Diffraction and Fourier optics

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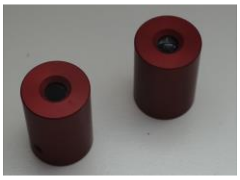
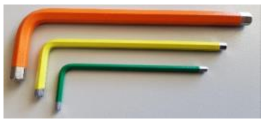
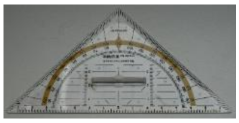
List of material for the TP

1x board	
1x linear stage 1x adapter plate 1x SM1 lens holder	  
1x camera holder 1x camera	 
1x laser	 
1x source holder	
1x lens cap with polarizer 1x slip cap	
1x diffraction grating array (glass wafer)	
1x diffraction structures	

2x polarizer sheet	
4x large screws with cap 3x small screws with cap	 
4x small screws (triangular head) 4x large screws	 
3x Allen keys	
1x slotted screwdriver	
1x black cover	
1x ruler 1x triangle	 

EPFL

1 Background

1.1. Scalar Diffraction

The text below is a modified copy of the book from **David G. Voelz, Computational Fourier optics: a MATLAB tutorial, SPIE Bellingham, 2011**.

Diffraction refers to the behavior of an optical wave when its lateral extent is confined; for example, by an aperture. It accounts for the fact that light rays do not follow strictly rectilinear paths when the wave is disturbed on its boundaries. In our everyday experience we rarely notice diffractive effects of light. The effects of reflection (from a mirror), or refraction (due to a lens) are much more obvious. In fact, the effects of diffraction become most apparent when the confinement size is on the order of the wavelength of the radiation. Nevertheless, diffraction plays a role in many optical applications and it is a critical consideration for applications involving high resolution, such as astronomical imaging, or long propagation distances such as laser radar, and in applications involving small structures such as photolithographic processes. The propagation behavior of an optical wave is fundamentally governed by Maxwell's equations. In general, coupling exists between the wave's electric field E with components (Ex, Ey, Ez) and its magnetic field H with components (Hx, Hy, Hz). There is also coupling between the individual components of the electric field, as well as between the magnetic components. However, consider a wave that is propagating in a dielectric medium that is linear (field quantities from separate sources can be summed), isotroptic (independent of the wave polarization, i.e., the directions of E and H), homogeneous (permittivity of the medium is independent of position), nondispersive (permittivity is independent of wavelength), and nonmagnetic (magnetic permeability is equal to the vacuum permeability). In this case, Maxwell's vector expressions become decoupled, and the behavior of each component of the electric or magnetic fields can be expressed independently from the other components. Scalar diffraction refers to the propagation behavior of light under this ideal situation. The long list of assumptions for the medium suggests a rather limited application regime for scalar diffraction theory. However, scalar diffraction can clearly be used for describing free-space optical (FSO) propagation, which refers to transmission through space.

Monochromatic Fields and Irradiance

Let

$$U_1(x,y) = A_1(x,y)\exp\left[j\phi_1(x,y)\right] \qquad \text{Eq. 1}$$

be the field in the x–y plane is located at some position "1" on the z axis.
Detectors do not currently exist that can follow the extremely high-frequency oscillations ($>10^{14}$ Hz) of the optical electric field. Instead, optical detectors respond to the time-averaged squared magnitude of the field. So, a quantity of considerable interest is the irradiance, which is defined here as

$$I_1(x,y) = U_1(x,y)\,U_1(x,y)^* = \left|U_1(x,y)\right|^2 \qquad \text{Eq. 2}$$

Irradiance is a radiometric term for the flux (watts) per unit area falling on the observation plane. It is a power density quantity that in other laser and Fourier optics references is often called "intensity." Expression (2) actually represents a shortcut for determining the time-averaged square magnitude of the field and is valid when the field is modelled by a complex phasor.

Optical Path Length and Field Phase Representation

The refractive index *n* of a medium is the ratio of the speed of light in vacuum to the speed in the medium. For example, a typical glass used for visible light might have an index of about 1.6. For light propagating a distance *d* in a medium of index *n*, the *optical path length* (OPL) is defined as

$$\mathrm{OPL} = nd \qquad \text{Eq. 3}$$

The OPL multiplied by the wavenumber *k* shows up in the phase of the complex exponential used to model the optical field. Think of *k* as the "converter" between the distance spanned by one wavelength and 2 π (rad of the phase). If a plane wave propagates a distance *d* through a piece of glass with index *n*, then the OPL is as indicated in Eq. (3), and the field phasor representation is

$$U(d) = A\exp(jknd) \qquad \text{Eq. 4}$$

In effect, the wavelength shortens to λ/n in the glass. There are other variations of this theme; for example, exp(*jkr*), where *r* is a radial distance in vacuum. Phasor forms associated with the optical field can also be a function of transverse position *x* and *y*; for example,

$$\exp\left[j\frac{k}{2z}\left(x^2+y^2\right)\right] \qquad \text{Eq. 5}$$

This is known as a "chirp" term and indicates a field phase change as the square of the transverse position. This type of term appears in a variety of situations to model a contracting or expanding optical field.

1.2 Diffraction and Rayleigh–Sommerfeld solution

Consider the propagation of monochromatic light from a 2D plane (source plane) indicated by the coordinate variables η and ξ (Fig. 1). At the source plane, an area: defines the extent of a source or an illuminated aperture. The field distribution in the source plane is given by $U_1(\eta,\xi)$, and the field $U_2(x, y)$ in a distant observation plane can be predicted using the first Rayleigh–Sommerfeld diffraction solution

$$U_2(x,y) = \frac{z}{j\lambda}\iint_{\Sigma} U_1(\xi,\eta)\frac{\exp(jkr_{12})}{r_{12}^2}d\xi d\eta \qquad \text{Eq. 6}$$

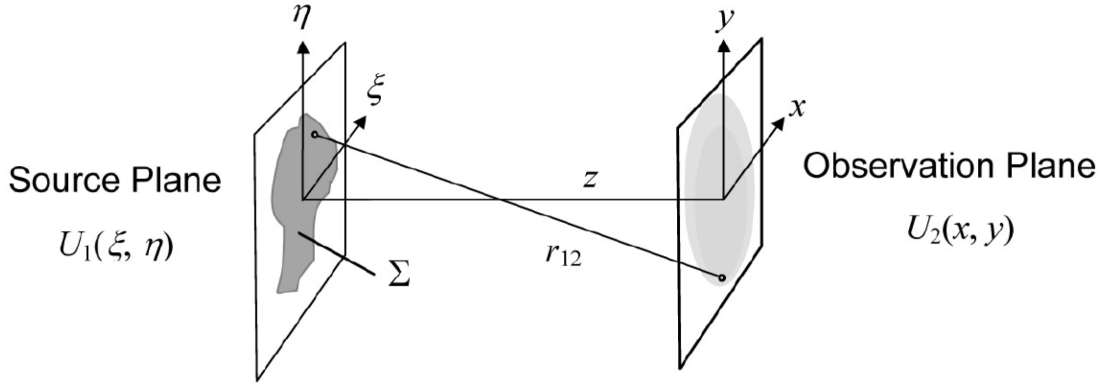


Figure 1 Geometry for a situation where light propagates from a source plane to an observation plane

Here, λ is the optical wavelength; k is the wavenumber, which is equal to $2\pi/\lambda$ for free space; z is the distance between the centres of the source and observation coordinate systems; and r_{12} is the distance between a position on the source plane and a position in the observation plane. η and ξ are variables of integration, and the integral limits correspond to the area of the source: With the source and observation positions defined on parallel planes, the distance r_{12} is

$$r_{12} = \sqrt{z^2 + (x-\xi)^2 + (y-\eta)^2} \qquad \text{Eq. 7}$$

Expression (6) is a statement of the Huygens–Fresnel principle. This principle supposes the source acts as an infinite collection of fictitious point sources, each producing a spherical wave associated with the actual source field at any position (ξ,η). The contributions of these spherical waves are summed at the observation position (x, y), allowing for interference. The extension of Eqs. (6) and (7) to nonplanar geometries is straightforward; for example, involving a more complicated function for r, but the planar geometry is more commonly encountered, and this is our focus here. Expression (6) is, in general, a superposition integral, but with the source and observation areas defined on parallel planes, it becomes a convolution integral, which can be written as

$$U_2(x,y) = \iint U_1(\xi,\eta)\, h(x-\xi, y-\eta)\, d\xi d\eta \qquad \text{Eq. 8}$$

where the general form of the Rayleigh–Sommerfeld impulse response is

$$h(x,y) = \frac{z}{j\lambda}\frac{\exp(jkr)}{r^2} \qquad \text{Eq. 9}$$

and

$$r = \sqrt{z^2 + x^2 + y^2} \qquad \text{Eq. 10}$$

The Fourier convolution theorem can be applied to rewrite the integral equation Eq. (8) as

$$U_2(x,y) = \Im^{-1}\left\{\Im\{U_1(x,y)\}\,\Im\{h(x,y)\}\right\} \qquad \text{Eq. 11}$$

For this convolution interpretation the source and observation plane variables are simply relabelled as x and y. An equivalent expression for Eq. (11) is

$$U_2(x,y) = \Im^{-1}\left\{\Im\{U_1(x,y)\}\,H(f_x,f_y)\right\} \qquad \text{Eq. 12}$$

where H is the Rayleigh–Sommerfeld transfer function given by

$$H(f_x,f_y) = \exp\left(jkz\sqrt{1-(\lambda f_x)^2-(\lambda f_y)^2}\right) \qquad \text{Eq. 13}$$

Strictly speaking, $\sqrt{f_x^2+f_y^2} < 1/\lambda$ must be satisfied for propagating field components. An angular spectrum analysis is often used to derive Eq. (13). **This solution only requires that *r* >> λ, the distance between the source and the observation position, be much greater than the wavelength.**

Fresnel approximation

The square root in the distance terms of Eq. (7) or (10) can make analytic manipulations of the Rayleigh–Sommerfeld solution difficult and add execution time to a computational simulation. By introducing approximations for these terms, a more convenient scalar diffraction form is developed. Consider the binomial expansion

$$\sqrt{1+b} = 1+\frac{1}{2}b-\frac{1}{8}b^2+\ldots \qquad \text{Eq. 14}$$

where *b* is a number less than 1, then expand Eq. (7) and keep the first two terms to yield

$$r_{12} \approx z\left[1+\frac{1}{2}\left(\frac{x-\xi}{z}\right)^2+\frac{1}{2}\left(\frac{y-\eta}{z}\right)^2\right] \qquad \text{Eq. 15}$$

This approximation is applied to the distance term in the phase of the exponential in Eq. (8), which amounts to assuming a parabolic radiation wave rather than a spherical wave for the fictitious point sources. Furthermore, use the approximation $r_{12} = z$ in the denominator of Eq. (6) to arrive at the *Fresnel diffraction* expression:

$$U_2(x,y) = \frac{e^{jkz}}{j\lambda z}\iint U_1(\xi,\eta)\exp\left\{j\frac{k}{2z}\left[(x-\xi)^2+(y-\eta)^2\right]\right\}d\xi d\eta \qquad \text{Eq. 16}$$

This expression is also a convolution of the form in Eq. (8), where the impulse response is

$$h(x,y) = \frac{e^{jkz}}{j\lambda z}\exp\left[\frac{jk}{2z}\left(x^2+y^2\right)\right] \qquad \text{Eq. 17}$$

and the transfer function is

$$H\left(f_x,f_y\right) = e^{jkz}\exp\left[j\pi\lambda z\left(f_x^2+f_y^2\right)\right] \qquad \text{Eq. 18}$$

The expressions in Eqs. (11) and (12) are again applicable in this case for computing diffraction results.

Another useful form of the Fresnel diffraction expression is obtained by moving the quadratic phase term that is a function of *x* and *y* outside the integrals:

$$U_2(x,y) = \frac{\exp(jkz)}{j\lambda z}\exp\left[j\frac{k}{2z}\left(x^2+y^2\right)\right]\times$$
$$\iint\left\{U_1(\xi,\eta)\exp\left[j\frac{k}{2z}\left(\xi^2+\eta^2\right)\right]\right\}\exp\left[-j\frac{2\pi}{\lambda z}(x\xi+y\eta)\right]d\xi d\eta \qquad \text{Eq. 19}$$

Along with the amplitude and chirp multiplicative factors out front, this expression is a Fourier transform of the source field times a chirp function where the following frequency variable substitutions are used for the transform:

$$f_\xi \to \frac{x}{\lambda z} \qquad f_\eta \to \frac{y}{\lambda z} \qquad \text{Eq. 20}$$

The accuracy of the Fresnel expression when modelling scalar diffraction at close ranges suffers as a consequence of the approximations involved. A criterion which is commonly used for determining when the Fresnel expression can be applied is the Fresnel number. The Fresnel number is given by

$$N_F = \frac{w^2}{4\lambda z}$$

Eq. 21

where ***w* is the width of a square aperture in the source plane**, or the diameter of a circular aperture, and *z* is the distance to the observation plane. If **N_F is larger than 1** for a given scenario, then it is commonly accepted that the observation plane is in the near field region, where the Fresnel approximations, typically, lead to useful results. However, for relatively “smooth” fields over the source aperture, the Fresnel expression can be applicable up to Fresnel numbers of even 20 or 30. In a geometrical optics context, the Fresnel expression describes diffraction under the paraxial assumption, where only rays that make a small angle (< ~0.1 rad) relative to the optical axis are considered.

Fraunhofer approximation

Fraunhofer diffraction, which refers to diffraction patterns in a regime that is commonly known as the “far field,” is arrived at mathematically by approximating the chirp term multiplying the initial field within the integrals of Eq. 19 as unity. The assumption involved is

$$z >> \left(\frac{k\left(\xi^2+\eta^2\right)}{2} \right)_{max} \qquad \text{Eq. 22}$$

and results in the Fraunhofer diffraction expression:

$$U_2(x,y) = \frac{\exp(jkz)}{j\lambda z} \exp\left[j\frac{k}{2z}\left(x^2+y^2\right)\right] \iint U_1(\xi,\eta) \exp\left[-j\frac{2\pi}{\lambda z}(x\xi + y\eta)\right] d\xi d\eta \qquad \text{Eq. 23}$$

The condition of Eq. (22), typically, requires very long propagation distances relative to the source support size. However, a form of the Fraunhofer pattern also appears in the propagation analysis involving lenses. The Fraunhofer diffraction expression is a powerful tool and finds use in many applications such as laser beam propagation, image analysis, and spectroscopy.
Along with multiplicative factors out front, the Fraunhofer expression can be recognized simply as a Fourier transform of the source field with the variable substitutions

$$f_\xi \to \frac{x}{\lambda z} \qquad\qquad f_\eta \to \frac{y}{\lambda z} \qquad \text{Eq. 24}$$

The Fraunhofer expression cannot be written as a convolution integral, so there is no impulse response or transfer function. But, since it is a scaled version of the Fourier transform of the initial field, it can be relatively easy to calculate, and as with the Fresnel expression, the Fraunhofer approximation is often used with success in situations where Eq. (22) is satisfied. For simple source structures such as a plane-wave illuminated aperture, the Fraunhofer result can be useful even when Eq. (22) is violated by more than a factor of 10, particularly if the main quantity of interest is the irradiance pattern at the receiving plane. Using the Fresnel number N_F, the commonly accepted requirement for the Fraunhofer region is $N_F << 1$.
Looking at the equations above one can see that in this region (Fraunhofer region), **$U_2(x,y)$ is just the two-dimensional Fourier transform of $U_1(x,y)$ except for a multiplicative phase factor which does not affect the intensity of the light.** This regime is also called Fraunhofer Diffraction or Fraunhofer Approximation. The examples presented here are calculated numerically assuming Fraunhofer approximation, i. e. simple two-dimensional Fourier transform.

3.2 Gratings and Periodic Functions

The following explanations are a shortened version of the document: Fourier Optics in Examples by Klaus Betzler, Fachbereich Physik, Universität Osnabrück.

When the two-dimensional pattern is only structured in one dimension, that also shows up in the Fourier transform, yet in a reciprocal meaning. This is visualized by Figs. 2 and 3.

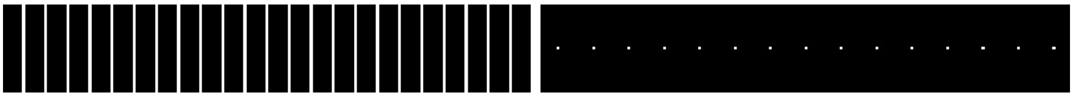


Figure 2: Array of lines (left) and the corresponding two-dimensional Fourier transform (right).

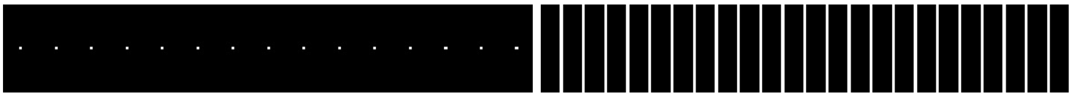


Figure 3: Array of points (left) and the corresponding two-dimensional Fourier transform (right).

In Fig. 2, the pattern is constant in the vertical dimension, its Fourier transform shows a delta function behavior in this dimension, yielding a linear array of points. Vice versa for Fig. 3. That's due to the fact that the Fourier transform of a constant is the delta function and vice versa.
The number of elements in the original pattern strongly determines the sharpness of the diffraction pattern. Fig. 4 demonstrates this using a one-dimensional regular structure of points as source pattern. Depending on the number of points used, the diffraction pattern varies in sharpness.

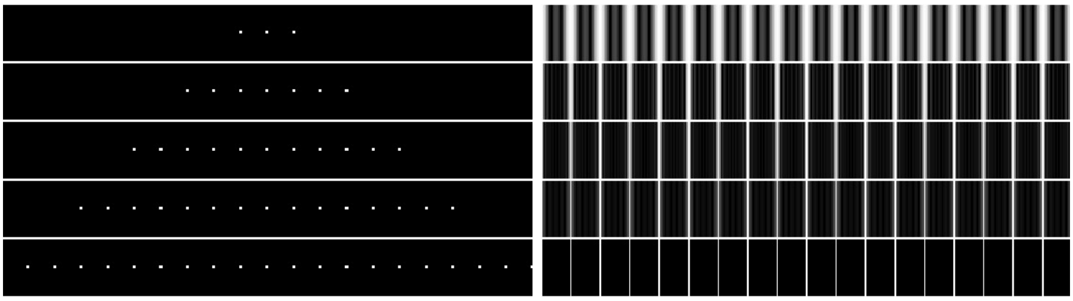


Figure 4: Dependence of the diffraction pattern on the number of source objects used. The sharpness increases with this number (from top to bottom: 3, 7, 11, 15, 19).

A special problem in Fourier transform is the fact that one always has to deal with limited data, albeit theory assumes unlimited data to be transformed. Using limited data means to make a transformation of the product of the unlimited data with a rectangular function. The Fourier transform in that case is the convolution of the two transforms. As the transform of a rectangular function shows side wings (sinus cardinal), these also show up in the transform of the product, mainly convoluted to each of the peaks of the transform. This spurious additional intensity may affect the pattern of the Fourier transform producing fictitious information. The effect is shown in Fig. 5, the peaks are smeared out, surrounded by undesired side wings.

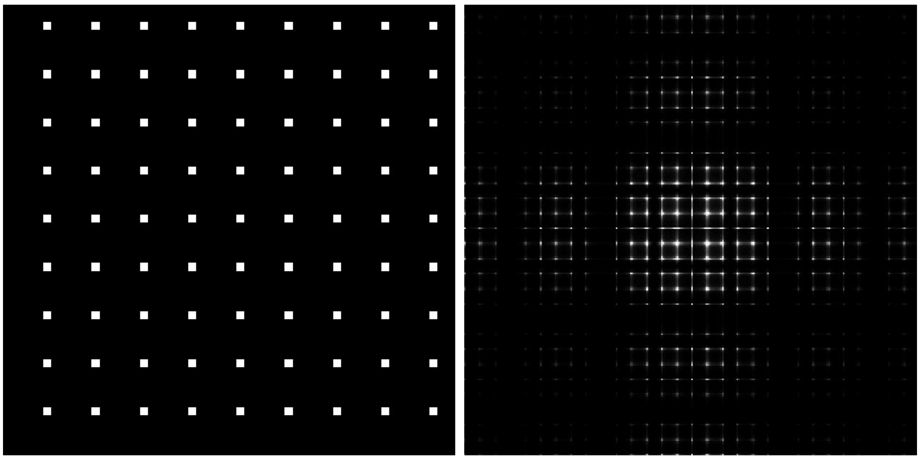


Figure 5: Sharply limited pattern (left) and its Fourier transform (right). Side wings appear at the peaks.

The effect can by reduced by smoothing the sharp edges of the original pattern. This treatment is called apodization or windowing. The pattern is multiplied by an appropriate apodization or windowing function. In Fig. 6 this is done using $\sin^2$ functions

$$\text{pattern} = \text{pattern}\,\sin^2\left(\frac{x\pi}{L_x}\right)\sin^2\left(\frac{y\pi}{L_y}\right) \qquad \text{Eq. 25}$$

L_x and L_y respectively, are the sizes of the original pattern.

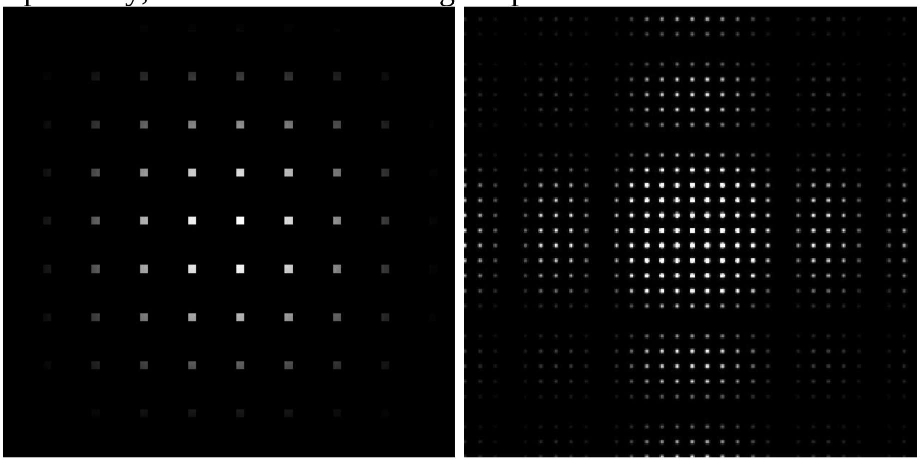


Figure 6: The pattern of Fig. 5 multiplied by an apodization function. The side wings in the Fourier transform disappear.

The resulting Fourier transform is now free from any side wings, yet the peaks are broadened slightly. This is due to the suppression of the outer parts of the pattern, which corresponds to an overall size and thus information reduction.
According to the Sampling Theorem the sampling frequency in discrete Fourier transform (DFT) must be at least twice the highest frequency to be detected. If this requirement cannot be met, Aliasing occurs, i. e., frequencies above this limit are aliased by corresponding lower frequencies.

The extent of the diffraction pattern is complementary to the size of the single diffracting elements. Fig. 7 shows this reciprocal behavior using a single circular shape as example.

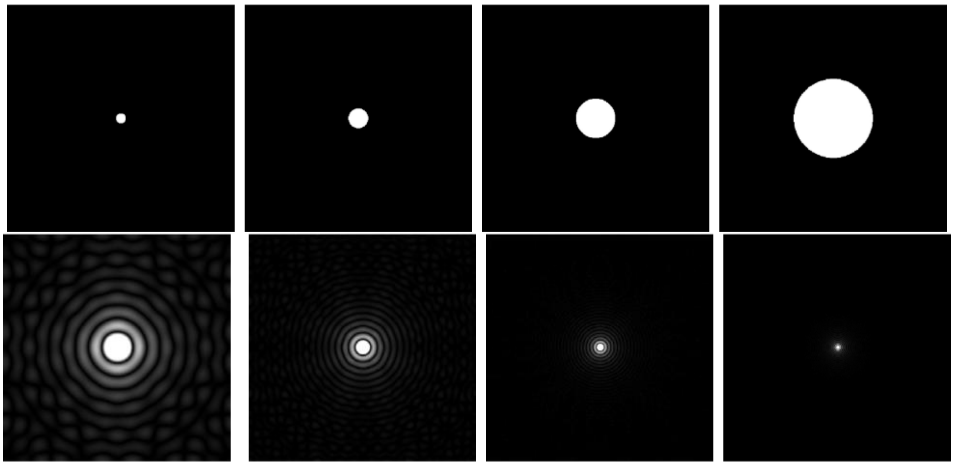


Figure 7: Circular apertures of different size (upper row) and their corresponding Fourier transforms (lower row). The intensities are normalized to their respective maximum value.

Similar results are produced by a two-dimensional regular array of objects.
A similar reciprocity as for the element size and the extent of the diffraction pattern of course must be valid for the periodicity of the original and transformed functions. This is one of the essentials of the Fourier transform.
In Fourier theory, convolution and product are complementary mathematical operations. The Fourier transform of a product of two functions equates the convolution of the Fourier transforms of the two functions. Vice versa, the Fourier transform of a convolution of two functions equates the product of the two Fourier transforms of the single functions. **A regular array of identical elements can be treated as a convolution of an array of corresponding points and a single element**. The Fourier transform then must equate the product of the two elementary transforms. For diffraction optics, this means that the diffraction pattern of a regular array can be calculated as the product of the diffraction pattern of a single element and the interference pattern of the point array. Figs. 8 – 10 visualize this property of the Fourier transform.

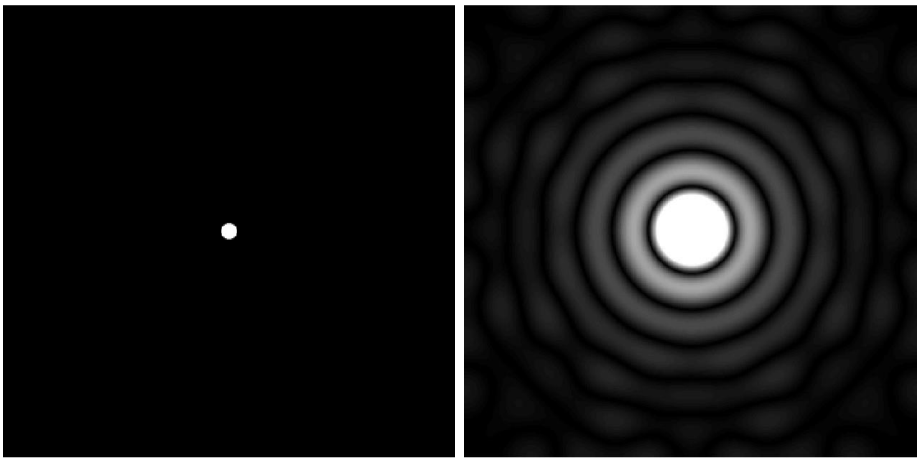


Figure 8: Single circular aperture and its Fourier transform.

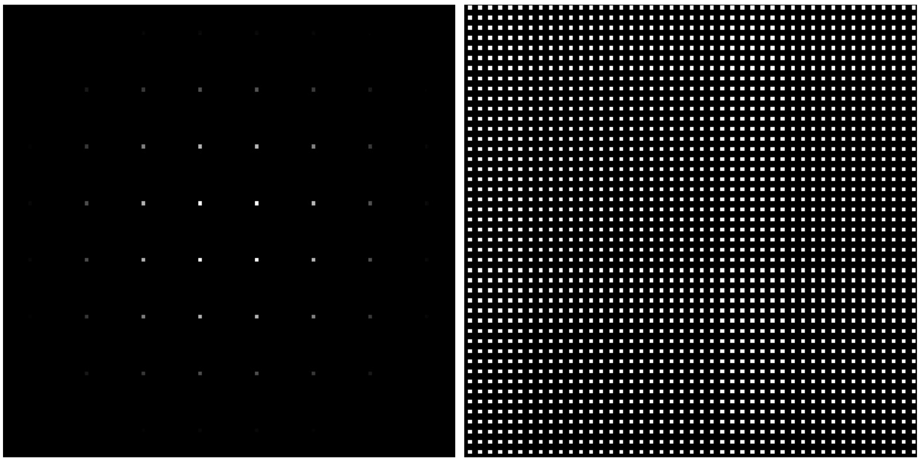


Figure 9: Regular array of points and corresponding Fourier transform

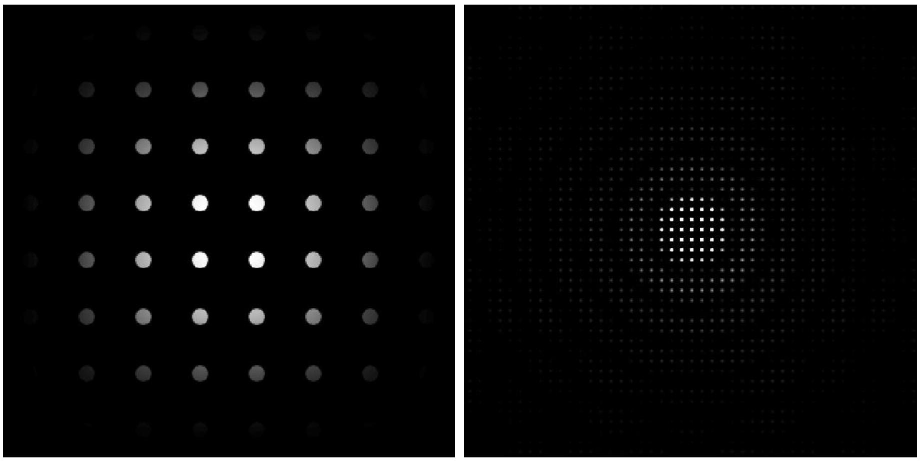


Figure 10: Regular array of circular apertures (*convolution* of single aperture and point array) and its corresponding Fourier transform (*product* of the respective Fourier transforms).

Note in the simulations in Fig. 8.-10. apodization was applied.
A diffraction grating is a (one-dimensional) array of identical slits or mirror elements. The diffraction pattern can be calculated by one or two-dimensional Fourier transform in a similar way as discussed above if we can assume Fraunhofer Approximation. Various typical features are shown in the following figures (appropriate aliasing is used to get sharp diffraction patterns)

Figure 11: Ideal grating (narrow slits) and corresponding diffraction pattern.

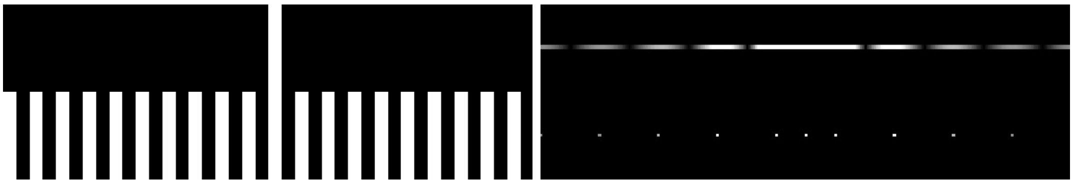


Figure 12: Slit width equates one half of the period: The minima of the slit diffraction function (top right) correspond to the maxima N=±2, ±4,...of the grating diffraction resulting in missing maxima (bottom right).

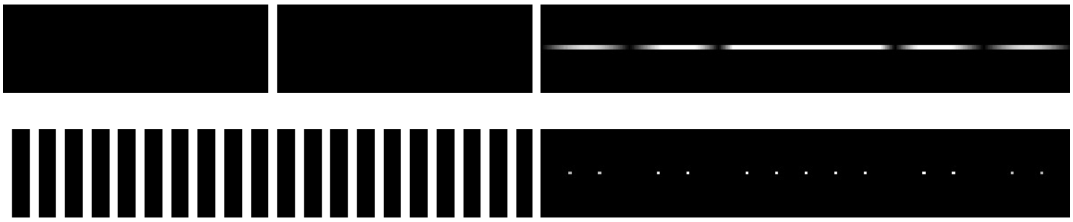


Figure 13: Slit width equates one third of the period: The minima of the slit diffraction function (top right) correspond to the maxima N=±3, ±6,...(result bottom right).

Fraunhofer Diffraction Example

It is extremely difficult (impossible?) to find closed-form diffraction solutions using the Rayleigh–Sommerfeld expression for most apertures. The Fresnel expression is more tractable, but solutions are still complicated even for simple cases such as a rectangular aperture illuminated by a plane wave. Analytic Fraunhofer diffraction analysis is easier and, for our purposes, serves as a check on some of the computer results. Consider a circular aperture illuminated by a unit amplitude plane wave. The complex field immediately beyond the aperture plane is

$$U_1(\xi,\eta) = circ\left(2\frac{\sqrt{\xi^2+\eta^2}}{w}\right) \qquad \text{Eq. 26}$$

To find the Fraunhofer diffraction field, the Fourier transform is taken as

$$\Im\{U_1(\xi,\eta)\} = w^2\frac{J_1\left(\pi w\sqrt{f_\xi^2+f_\eta^2}\right)}{2w\sqrt{f_\xi^2+f_\eta^2}} \qquad \text{Eq. 27}$$

Then, with the substitutions in Eq. (24), and applying the leading amplitude and phase terms of Eq. (23), the field is found with

$$I_2(x,y) = \left(\frac{w^2}{4\lambda z}\right)^2\left[\frac{J_1\left(\pi\frac{w}{\lambda z}\sqrt{x^2+y^2}\right)}{\frac{w}{2\lambda z}\sqrt{x^2+y^2}}\right]^2 \qquad \text{Eq. 28}$$

2 Setup and equipment

2.1 Analytical calculation of Fraunhofer diffraction

Let's use MATLAB to display an irradiance pattern. We want to calculate

$$I(x,y) = \left(\frac{w^2}{4\lambda z}\right)^2 \left[\frac{J_1\left(\pi\frac{w}{\lambda z}\sqrt{x^2+y^2}\right)}{\frac{w}{2\lambda z}\sqrt{x^2+y^2}}\right]^2 \qquad \text{Eq. 28}$$

For the example below, I have chosen arbitrary parameters. You should use the code and change the parameters to the actual setting in your experiment!

The following parameters have been used:

Aperture (round or square) width	***w* = 2 mm**
Wavelength	**λ = 633 nm (He–Ne laser wavelength)**
Distance from aperture	**z = 50 m**

The Fresnel number constraint will assure that we can use Fraunhofer diffraction formula (Fourier transform) and requires that $w^2/4\lambda z < 0.1$ or $z > 10w^2/4\lambda$, which leads to $z > 15.8$ m. We'll use $z = 50$ m.

Now, let's choose the other parameters. A first parameter is the **size of the screen L,** which would in our case correspond to the **detector size**!
A good display size for the function is if the array side length is perhaps five times wider than the pattern's central lobe. The Bessel function J_1 has a first zero when the argument is equal to 1.22. If $y = 0$, then the first zero in the pattern occurs when

$$2\pi\frac{w}{2\lambda z}x = 1.22\pi \qquad \text{Eq. 29}$$

Solve for x to get half the center lobe width and double this result to get the full width of the center lobe

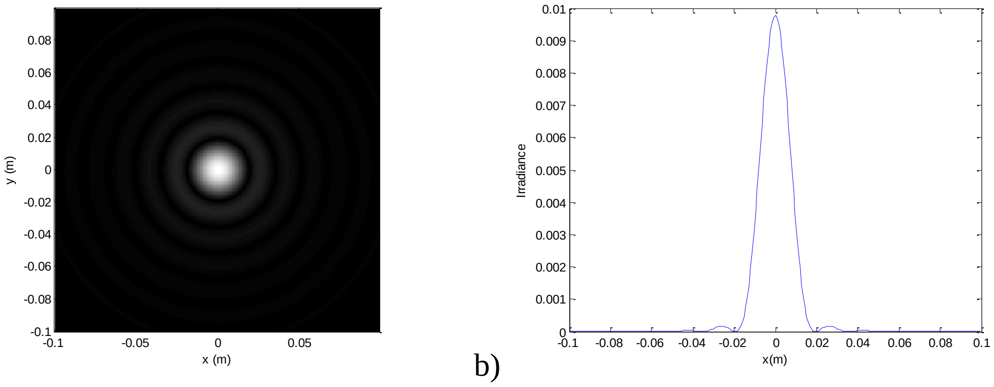
$$D_{\mathrm{lobe}} = 2.44\frac{\lambda z}{w} \qquad \text{Eq. 30}$$

We will choose $L = 5 \times 2.44\,\lambda z/w = 0.2$ m.

Screen or detector size L	**0.2 m**

A possible implantation of the code is available in the file "fraun_circ.m".

Running this script produces the results in Fig. 14. The Fraunhofer pattern of a circular aperture is commonly known as the ***Airy pattern***. The central core of this pattern, whose width is given in Eq. (30), is known as the ***Airy disk***.



a) b)

Figure 14 Fraunhofer irradiance (a) image pattern and (b) x-axis profile for a circular aperture. This is known as the Airy pattern.

2.2 Diffraction with a collimated beam setup and verification

Mechanical setup:

Start with the breadboard and mount the translation stage.

Figure 15: Breadboard with translation stage

The stage is fixed with screws arranged asymmetrically (see the right picture). Continue by mounting the adapter plate and the intermediate piece as below.

Figure 16: Adapter plate and intermediate fixing have to be screw together.

The cameras PCB (printed circuit board) special holder is mounted as shown below.

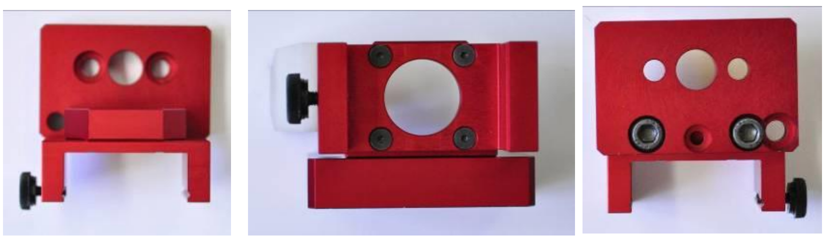


Figure 17: Camera holder (left) mounted on the adapter plate with the intermediate piece. Right: as seen from below.

The assembly has to be fixed on the translation stage before the camera PCB is put into place.

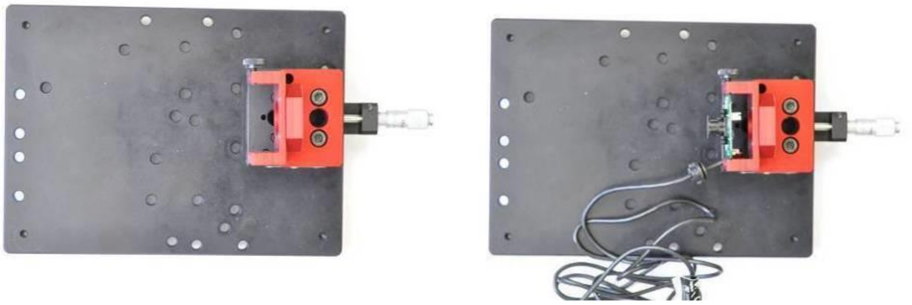


Figure 18 Mechanical holder to fix the camera

Figure 19: Camera PCB in the mount ready for shooting.

Mount the source holder at the other side of the breadboard so that the source is in front of the camera and put the laser in place. Switch on the laser (just connect to USB of the PC) and apply the lens cap.

Figure 20: Source mounted with the lens cap (on the right) to achieve focussing and collimation

You should have a setup similar to that shown below.

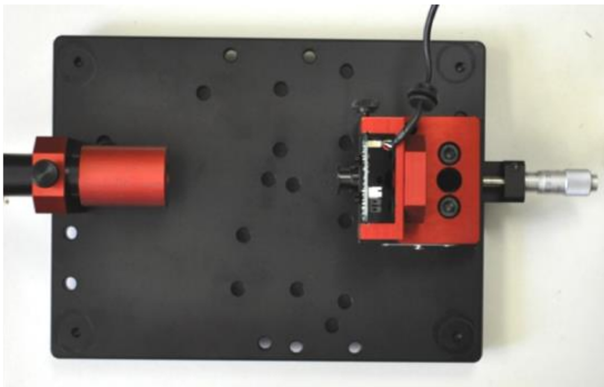


Figure 21: Collimated beam setup. The laser with the lens cap on the left illuminates the camera to the right.

The red tube fixed over the source carries a plano-convex lens (f=12 mm,Ø=9 mm) made from BK7 (Thorlabs L1576). In addition, a polarizer is fixed inside the tube (Edmund Scientific part number ES45668). Moving the tube longitudinally (along the optical axis) focuses the light. Turning the tube adjusts the intensity (because there is a polarizer in the tube and the source is polarized too).

Figure 22. Focalisation (black arrow) and intensity adjustment with the lens cap (white arrow).

A collimated beam shows nearly no change of size when seen at different distances. This can be probed with a sheet of paper, as shown in the image sequence below in Fig. 23.

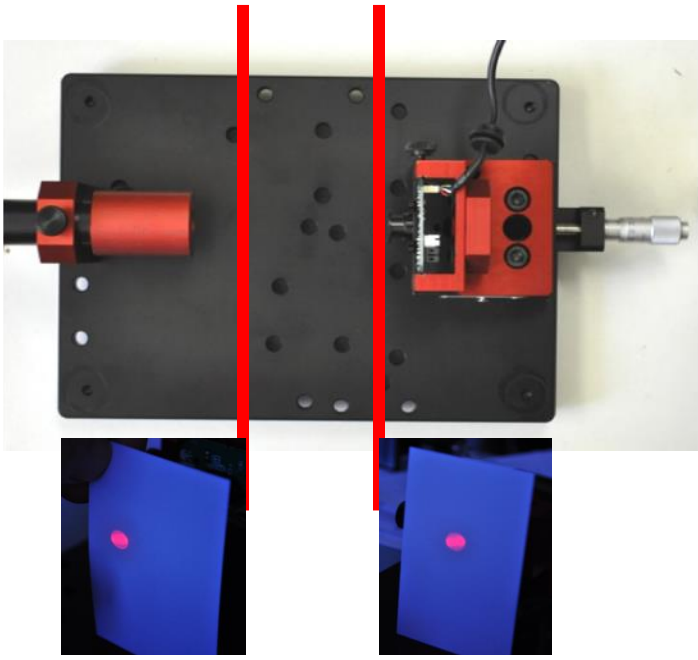


Figure 23: Light spot at different distances from the collimated source. There is nearly no change.

Adjust the lens cap to have the best possible collimation. To make it happen the lens cap has to be moved along the optical axis to adjust the focus and the sheet of paper has to be used regularly to probe the collimation quality.
For our measurements, we will use two different arrangements: without camera objective for direct observation of diffraction and with objective to observe grating diffraction. The infrared (IR) filter is a part of the objective holder of the camera objective. Use the sheet polarizer for additional intensity adaption. The sheet polarizer can be put just between the lens and the camera.

- Remove the camera objective **but leave the IR filter!**
- Set the linear stage at the middle position (5 mm).

NOTE: The IR filter has to be left mounted in front of the camera.

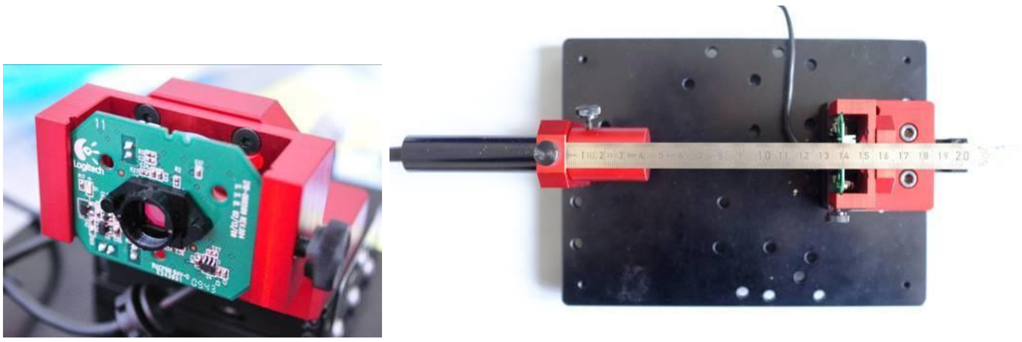


Figure 24. Details of the direct diffraction setup. The objective is removed and the distance between the detector surface and lens cap is set to a convenient value (e.g. 10 cm).

The diffractive structures will be put in front of the lens cap. So, the object plane is the end of the lens cap. It is important to know the distance between the object and the observation plane. Measure this carefully with the ruler and adjust with the linear stage to a convenient position. Put different diffracting objects in front of the lens cap and take images of their diffraction patterns. Analyse the results with the scripts discussed in the next sections.

Diffraction of a circular aperture

Place the holder containing the diffraction structures shown in Fig. 25 in front the lens cap. Slide it gently along the lens cap until the desired diffraction pattern appears on the camera. For the circular aperture, it should look like the image on the left of Fig. 26.

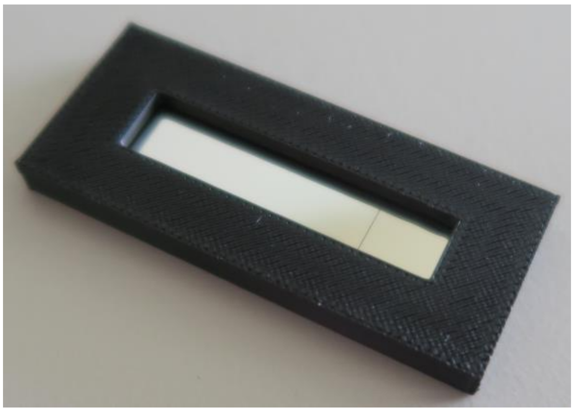


Figure 25 Holder containing diffraction structures: a 100-microns wide slit, a 100-microns wide rectangular aperture and a circular aperture with a diameter of 100 microns.

Use the script “fraun_circ.m” to analytically compute the diffraction pattern**. You need to adjust the parameters in the script to your experimental setup**. Be careful that all the parameters are correctly chosen and note the parameters in your report.

Remember: The aperture width is 100 microns. Our wavelength is 635 nm and a typical propagation distance is 7 cm (to be measured by you!). Now, use your parameters for the simulations and compare with measurements.

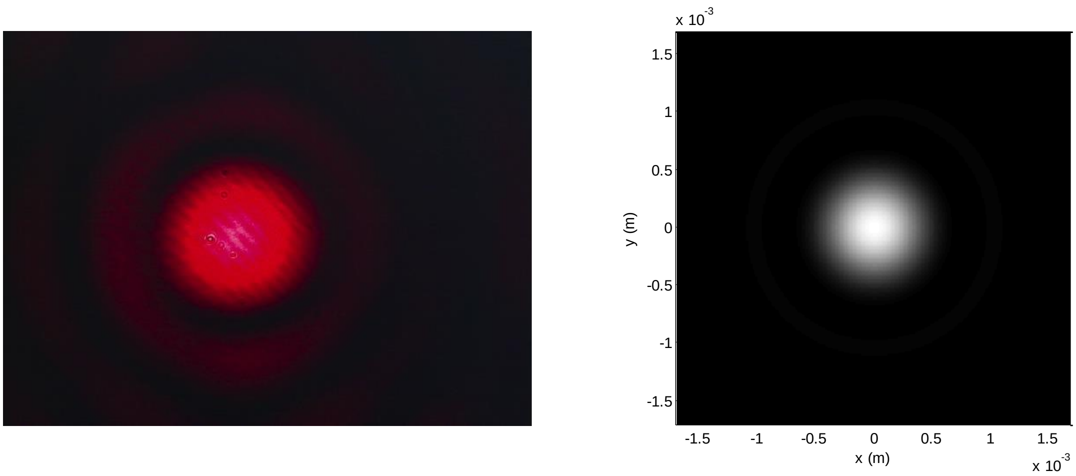


Figure. 26 Diffraction patterns measured and simulated. The height of the simulation field matches the detector height. Propagation distance in this case is 7 cm (you might have different values!!!). Pinhole diameter 100 microns.

Rectangular aperture diffraction

As before, use the holder in Fig. 25. Slide it along the lens cap until you see a diffraction pattern as in the image on the left of Fig. 27.

Again, you have to compare the diffraction pattern with the simulation. Use the script “fraun_square.m”. **You again need to adjust the parameters in the script to the current problem**. Be careful that all the parameters are correctly chosen and note the parameters in your report.

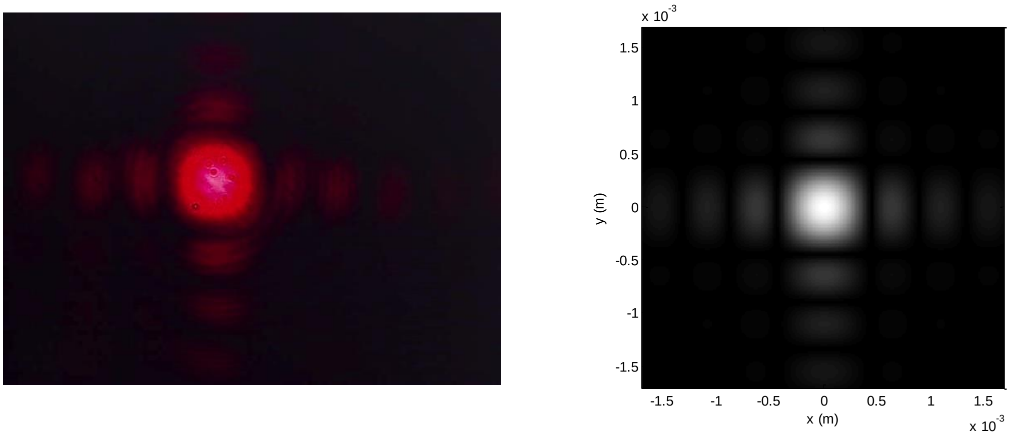


Figure 27. Diffraction of a square aperture measured and calculated. Left measured, right simulated

2.3 Fraunhofer diffraction for a slit

In this experiment, we want to measure the light intensity diffracted by a slit and simulate the corresponding diffraction pattern with Fraunhofer propagation.

Measurement of Slit diffraction

For the measurement, use the diffraction slit in the holder of Fig. 25.

- Align the slit
- Adjust intensity to avoid saturation with the polarizer
- Take images with the slit aligned **vertically**

An example of the measurement is shown below. Make a measurement and plot the images as shown below:

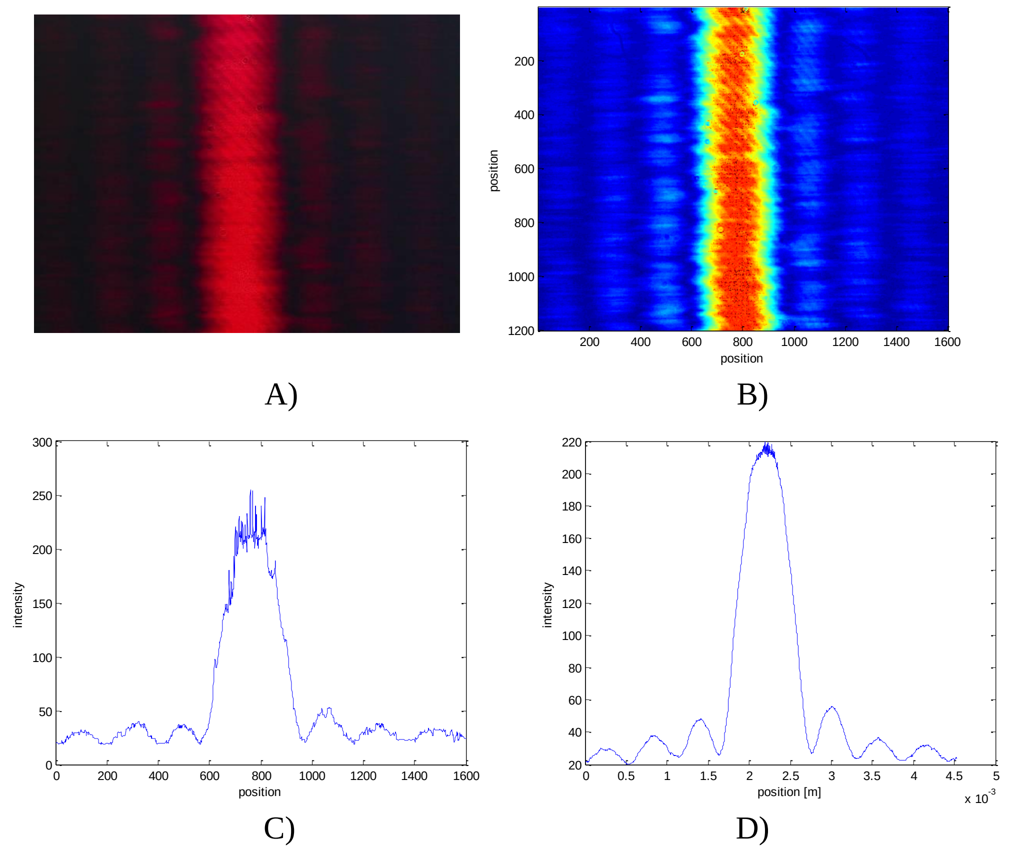


Figure 28 Evaluation of the intensity pattern for slit diffraction. A) Image, B) 2d intensity as seen in Matlab, C) Single line intensity and D) averaged line intensity.

You can use the script “AVG_line_ROI_red_single.m” to create the plots above.

Take care to have a well aligned (vertical!!!) diffraction pattern that allows averaging procedures and avoid saturation.

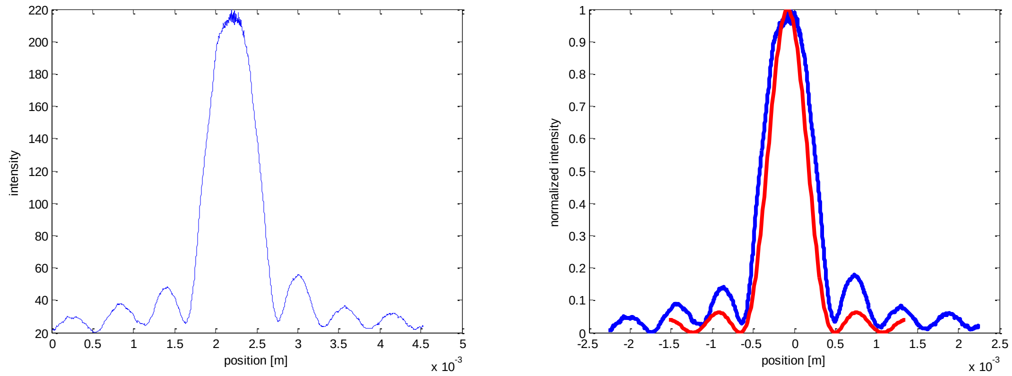


Figure 29 Example plot with matching parameters between measurement and simulation. Such plots could be created with a modified script (AVG_line_ROI_red_compare.m).

To assure the validity of the Fraunhofer approximation the calculation of the Fresnel number has to be done with the following formula

$$N_F = \frac{w^2}{4\lambda z}$$

For the slit, we can use the analytical diffraction equation to control our results. The zeros of intensity for the **first order** minima of a slit are given by the equation

$$\sin\theta = \frac{\lambda}{w}$$

where θ is the diffraction angle, w is the slit width and λ is the wavelength.

In the experiment, the distance from the slit to the camera is z. On our camera image, the positions x of the minima are given by the relation

$$\tan \mathrm{q} = \frac{\mathrm{x}}{\mathrm{z}}$$

and with the diffraction angle θ from above, one finds

$$x = z\tan\theta = z\tan\left(\arcsin\left(\frac{\lambda}{w}\right)\right)$$

2.4 Grating diffraction and Fourier transform

Grating diffraction is observed by illuminating a grating with a plane wave and focusing the light onto the detector. For the measurements, **we are using the original camera objective**. One uses collimated laser light that leads to extremely small spot sizes. Saturation is an issue and needs to be carefully controlled.
The sheet polarizer is used to adjust the light intensity. It is placed in front of the collimation lens. By rotating the source in its mount, the light intensity is varied. In addition, the settings of the camera are changed to prevent overexposure. We are looking for the smallest focus size that can be achieved for point sources located away from the detector.

The laser diode is itself a polarized source. Sources are polarized in a certain direction and rotation of the source with respect to the sheet polarizer allows intensity adjustment (cosine square law when rotated by a certain angle Θ). To measure the focalisation properties, follow the following procedure:

- Mount the objective in front of the camera

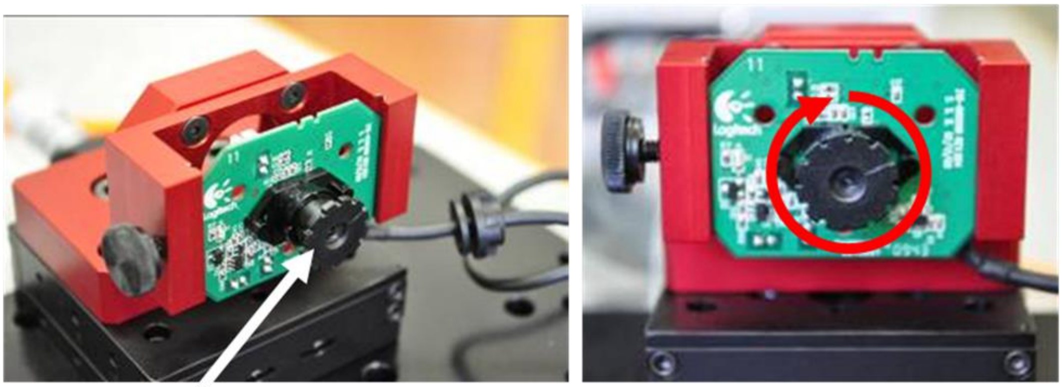


Figure 30 Mount the objective. Rotating allows to focus the collimated source to the smallest spot size

For convenience, we will not use the collimated laser beam as before but work with a situation where the laser is put as far as possible and imaged onto the detector by the objective.

- **REMOVE THE LENS CAP (!) from the source.**

- Adjust the intensity again using the sheet polarizer and rotate the source to lower the intensity
- Adjust the exposure settings (low gain, if possible, also play with exposure time)
- It is very important to avoid saturation because it falsifies the measurement. Note that this is not always possible

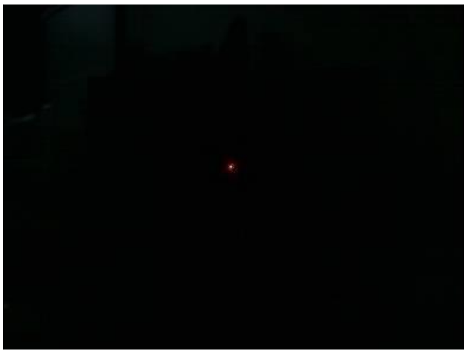


Figure 31. Example image for the smallest spot size of the laser. The intensity has to be adapted depending on the diffractive object. Avoid saturation (not always possible!).

Now the diffractive structures can be applied. Here, we will **put them directly in front of the camera objective.**

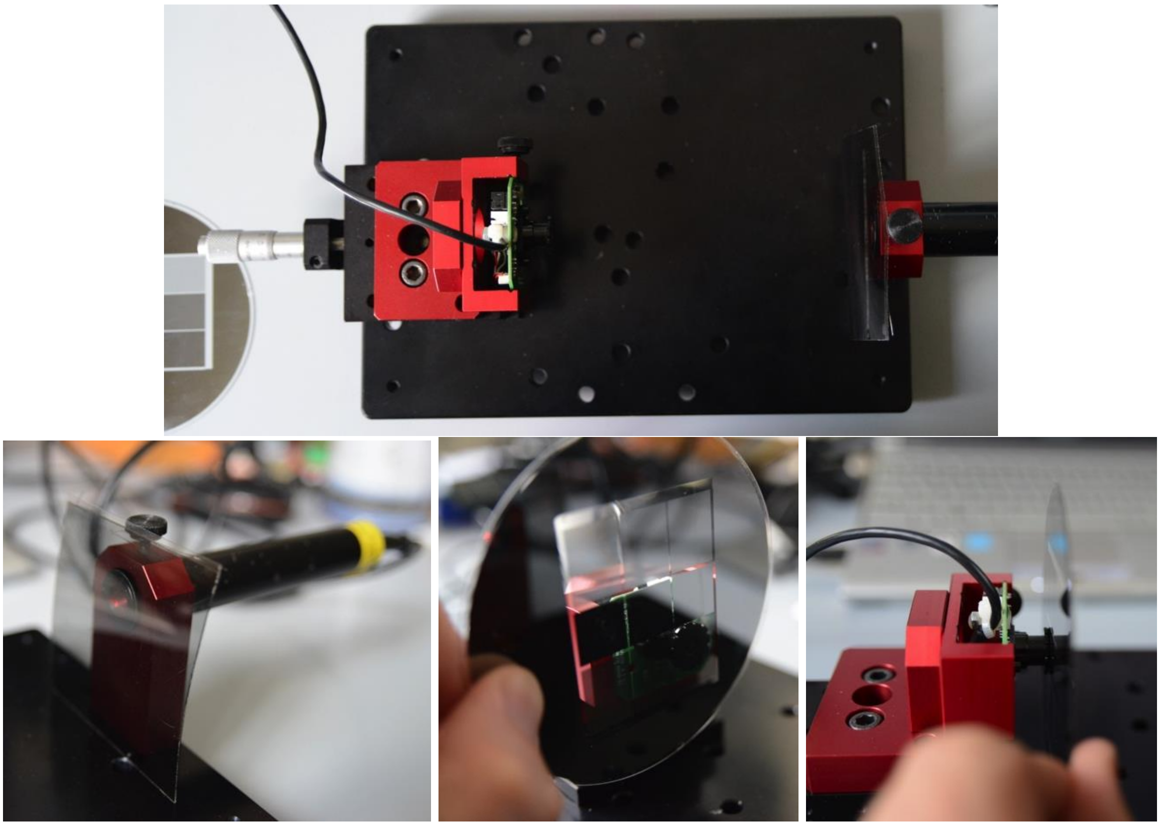


Figure 32. Setup with sheet polarizer and the application of the diffractive structure in front of the objective.

There are several structures implemented on the sample. We want to study the influence of the gratings period and aspect ratio on the diffraction pattern and compare this with simulations.

Structure type	Period (μm)	Slit width (μm)
Line grating	20	10
Line grating	40	10
Line grating	60	10
Line grating	60	20
Line grating	60	30
2d grating - square	50	10
2d grating - square	50	25
2d grating - triangle	50	25
2d grating - circle	50	25

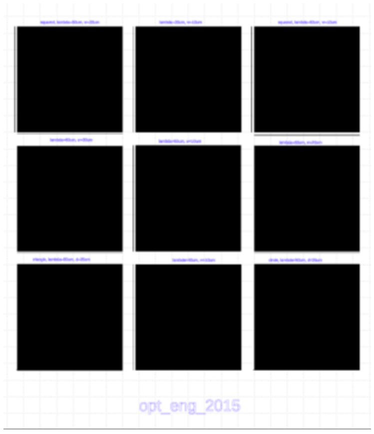


Figure 33 Diffractive structures present on the high-resolution wafer used in our experiment. A larger print out is given in the appendix at the end of this document.

In our experiment, we will put the diffractive structure in front of the lens. Because of the lens, the formula describing the diffraction properties for Fraunhofer diffraction will change. One finds

$$U_2(x,y) = \frac{\exp(jkf)}{j\lambda f}\exp\left[j\frac{k}{2f}\left(x^2+y^2\right)\right]\iint U_1(\xi,\eta)\exp\left[-j\frac{2\pi}{\lambda f}(x\xi+y\eta)\right]d\xi d\eta \quad \text{Eq. 34}$$

where f is the focal length of the lens. The equation is similar to expression Eq. (23) above but the definition of the spatial frequency has changed. Instead of Eq. 24 one finds now

$$f_x = \frac{x}{\lambda f} \qquad f_y = \frac{y}{\lambda f} \qquad \text{Eq. 35}$$

So, everything is expressed in terms of the focal lengths.
This will have consequences on the calculation of the position of the diffraction spots on the detector.

Do the following:

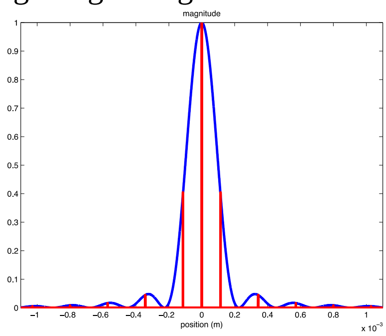
- Determine the centre peak position by taking an image without diffractive structures.
- Introduce the diffractive structure **in front of the objective lens**
- Adjust intensity
- Take images for all gratings and name them correctly
- **Take care that the images are aligned horizontally for further processing!!**

The resulting images are plotted below.

Figure 34. Diffraction patterns of gratings with different parameters. From top: Λ=20μm, w=10 μm, Λ=40μm, w=10 μm, Λ=60μm, w=10 μm, Λ=60μm, w=20 μm, Λ=60μm, w=30 μm. The pictures need to be aligned perfectly horizontal which is not the case in this example.

Diffraction pattern simulation

Next, we will compare a simulated diffraction pattern with measurements using the script “fft_example_1d_grating_compare.m”. We want to visualize the effect of the slit/rectangle and the grating period and this is best done by plotting two graphs in each simulation plot – one for the slit alone and one for the grating. Note that w is the **rectangle widths**. The image below shows a simulation of selected a grating configuration.



Λ=20μm, w=10 μm

Figure 35. Simulated diffraction patterns for different situations. In Blue, the diffraction of a single rectangle is shown. In red, the grating diffraction is given.

Next, the simulation can be compared with experiment. To do so, a line plot of the measurement as shown in Fig. 36 should be used. The script will help you. It plots an image and a subplot with the diffraction pattern. You need to change to parameters in the script to your current situation. Determine the centre of your plot by introducing the name of your files and utilizing the zoom and curser function as in the MATLAB figure below.

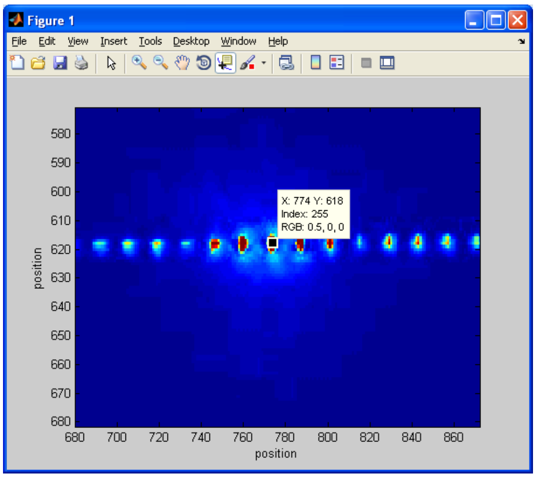
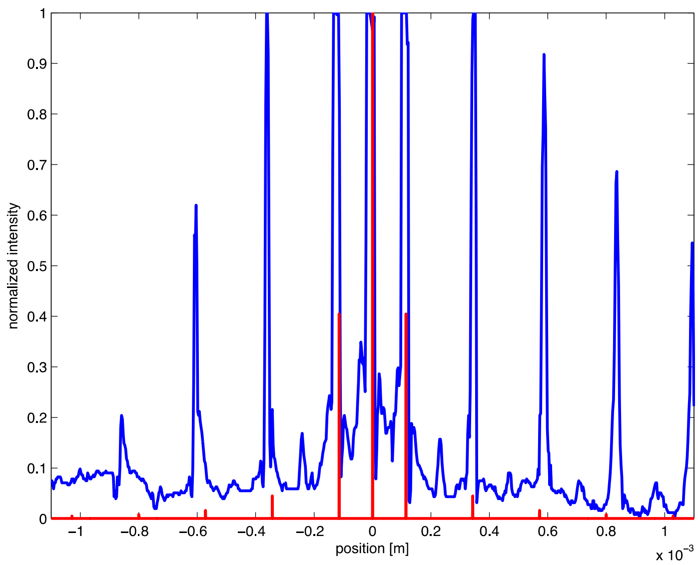


Figure 36. The centre spot of the diffraction image is best found visually. In this example, it is x_center = 773 and y_center = 617.

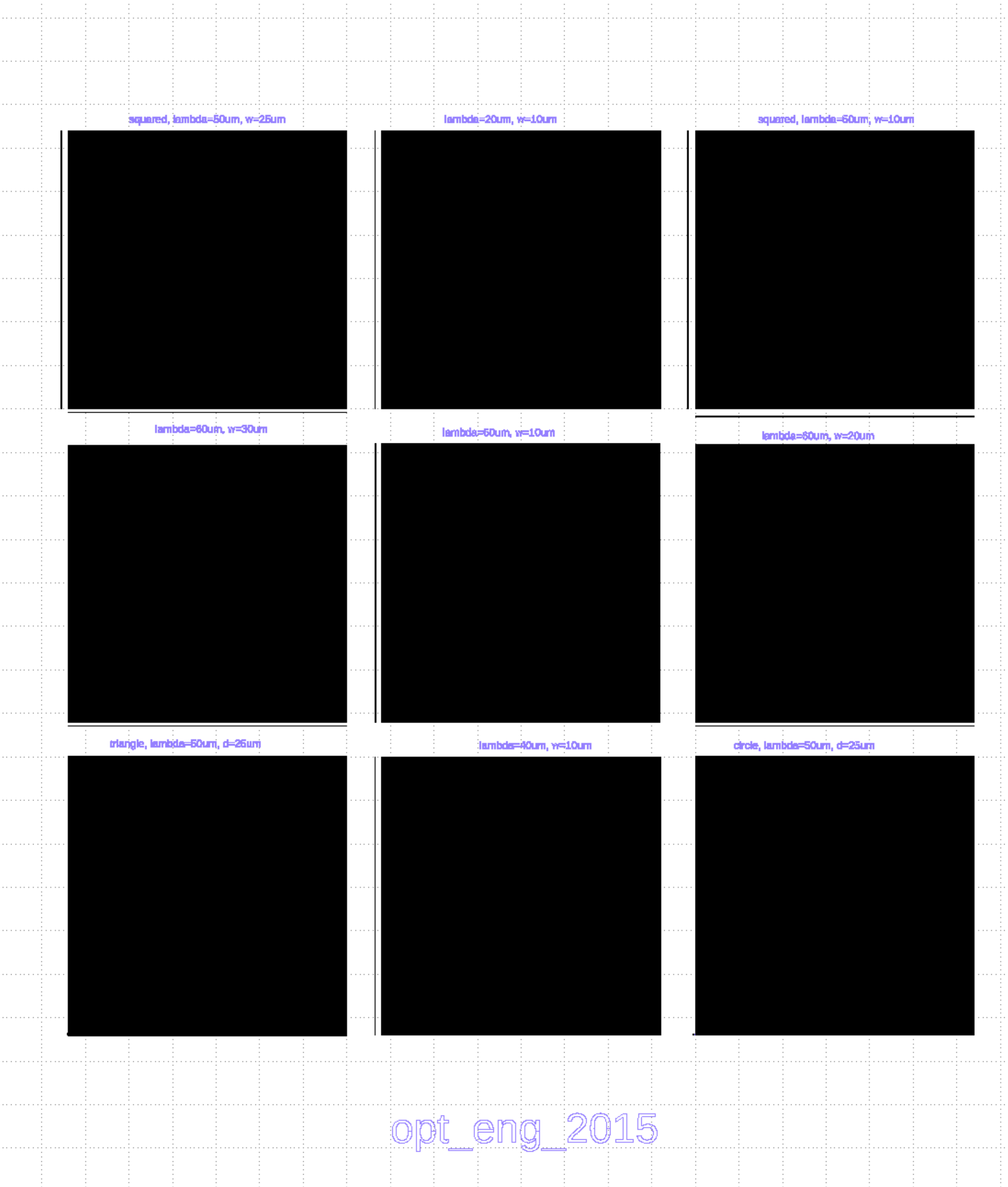
Change the script at the corresponding lines and run it again. You will get results similar to the plot below.



Λ=20µm and w=10 µm

Figure 37. Comparison of measured and simulated plots for a given grating. In red is the simulated diffraction pattern and in blue the measurement

3. Appendix



squared, lambda=50um, w=25um
lambda=20um, w=10um
squared, lambda=60um, w=10um
lambda=60um, w=30um
lambda=60um, w=10um
lambda=60um, w=20um
triangle, lambda=50um, d=25um
lambda=40um, w=10um
circle, lambda=50um, d=25um
opt_eng_2015