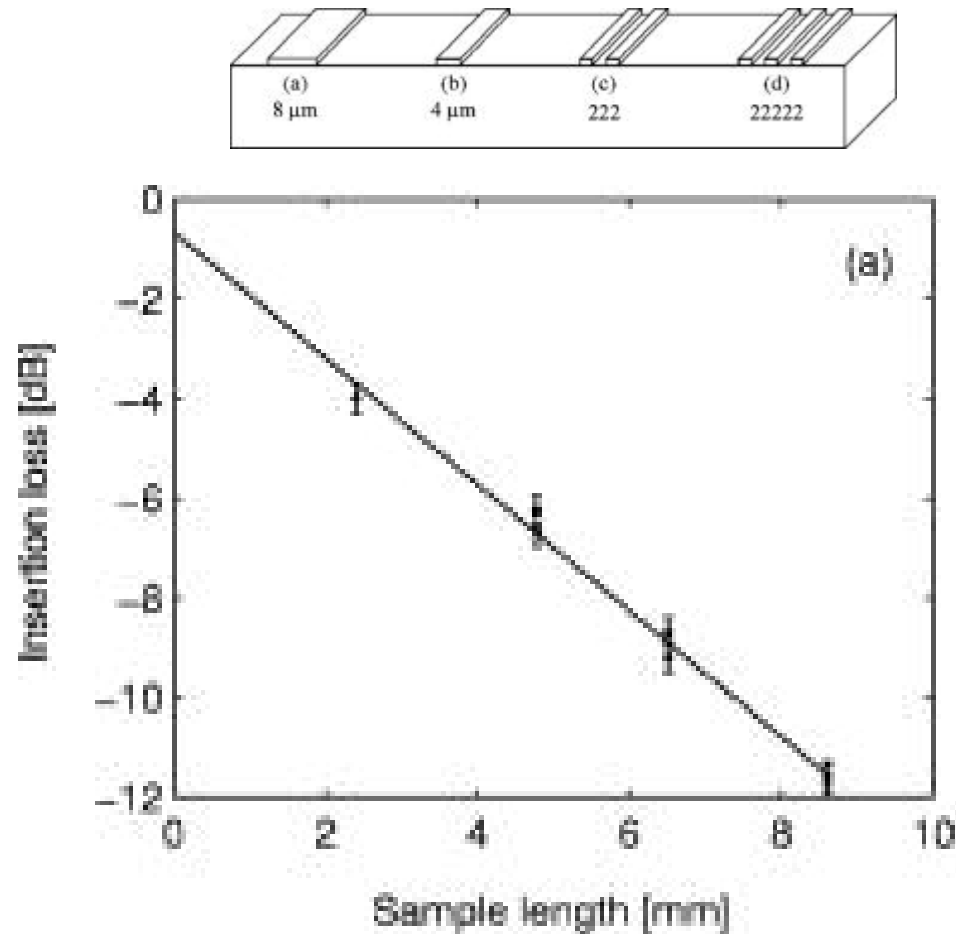
Poynting vector



Contour and 3D plots of  $\text{Re}\{S_z\}$  of the  $ss_b^0$  mode for  $w=1 \mu\text{m}$ . (a)  $t=80 \text{ nm}$ , (b)  $t=20 \text{ nm}$ , (c)  $t=100 \text{ nm}$ , (d)  $t=40 \text{ nm}$ . All other parameters are the same as in Fig. 17 ( $\lambda_0=633 \text{ nm}$ ,  $\epsilon_{r,2}=-19-j0.53$  and  $\epsilon_{r,1}=\epsilon_{r,3}=4$ ). The outline of the metal film is shown as the rectangular dashed contour. Adapted from Fig. 2 of [246], © 1999 Optical Society of America, and Fig. 7 of [248], © 2000 American Physical Society.

P. Berini et al., “Plasmon-polariton waves guided by thin lossy metal films of finite width: Bound modes of asymmetric structures”, *Phys. Rev. B*, vol. 63, p. 125417 (2001).

# Characterization of long-range strip waveguides



P. Berini et al., "Characterization of long-range surface-plasmon-polariton waveguides", *Journal of Applied Physics*, vol. 98, p. 43109 (2005).

## Dielectric loaded plasmon waveguide

- The dielectric load confines the plasmon that propagates on the metal film
- The more field in the metal, the better light confinement but the shorter propagation distance

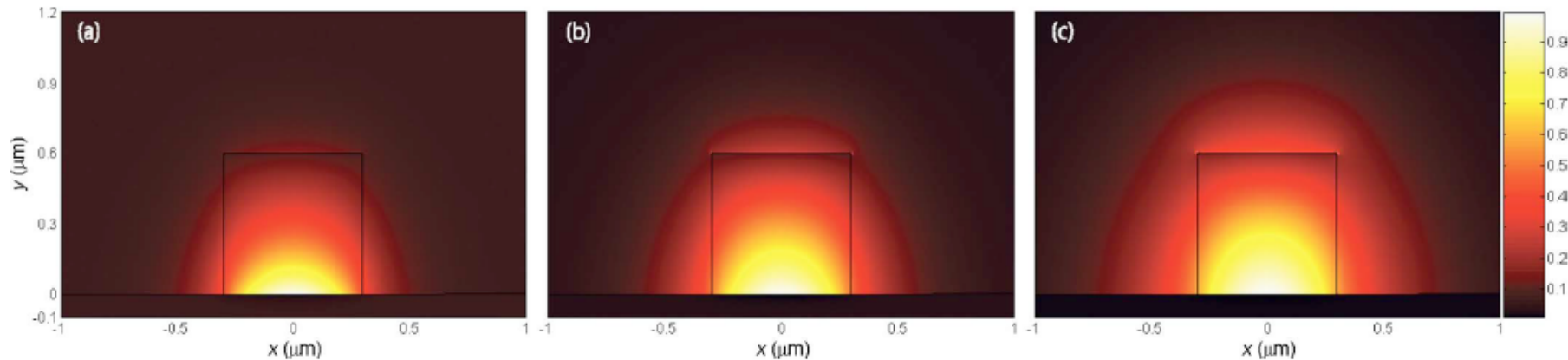


FIG. 7. (Color online) Field distribution plots of the magnitude of the vertical component of the electric field  $|E_y|$  for ridge dimension  $t=600$  nm,  $w=600$  nm, calculated by application of the FEM. In all three plots, the fundamental  $TM_{00}$  mode is depicted. (a) shows the field for  $\lambda=893$  nm ( $n_3=0.21+i5.94$ ) where  $N_{eff}=1.49$  and  $L=16.1$   $\mu\text{m}$ , (b) for  $\lambda=1.22$   $\mu\text{m}$  ( $n_3=0.36+i8.60$ ) where  $N_{eff}=1.39$  and  $L=29.8$   $\mu\text{m}$ , and (c) for  $\lambda=1.55$   $\mu\text{m}$  ( $n_3=0.55+i11.5$ ) where  $N_{eff}=1.29$  and  $L=44.4$   $\mu\text{m}$  (Ref. 28).

T. Tobias Holmgaard et al., “Theoretical analysis of dielectric-loaded surface plasmon-polariton waveguides,” *Phys. Rev. B* **75**, 245405 (2007).

# Dielectric loaded plasmon waveguide

- Dielectric strip made of PMMA

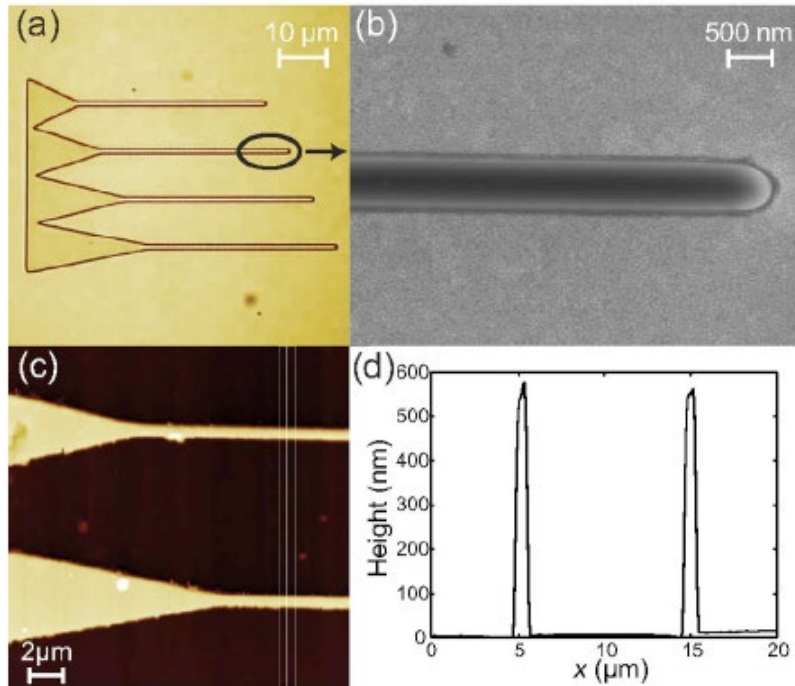


FIG. 1. (Color online) (a) Microscope image of a block with four waveguides with in coupling funnel of different dimensions. (b) SEM image of the termination region of a straight waveguide, revealing a waveguide width of  $\sim 500$  nm. (c) AFM image of the tapering region and (d) cross section of the AFM image revealing a waveguide height of  $\sim 550$  nm.

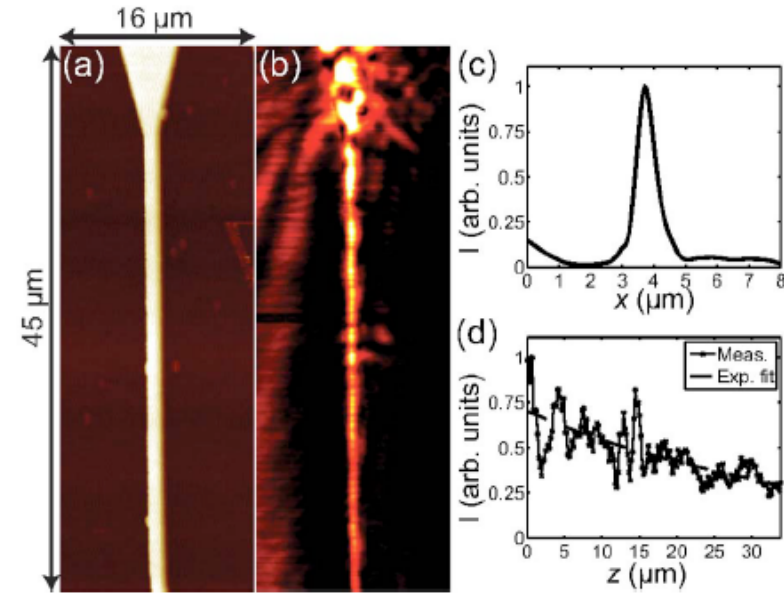
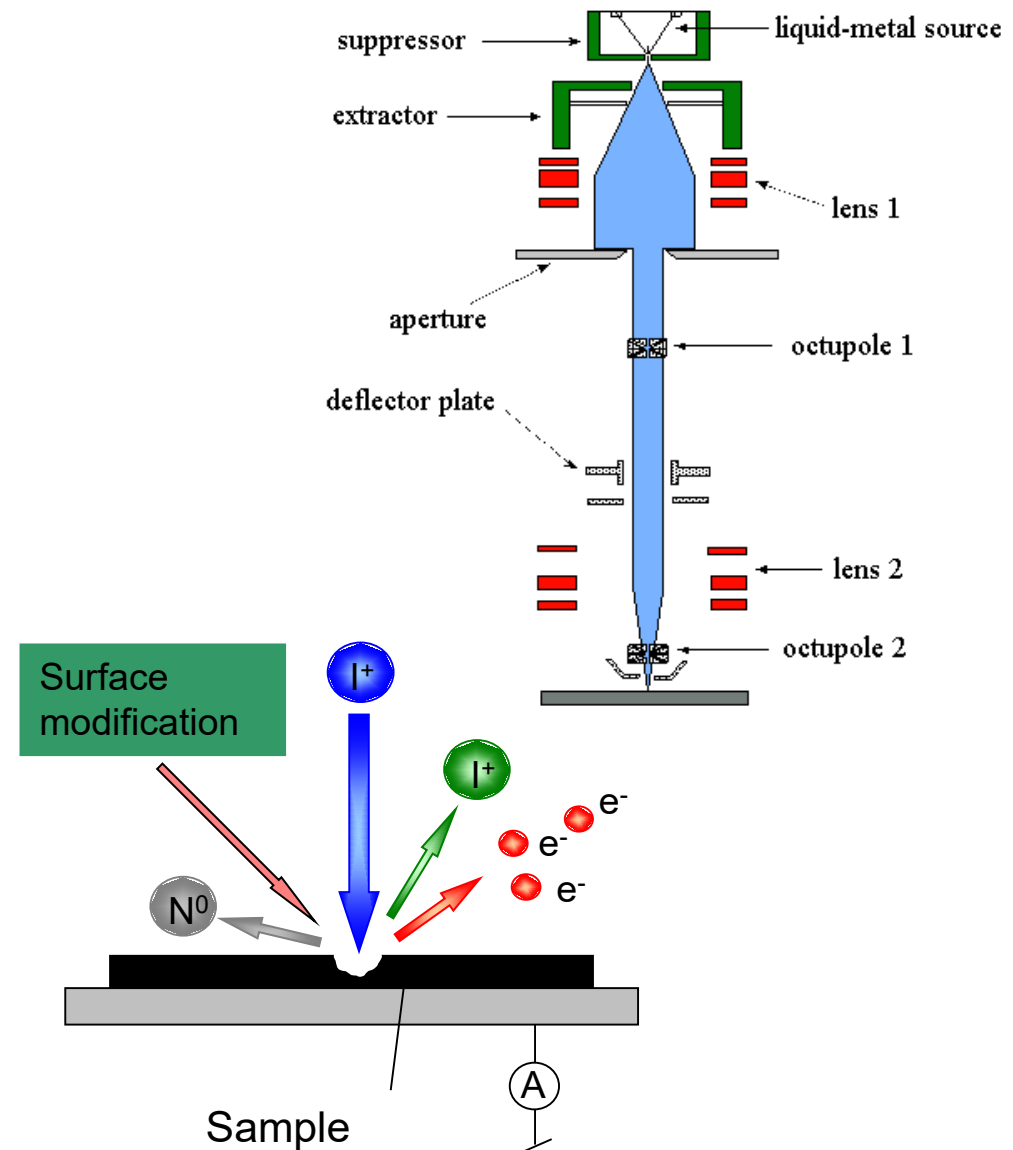


FIG. 3. (Color online) Near-field optical measurements of a straight DL-SPPW with a  $10 \times 25 \mu\text{m}$  in coupling funnel. [(a) and (b)] Topographical and near-field optical signal images, respectively, for  $\lambda = 1550$  nm. (c) Cross section of the optical image perpendicular to the waveguide and (d) averaged profile of the near-field optical signal recorded along the waveguide.

T. Tobias Holmgaard et al., “Dielectric-loaded surface plasmon-polariton waveguides at telecommunication wavelengths: Excitation and characterization,” *Appl. Phys. Lett.* **92**, 011124 (2008).

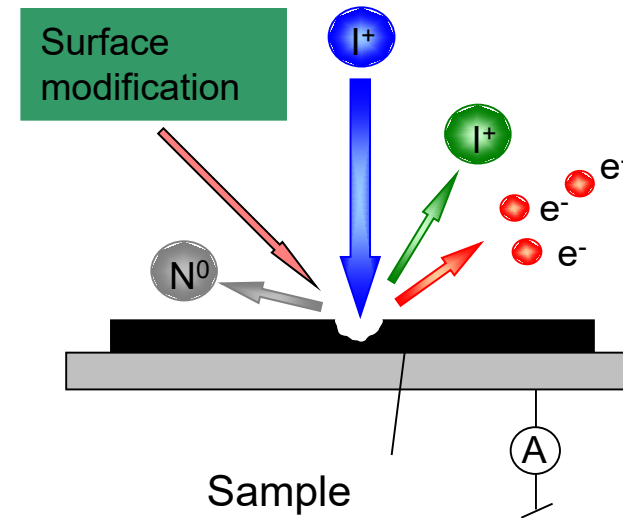
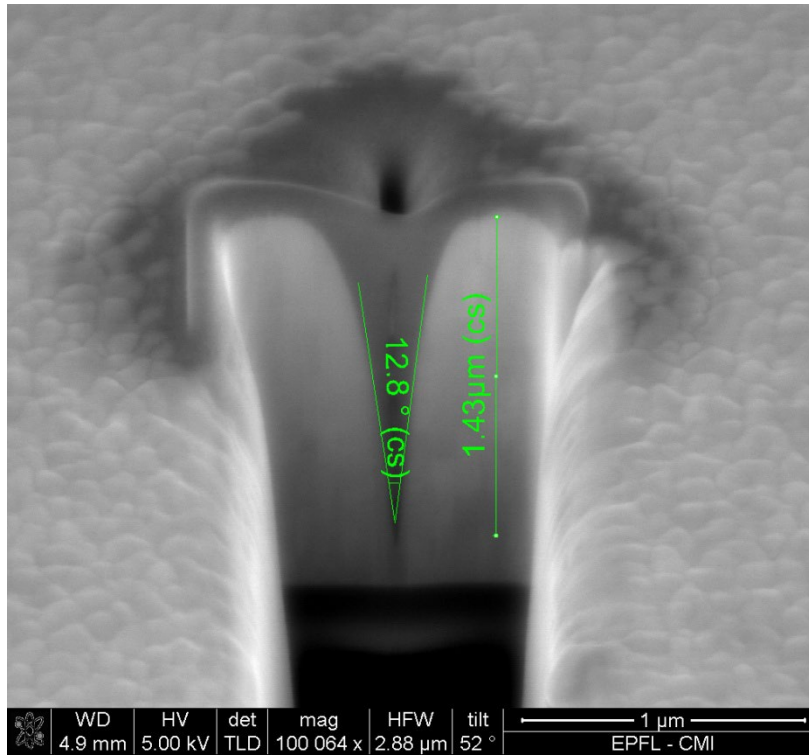
# FIB fabrication for groove (channel) waveguides

- FEI Nova 600 Nanolab dual-beam (FIB/SEM)

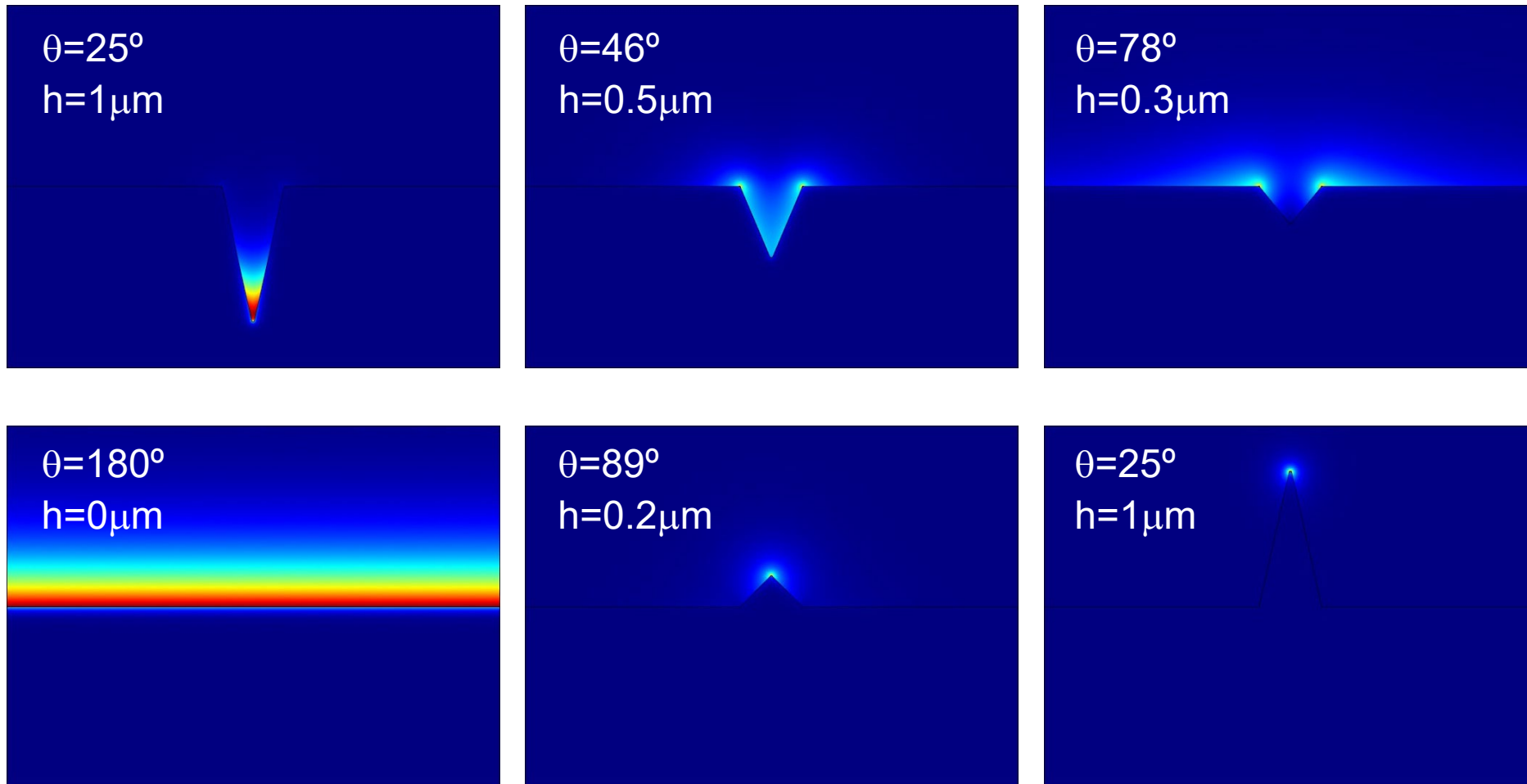


# FIB fabrication for groove (channel) waveguides

- Only relatively narrow grooves can be milled easily...

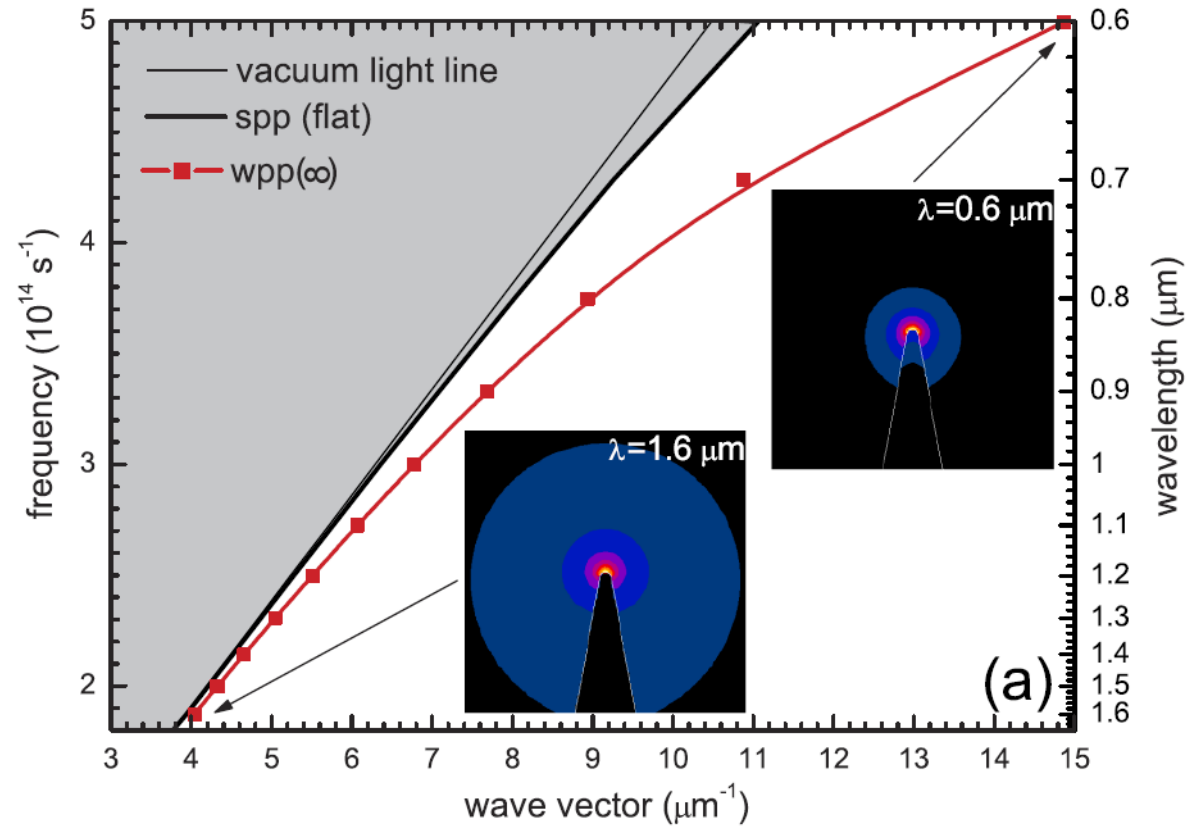
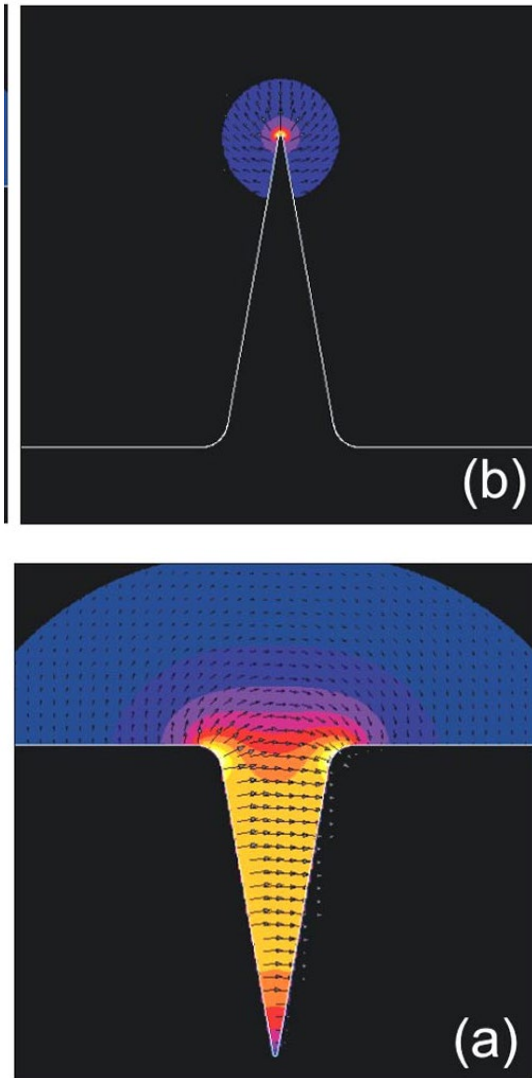


# Plasmonic modes of groove/wedge waveguides



• Au film,  $\lambda=1.55\mu\text{m}$

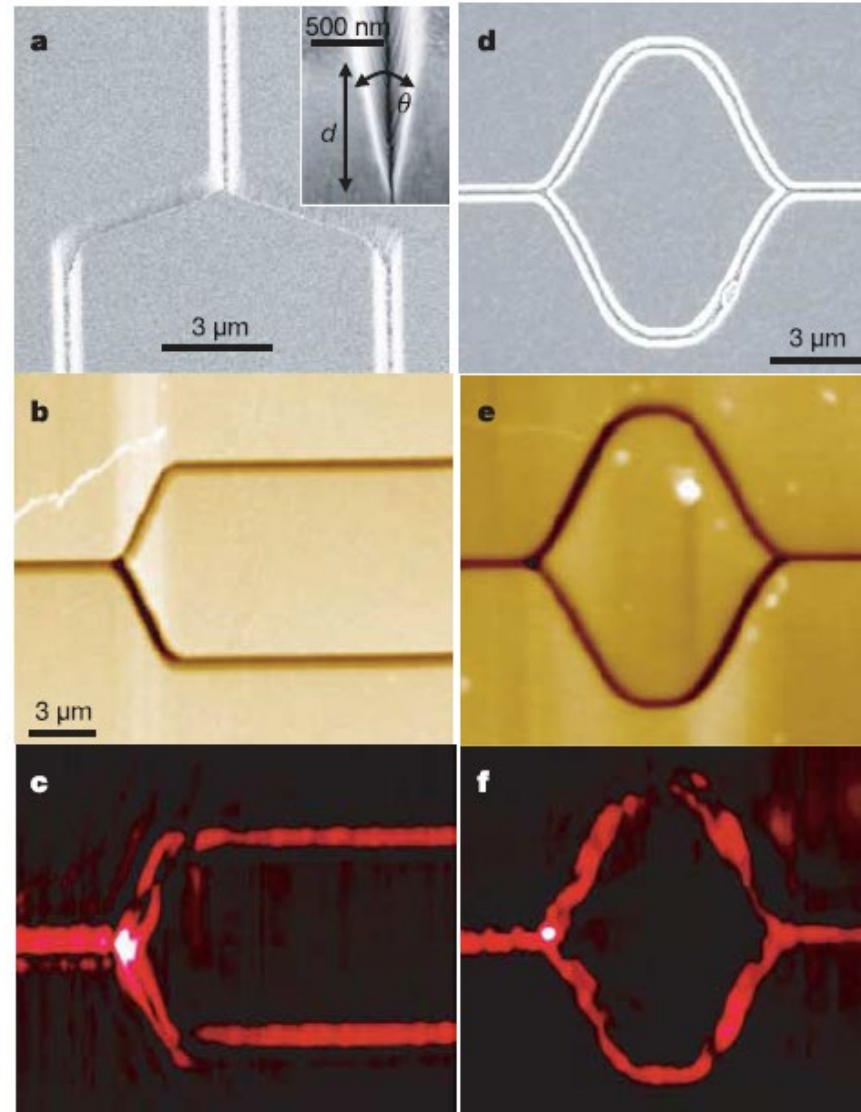
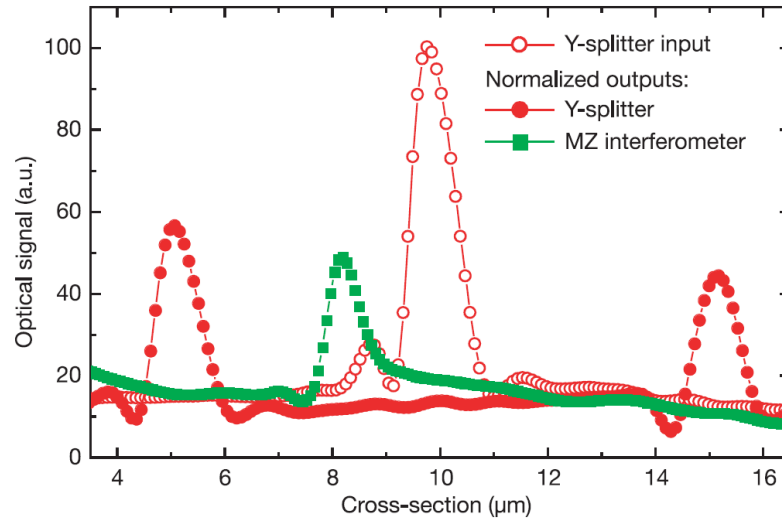
# Groove/wedge waveguides



E. Moreno et al., "Guiding and Focusing of Electromagnetic Fields with Wedge Plasmon Polaritons", *Physical Review Letters* vol. 100, p. 023901 (2008).

# Groove (channel) waveguides

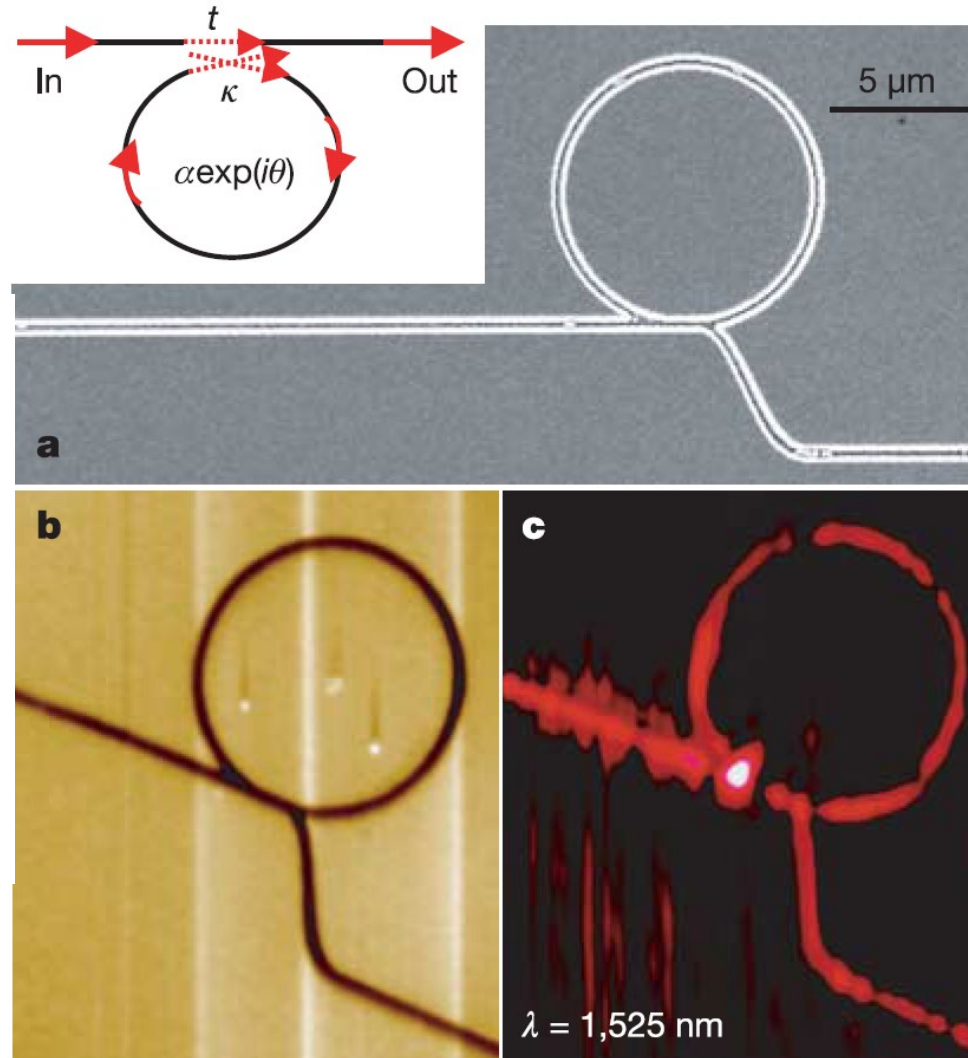
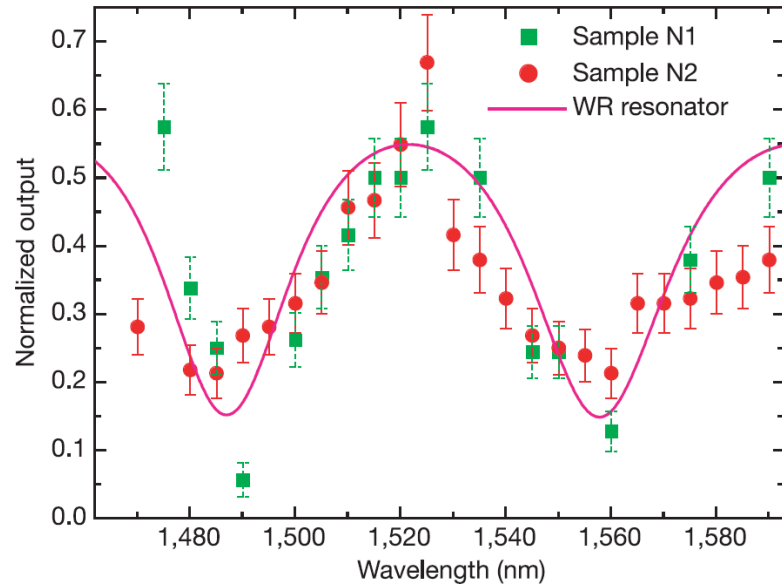
Y-splitter



Mach-Zehnder  
interferometer

S.I. Bozhevolnyi, "Channel plasmon subwavelength waveguide components including interferometers and ring resonators", Nature vol. 440, p. 508 (2006).

# Groove (channel) waveguides



Ring resonator

S.I. Bozhevolnyi, "Channel plasmon subwavelength waveguide components including interferometers and ring resonators", Nature vol. 440, p. 508 (2006).