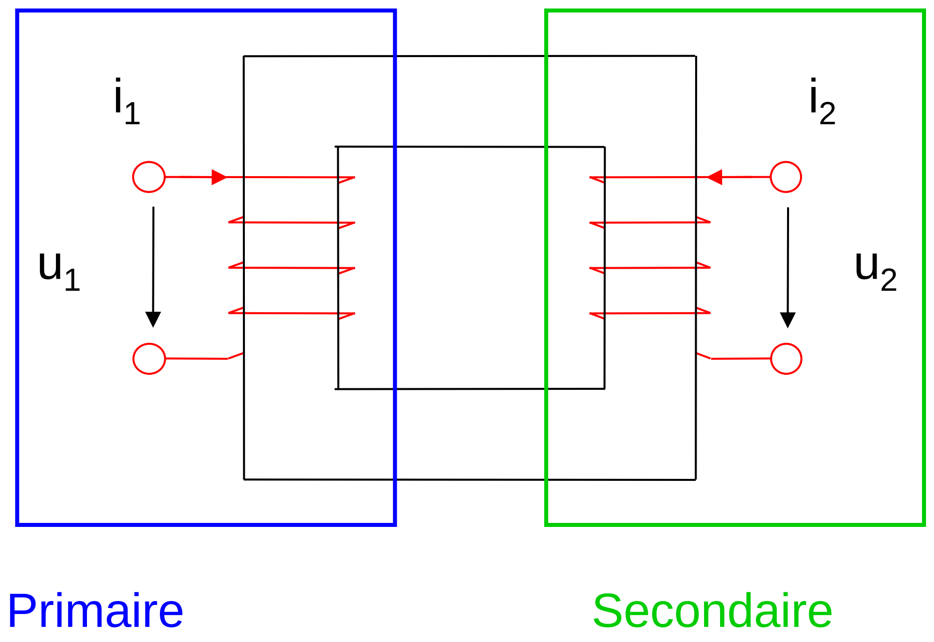
# Transformateur monophasé

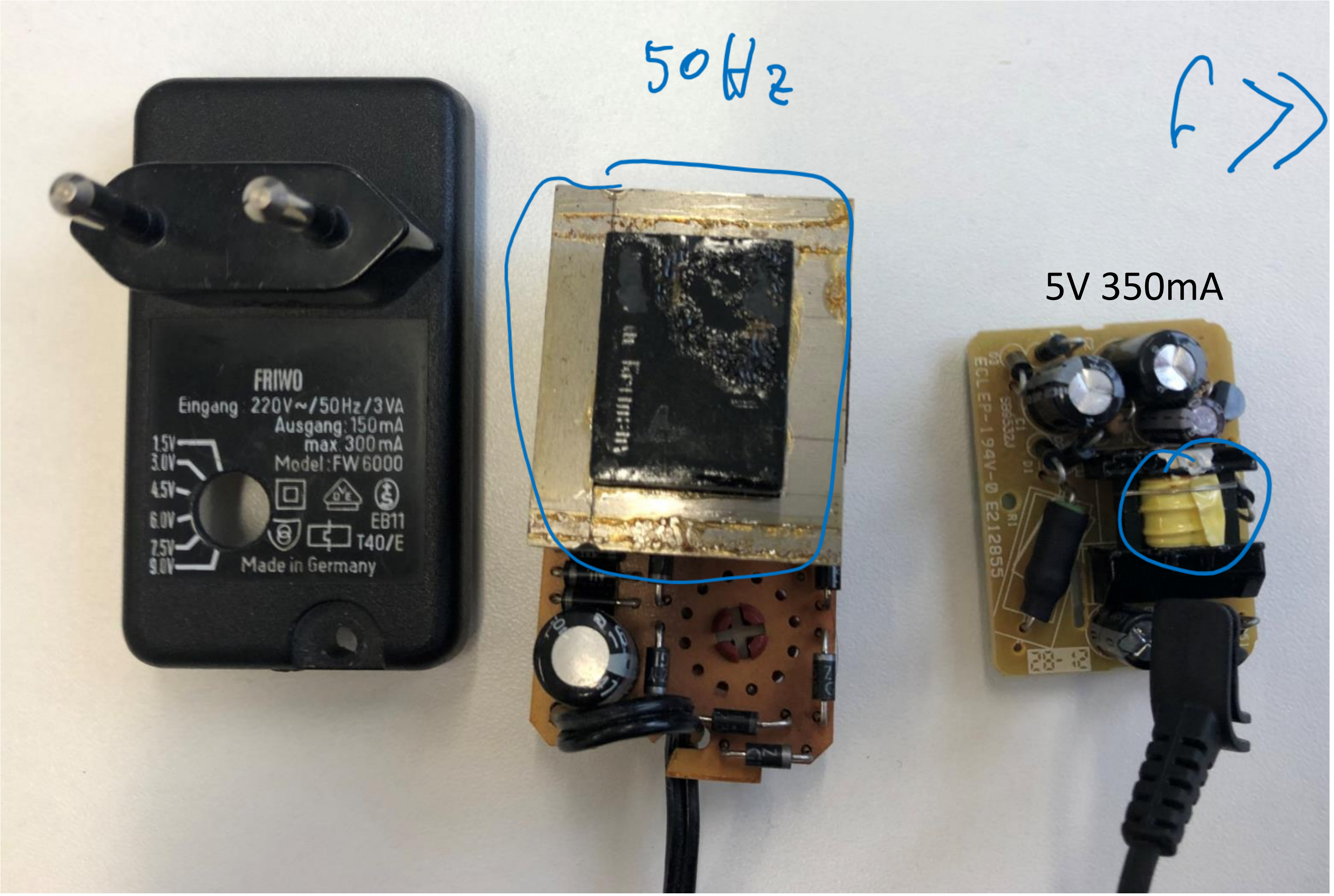
## Actionneurs et systèmes électromagnétiques

# Fonctionnalité

# Principe

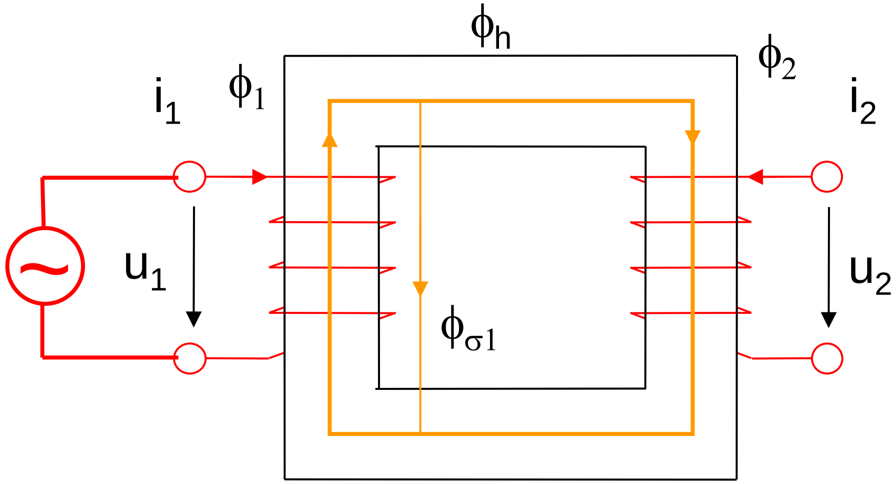


# Evolution




$f \gg 50$ Hz

# Principe de fonctionnement




$$u_1 = R_1 i_1 + \frac{d\psi_1}{dt} = R_1 i_1 + N_1 \frac{d\phi_1}{dt}$$

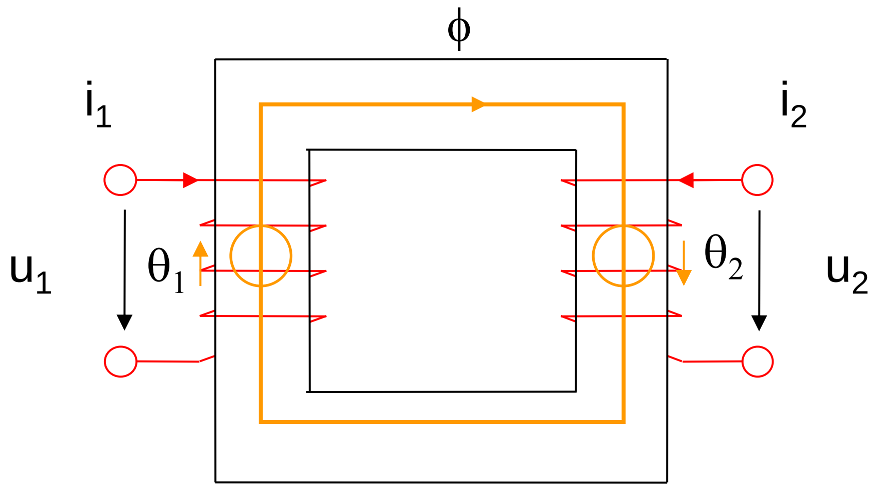
$$u_2 = \cancel{R_2 i_2} + \frac{d\psi_2}{dt}$$

$$\psi_2 = L_{22} i_2 + L_{12} i_1$$

$i_2 = 0$

$$u_2 = \frac{d\psi_2}{dt} = \frac{d\left(L_{12}\, i_1\right)}{dt}$$

# Transformateur Idéal




$\mu_{fer}=\infty$

Pas de pertes fer $\longrightarrow$ $\phi_1 = \phi_2 = \phi$

$R_1=R_2=0$ $\longrightarrow$ $u_1 = \dfrac{d\psi_1}{dt}$, $\quad u_2 = \dfrac{d\psi_2}{dt}$

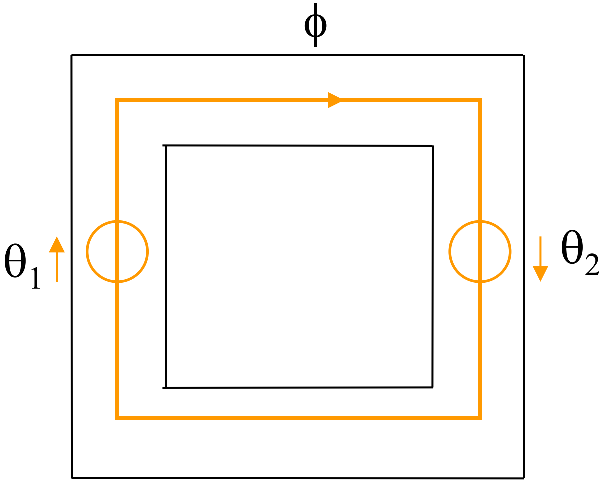
$$\psi_1 = N_1\phi_1 = N_1\phi \quad \rightarrow u_1 = \frac{d\psi_1}{dt} = N_1\frac{d\phi}{dt}$$

$$\psi_2 = N_2\phi_2 = N_2\phi \quad \rightarrow u_2 = \frac{d\psi_2}{dt} = N_2\frac{d\phi}{dt}$$

$$\Rightarrow \frac{d\phi}{dt} = \frac{U_1}{N_1} = \frac{U_2}{N_2}$$

$$\boxed{\frac{U_1}{U_2} = \frac{N_1}{N_2}}$$

# Transformateur Idéal




$\mu_{fer}=\infty$

Pas de pertes fer $\rightarrow$ $\theta_1 + \theta_2 = 0$

$$\boxed{\frac{u_1}{u_2} = \frac{N_1}{N_2}}$$

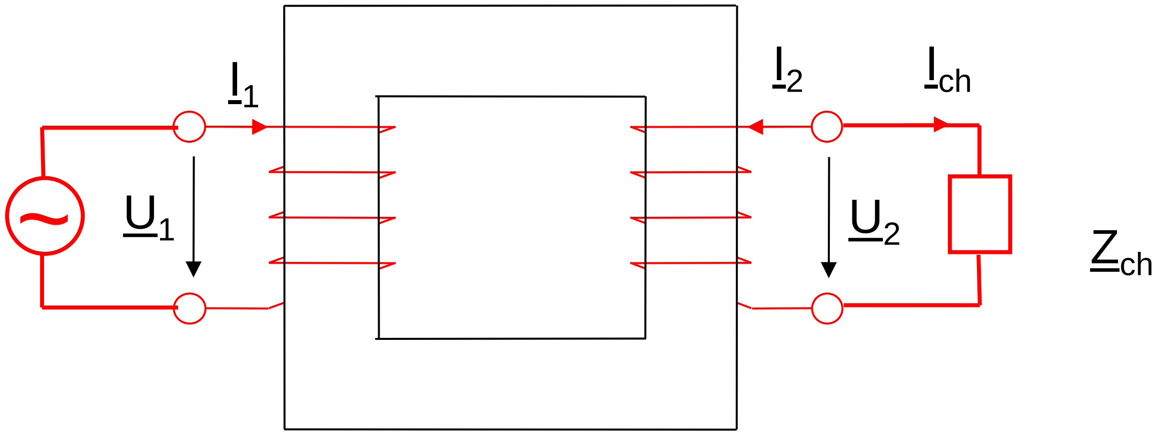
$$\theta_1 + \theta_2 = 0 \Rightarrow N_1 i_1 + N_2 \cdot i_2 = 0$$

$$N_1 i_1 = -N_2 i_2 \qquad -\frac{i_1}{i_2} = \frac{N_2}{N_1}$$

$$\frac{N_1}{N_2} = \ddot{u}$$ rapport de transformation

$$\ddot{u} = \frac{u_1}{u_2} = -\frac{i_2}{i_1}$$

# Alimentation sinusoïdale




$$\frac{N_1}{N_2} = \frac{u_1}{u_2} = -\frac{i_2}{i_1} = ü \quad \Longrightarrow \quad \frac{N_1}{N_2} = \frac{\underline{U}_1}{\underline{U}_2} = -\frac{\underline{I}_2}{\underline{I}_1} = ü$$

$$\frac{N_1}{N_2} = \frac{\underline{U}_1}{\underline{U}_2} = \frac{-\underline{I}_2}{\underline{I}_1}$$

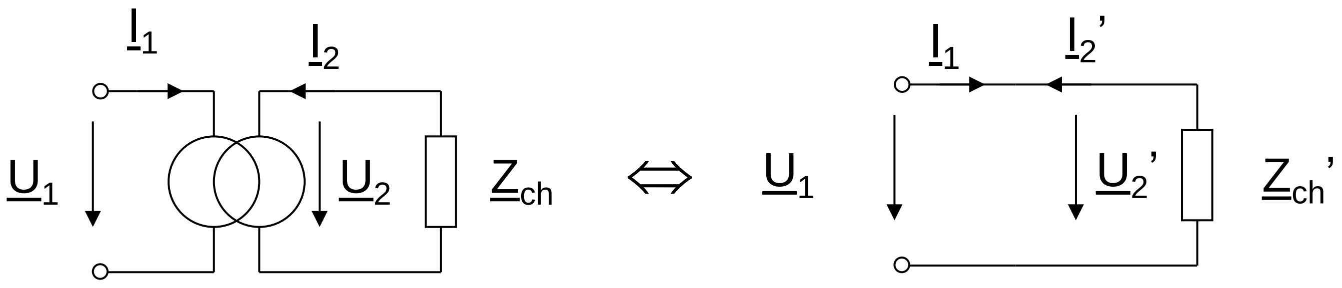
$$\frac{\underline{U}_1}{\underline{I}_1} = \frac{\frac{N_1}{N_2}\,\underline{U}_2}{N_2/N_1 \, (-\underline{I}_2)} = \left(\frac{N_1}{N_2}\right)^2 \frac{\underline{U}_2}{-\underline{I}_2} = ü^2 \frac{\underline{U}_2}{\underline{I}_{ch}} = ü^2 \underline{Z}_{ch}$$

# Grandeurs rapportées

Tension secondaire rapportée au primaire $\underline{U}_2' = \ddot{u}\underline{U}_2$

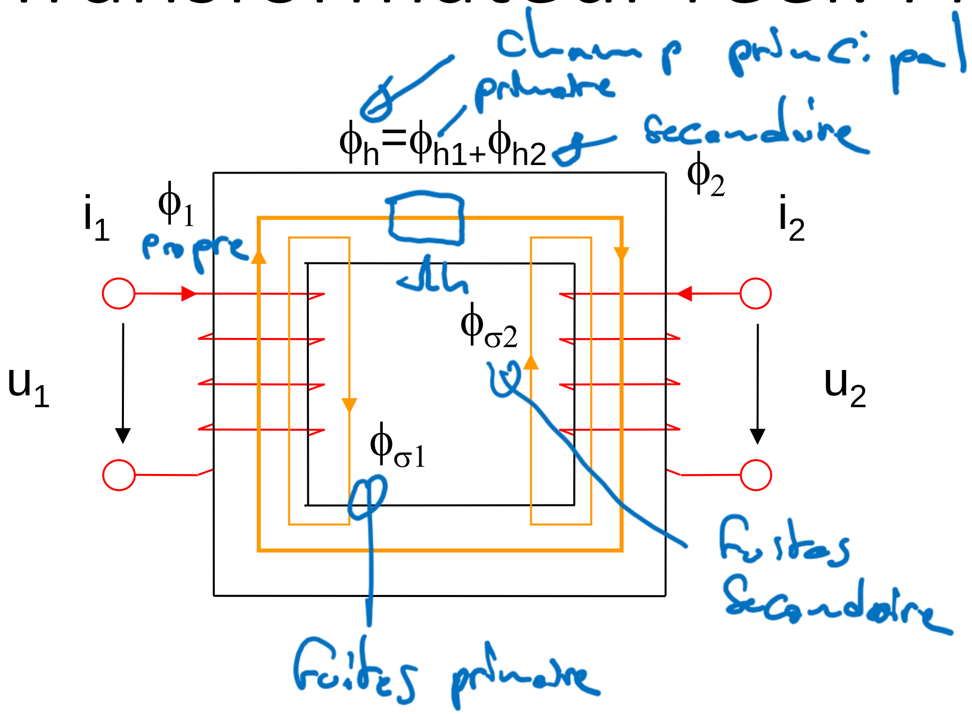
Courant secondaire rapportée au primaire $\underline{I}_2' = \dfrac{\underline{I}_2}{\ddot{u}}$

Impédance secondaire rapportée au primaire $\underline{Z}_{ch}' = \ddot{u}^2\,\underline{Z}_{ch}$




Dans un transformateur idéal $\underline{I}_2$'=- $\underline{I}_1$ et $\underline{U}_2$'= $\underline{U}_1$

# Transformateur réel: Flux




$$\phi_1 = \underline{\phi_{\sigma 1}} + \underline{\phi_h}$$

$$\phi_2 = \phi_{\sigma 2} + \phi_h$$

~~$\mu_{fer}=\infty$~~

Pas de pertes fer

~~$R_1=R_2=0$~~

Pas de saturation =>

$$\phi_h = \phi_{h1} + \phi_{h2}$$

Avec:

$$\phi_{h1} = \Lambda_h \underline{N_1 i_1} \quad \phi_1$$

$$\phi_{h2} = \Lambda_h N_2 i_2$$

$$\phi_h = \Lambda_h (N_1 i_1 + N_2 i_2)$$

# Inductances

$$\phi_h = \Lambda_h (N_1 i_1 + N_2 i_2)$$

$$\psi_1 = N_1 \phi_1 = N_1 \phi_{\sigma 1} + N_1 \phi_h$$

$$\psi_2 = N_2 \phi_2 = N_2 \phi_{\sigma 2} + N_2 \phi_h$$

Inductances de fuite $\phi_{\sigma 1}$

$$\boxed{L_{\sigma 1} = \frac{N_1 \phi_{\sigma 1}}{i_1}} = N_1^2 \Lambda_{\sigma 1}$$

$$L_{\sigma 2} = \frac{N_2 \phi_{\sigma 2}}{i_2} = N_2^2 \Lambda_{\sigma 2}$$

Inductances principales

$$L_{h1} = \frac{N_1 \phi_{h1}}{i_1} = N_1^2 \Lambda_h$$

$$L_{h2} = \frac{N_2 \phi_{h2}}{i_2} = N_2^2 \Lambda_h$$

mutuelle

$$L_{12} = \frac{N_1 \phi_{h2}}{i_2} = N_1 N_2 \Lambda_h$$

Equations de tension induite

$$\psi_1 = N_1 (\phi_{\sigma 1} + \phi_h)$$

$$u_1 = R_1 i_1 + \frac{d\psi_1}{dt}$$

$$u_1 = R_1 i_1 + N_1 \left( \frac{d\phi_{\sigma 1}}{dt} + \frac{d\phi_h}{dt} \right)$$

$$u_1 = R_1 i_1 + \cancel{N_1} \frac{L_{\sigma 1}}{\cancel{N_1}} \frac{di_1}{dt} + N_1 \frac{d\phi_h}{dt}$$

$$u_1 = R_1 i_1 + L_{\sigma 1} \frac{di_1}{dt} + N_1 \Lambda_h \frac{di_1}{dt} + N_1 N_2 \Lambda_h \frac{di_2}{dt}$$

$$u_1 = R_1 i_1 + L_{\sigma 1} \frac{di_1}{dt} + L_{h1} \left( \frac{di_1}{dt} + \frac{N_2}{N_1} \frac{di_2}{dt} \right)$$

# Equations de tension en grandeurs rapportées

$$\ddot{u} = \frac{N_1}{N_2}$$

$$u_2' = \ddot{u}\, u_2$$

$$i_2' = \frac{i_2}{\ddot{u}}$$

$$R_2' = \ddot{u}^2 R_2$$

$$L_{\sigma 2}' = \ddot{u}^2 L_{\sigma 2}$$

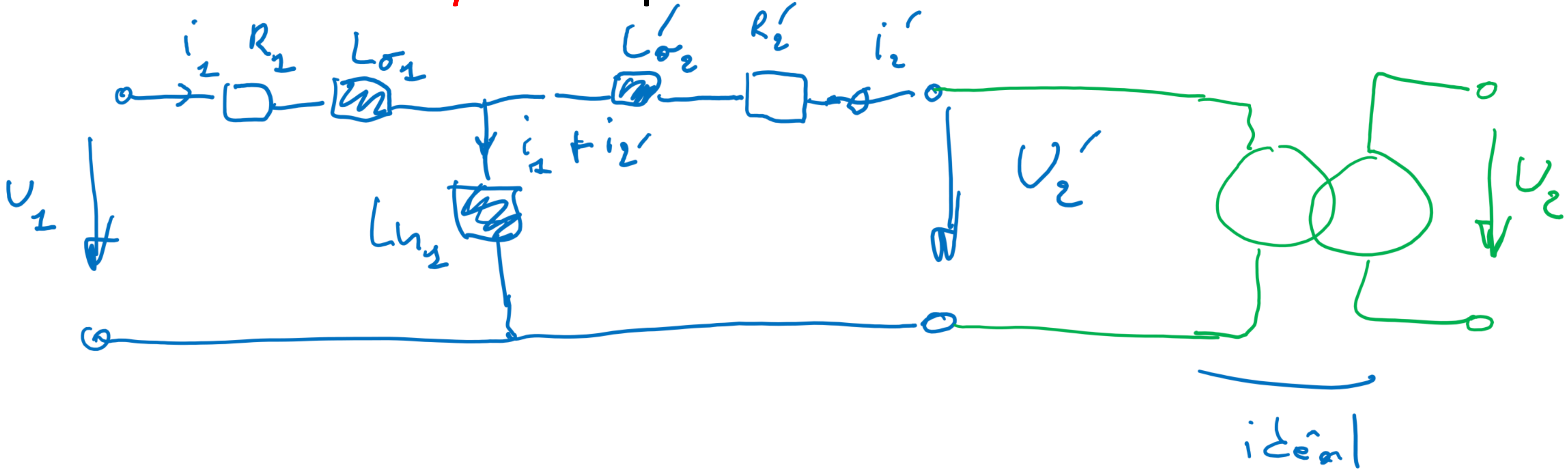
$$L_{h2}' = \ddot{u}^2 L_{h2} = L_{h1}$$

$$u_1 = R_1 i_1 + L_{\sigma 1}\frac{di_1}{dt} + L_{h1}\left(\frac{di_1}{dt} + \frac{N_2}{N_1}\frac{di_2}{dt}\right)$$

$$= R_1 i_1 + L_{\sigma 1}\frac{di}{dt} + L_{h1}\left(\frac{di_1}{dt} + \frac{di_2'}{dt}\right)$$

$$u_2' = R_2' i_2' + L_{\sigma 2}'\frac{di_2'}{dt} + L_{h1}\left(\frac{di_1}{dt} + \frac{di_2'}{dt}\right)$$

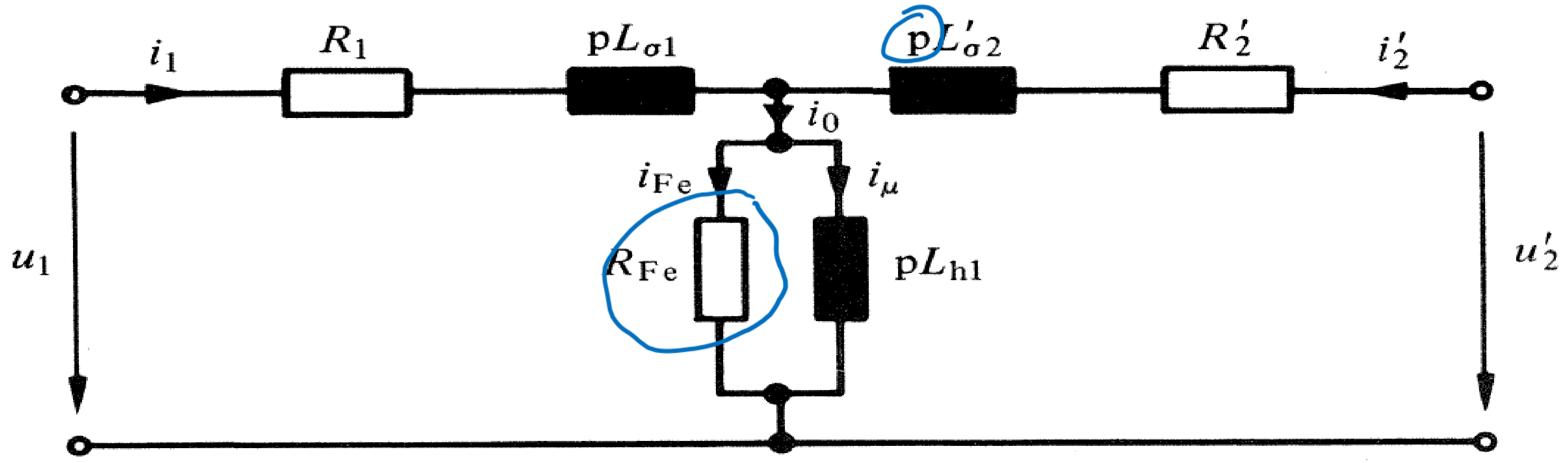
# Schéma *électrique* équivalent




$$u_1 = R_1 i_1 + L_{\sigma 1}\frac{di_1}{dt} + L_{h1}\left(\frac{di_1}{dt} + \frac{di'_2}{dt}\right)$$

$$u'_2 = R'_2 i'_2 + L'_{\sigma 2}\frac{di'_2}{dt} + L_{h1}\left(\frac{di_1}{dt} + \frac{di'_2}{dt}\right)$$

# Pertes fer



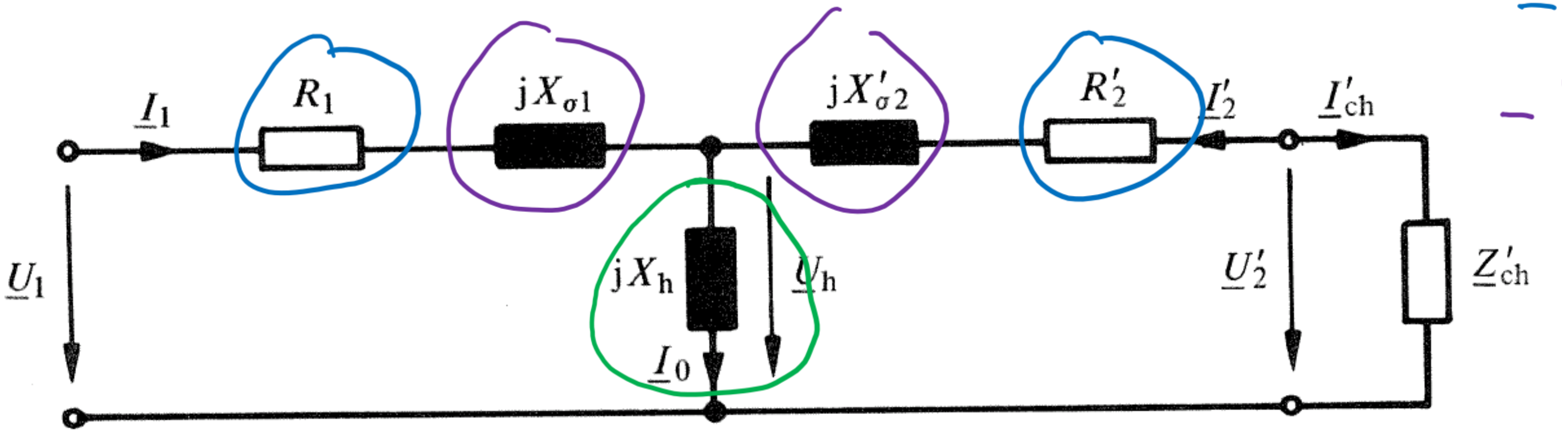
# Régime permanent (sinus)

$$\underline{U}_1 = R_1\underline{I}_1 + j\omega L_{\sigma 1}\underline{I}_1 + j\omega L_{h1}\left(\underline{I}_1 + \underline{I}_2'\right)$$

$$= R_1\underline{I}_1 + jX_{\sigma 1}\underline{I}_1 + jX_h\left(\underline{I}_1 + \underline{I}_2'\right)$$

$$\underline{U}_2' = R_2'\underline{I}_2' + j\omega L'_{\sigma 2}\underline{I}_2' + j\omega L_{h1}\left(\underline{I}_1 + \underline{I}_2'\right)$$

$$R_2'\underline{I}_2' + jX'_{\sigma 2}\underline{I}_2' + jX_h\left(\underline{I}_1 + \underline{I}_2'\right)$$




– Pertes

– Fuites

$X_h = \omega L_h$