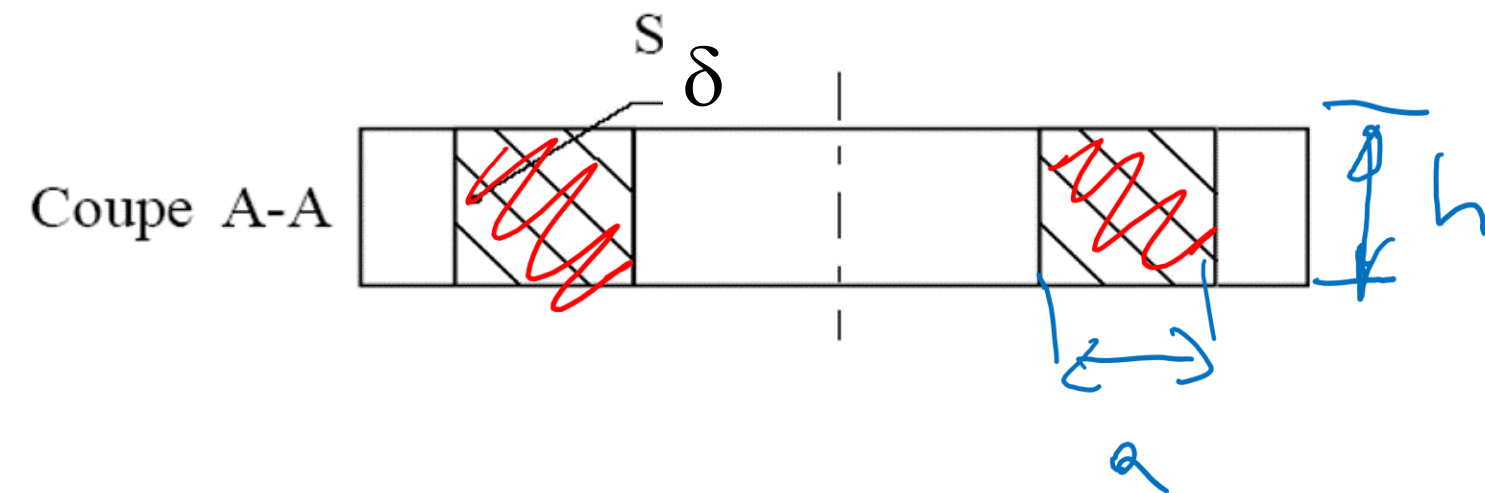
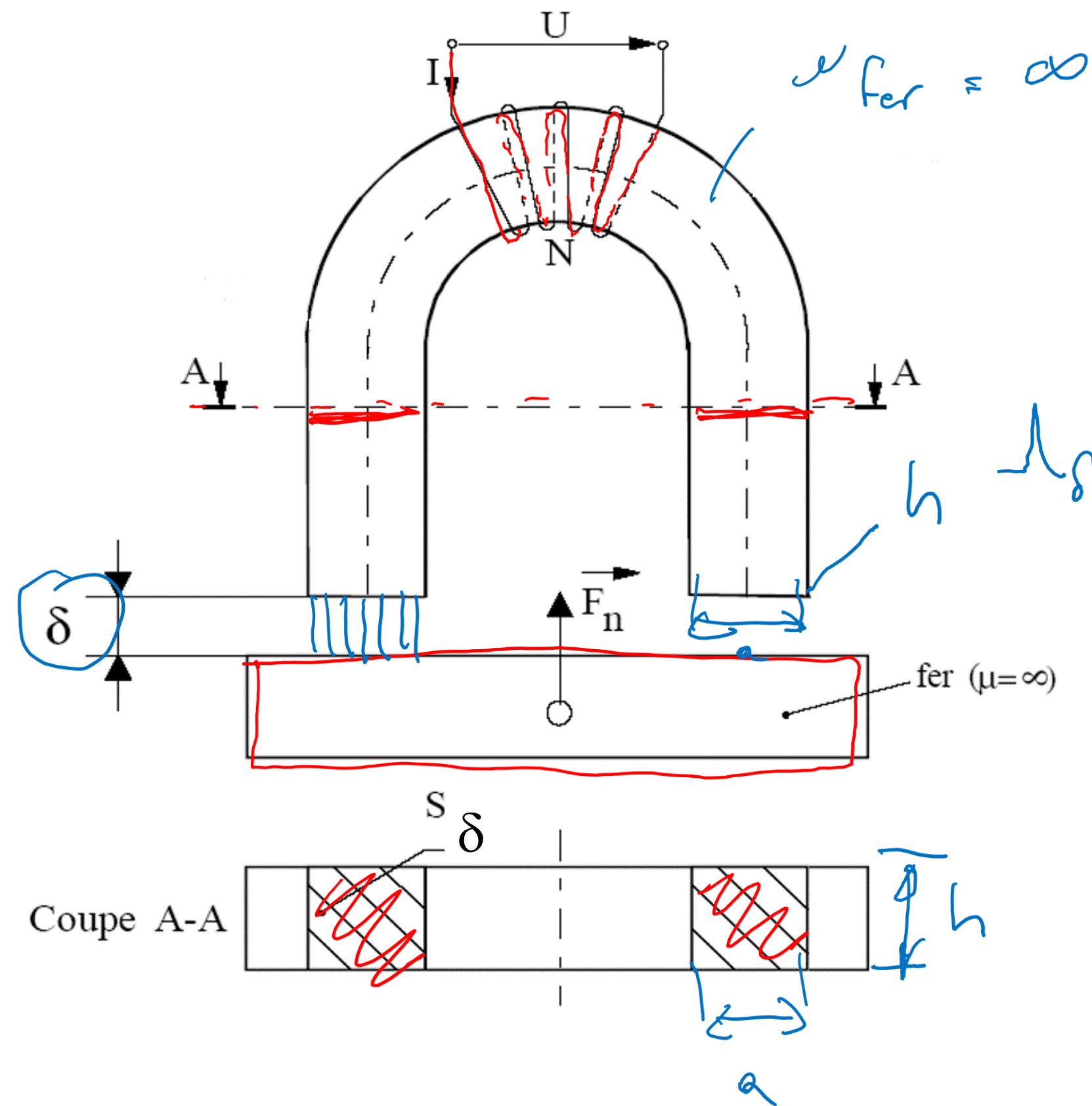


EPFL

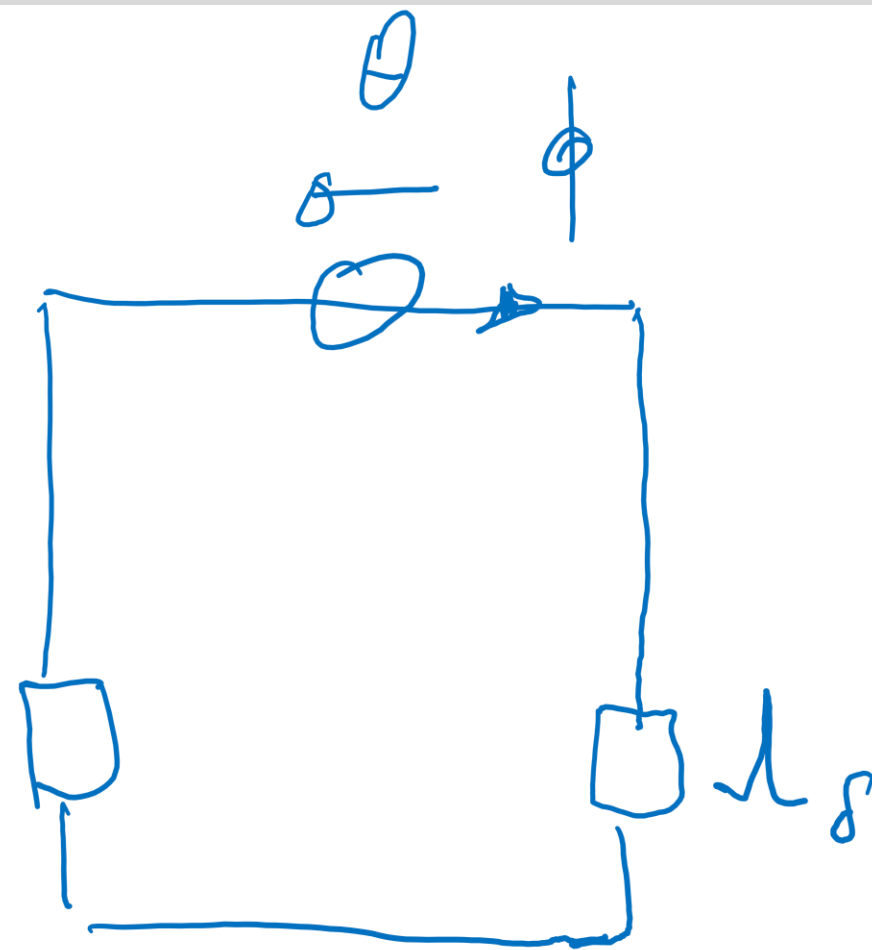
Conversion électromécanique

Actionneurs et systèmes électromagnétiques

Exemple: électroaimant



$$i = cte$$



$$\mathcal{L}_{eq} = \frac{\mu_0 \mu_r a}{2}$$

$$\Theta = Ni \quad \mathcal{L}_\delta = \frac{\mu_0 \mu_r a \cdot h}{\delta}$$

$$L_{11} = \frac{\Psi_{11}}{i_1} = \frac{N \Phi_{11}}{i_1} = \frac{N \mathcal{L}_{eq} N i}{i} = N^2 \mathcal{L}_{eq} \quad i_1 = i$$

$$L = L_{11}$$

Expression de la force

$$v = Ri + \frac{d\varphi}{dt} \quad \psi = Li$$

$$v = Ri + \frac{d}{dt}(Li) = Ri + \underbrace{L \frac{di}{dt}}_{=0} + i \frac{dL}{dt}$$

$$v = Ri + i \frac{dL}{dt} \parallel i \Rightarrow \underbrace{vi}_{P_{el}} = \underbrace{Ri^2}_{P_{joule}} + \underbrace{i^2 \frac{dL}{dt}}_{P_{mag} + P_{mec}}$$

$$W_{mag} + W_{mec} = \int_t i^2 \frac{dL}{dt} dt = \int i^2 dL = Li^2$$

$$W_{mag} = \frac{1}{2} Li^2$$

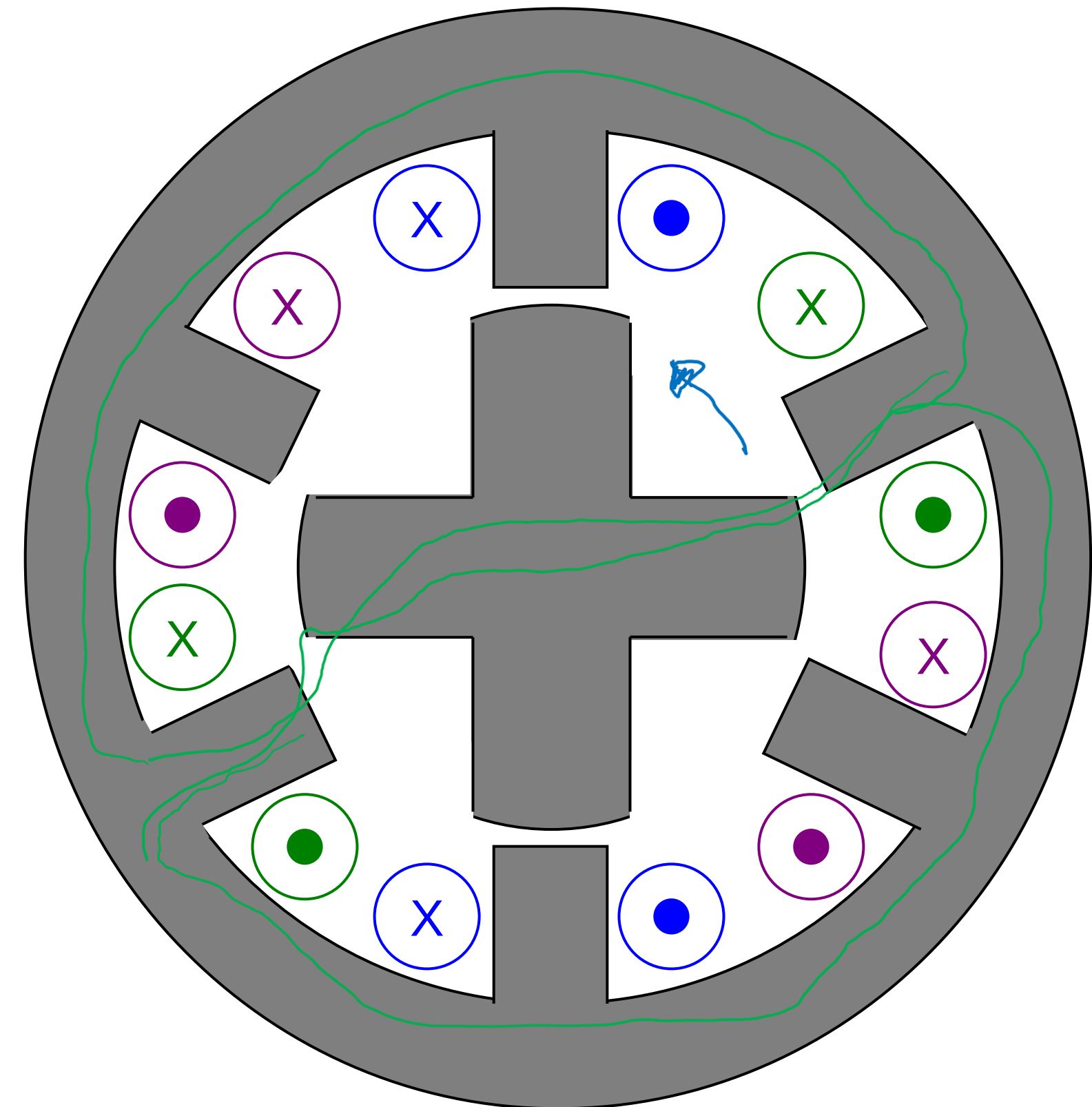
$$W_{mec} = Li^2 - \frac{1}{2} Li^2 = \frac{1}{2} Li^2 \Big|_{\delta_0}^{\delta_1}$$

$$W_{mec} = \int F(\delta) d\delta \Rightarrow F(\delta) = \frac{dW_{mec}}{d\delta} = \frac{1}{2} \frac{dL}{d\delta} i^2 = -\frac{1}{4} \frac{N^2 \omega_0 a^4}{\delta^2} i^2$$

Systeme électromécanique:

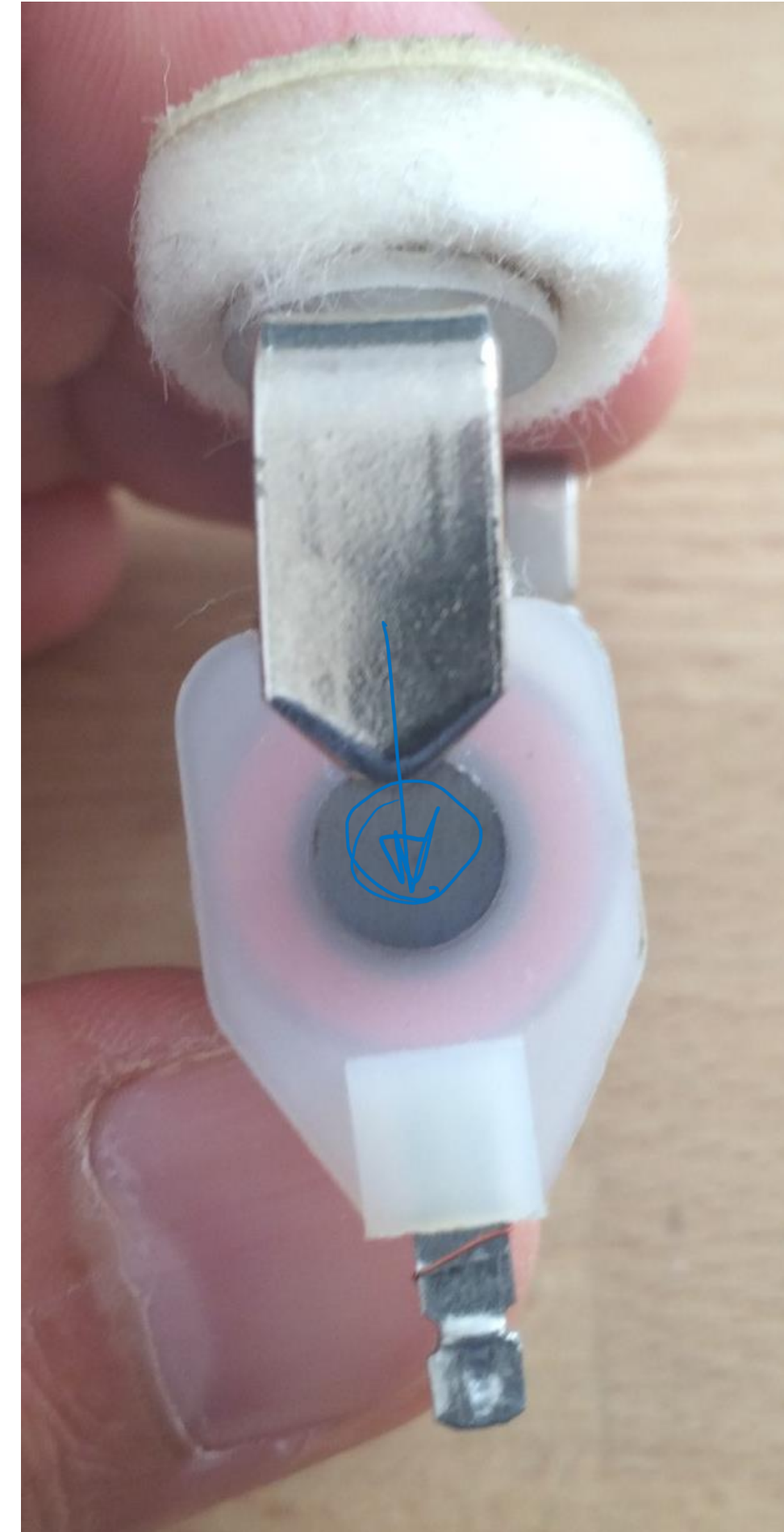
- ensemble de circuits électriques liés mécaniquement ou couplés magnétiquement;
- géométriquement déformable;
- possède un nombre variable n de degrés de liberté mécaniques.

Exemples de système électromécanique



Actionneur réluctant

$F \propto i^2$



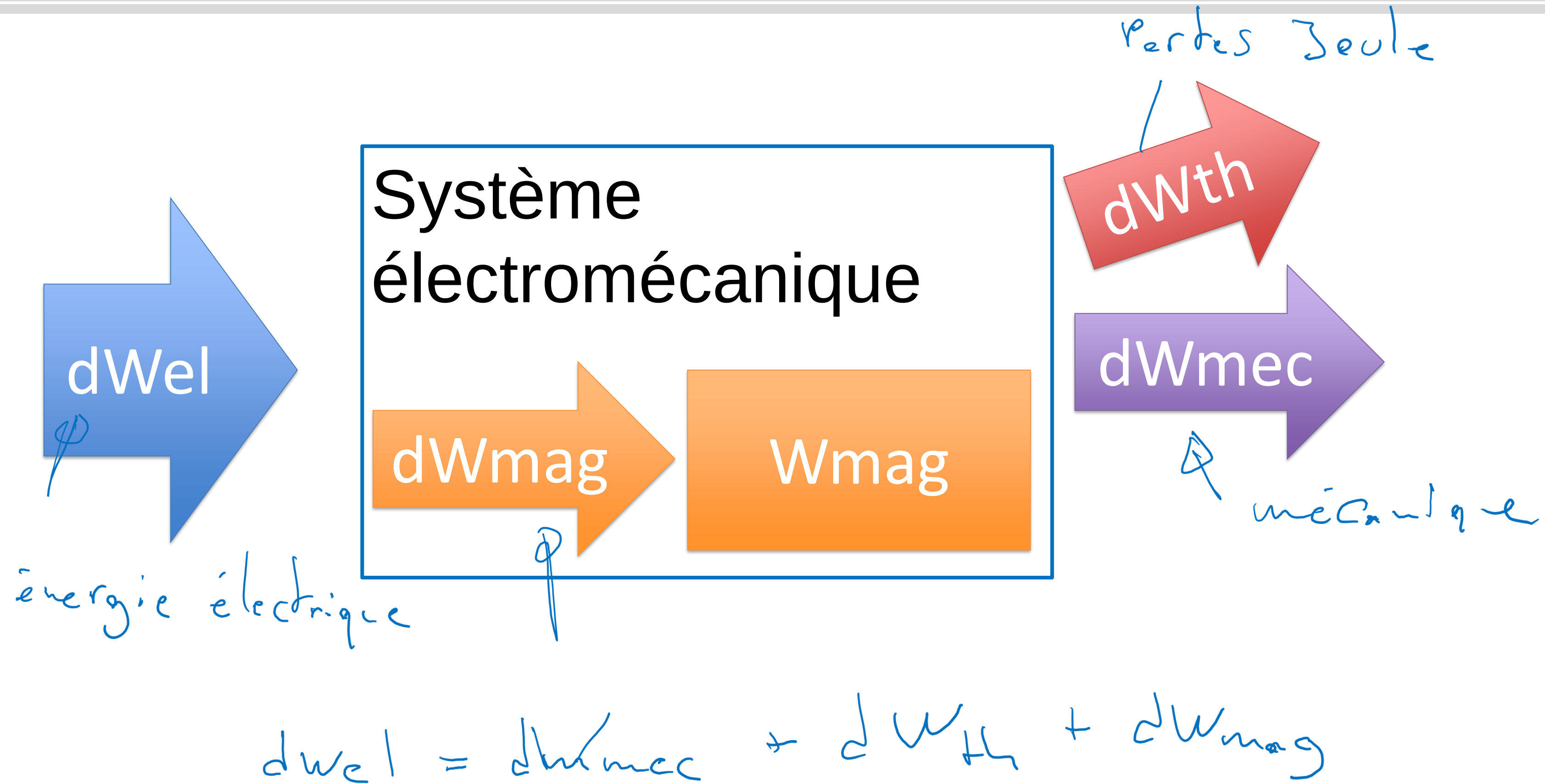
$$u_j = R_j i_j + \frac{d\Psi_j}{dt} \quad (\text{k circuits})$$

$$\Psi_j = \sum_{p=1}^k L_{jp} i_p$$

$$\sum F_m = m_m \frac{d^2 x_m}{dt^2} \quad (\text{n degrés de liberté})$$

$$\sum M_m = J_m \frac{d^2 \alpha_m}{dt^2}$$

Bilan d'énergie (moteur)



$$dW_{el} = dW_{mag} + dW_{th} + dW_{mec}$$

$$dW_{mec} = \sum_{m=1}^n F_m dx_m$$

$$dW_{el} = \sum_{j=1}^k U_j i_j dt = \sum_{j=1}^k R_j i_j^2 dt + \sum_{j=1}^k d\Psi_j i_j$$

$$dW_{th} = \sum_{j=1}^k R_j i_j^2 dt$$

$$W_{mag} = \sum_{j=1}^k \int_0^{\Psi_j} i_j d\Psi_j$$

$$dW_{mag}(\Psi_j, x_m) = \sum_{j=1}^k \frac{\partial W_{mag}}{\partial \Psi_j} d\Psi_j + \sum_{m=1}^n \frac{\partial W_{mag}}{\partial x_m} dx_m = \sum_{j=1}^k i_j d\Psi_j + \sum_{m=1}^n \frac{\partial W_{mag}}{\partial x_m} dx_m$$

Obtention d'une force

$$\sum_{m=1}^n \frac{\partial W_{mag}}{\partial x_m} dx_m + \sum_{m=1}^n F_m dx_m = 0$$

≈ 0

$$\sum_{m=1}^n \left(\frac{\partial W_{mag}}{\partial x_m} + F_m \right) dx_m = 0$$
$$F_m = - \frac{\partial W_{mag}}{\partial x_m} = - \frac{dW_{mag}}{dx_m} \Big|_{\Psi_j = \text{cte}}$$

$$F_m = \frac{\partial W'_{mag}}{\partial x_m} = \frac{dW'_{mag}}{dx_m} \Big|_{i_j = \text{cte}}$$

- Bilan d'énergie:

$$dW_{el} = dW_{mag} + dW_{th} + dW_{mec}$$

- Méthode de la dérivée de la coénergie

$$F_m = \frac{\partial W'_{mag}}{\partial x_m} = \left. \frac{dW'_{mag}}{dx_m} \right|_{i_j = \text{cte}}$$

Force électromagnétique

$$F_m = \frac{\partial W'_{mag}}{\partial x_m} = \left. \frac{dW'_{mag}}{dx_m} \right|_{i_j = \text{cte}}$$

Cas général

$$W'_{mag} = \sum_{j=1}^k \int_0^{i_j} \Psi_j di_j$$

$$F_m = \sum_{j=1}^k \int_0^{i_j} \frac{\partial \Psi_j}{\partial x_m} di_j$$

$$F_m = \frac{\delta W'_{mag}}{\delta x_m}$$

Milieu linéaire

$$W'_{mag} = \frac{1}{2} \sum_{j=1}^k \Psi_j i_j$$

$$\Psi_j = \sum_{p=1}^k L_{jp} i_p$$

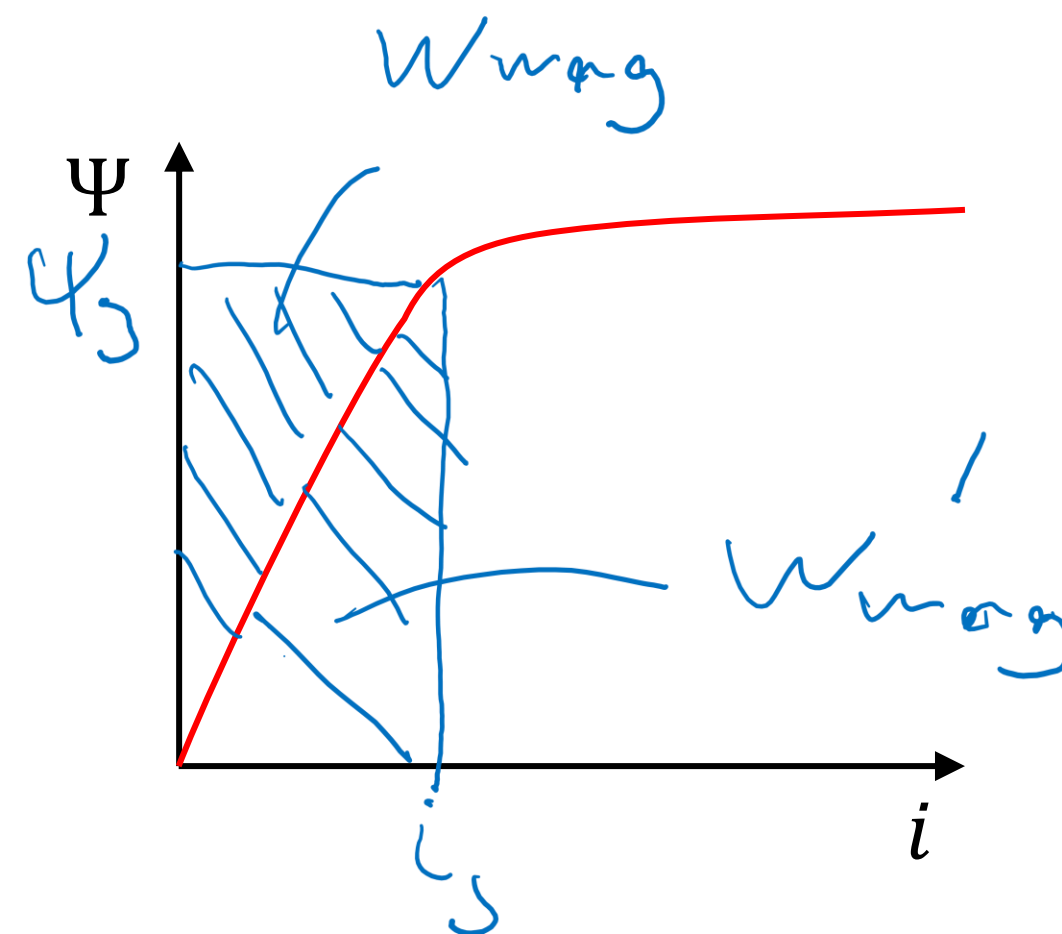
$$F_m = \frac{\partial W'_{mag}}{\partial x_m} = \frac{1}{2} \sum_{j=1}^k \sum_{p=1}^k \frac{dL_{jp}}{dx_m} i_j i_p$$

$$\Psi_1 = L_{11} i_1 + L_{12} i_2 + L_{13} i_3 + \dots$$

$$L_{jp} = N_j N_p \mathcal{L}_{jp} \quad \Theta_j = N_j i_j$$

$$\Theta_p = N_p i_p$$

$$F_m = \frac{1}{2} \sum_{j=1}^k \sum_{p=1}^k \frac{d\mathcal{L}_{jp}}{dx_m} \Theta_j \Theta_p$$



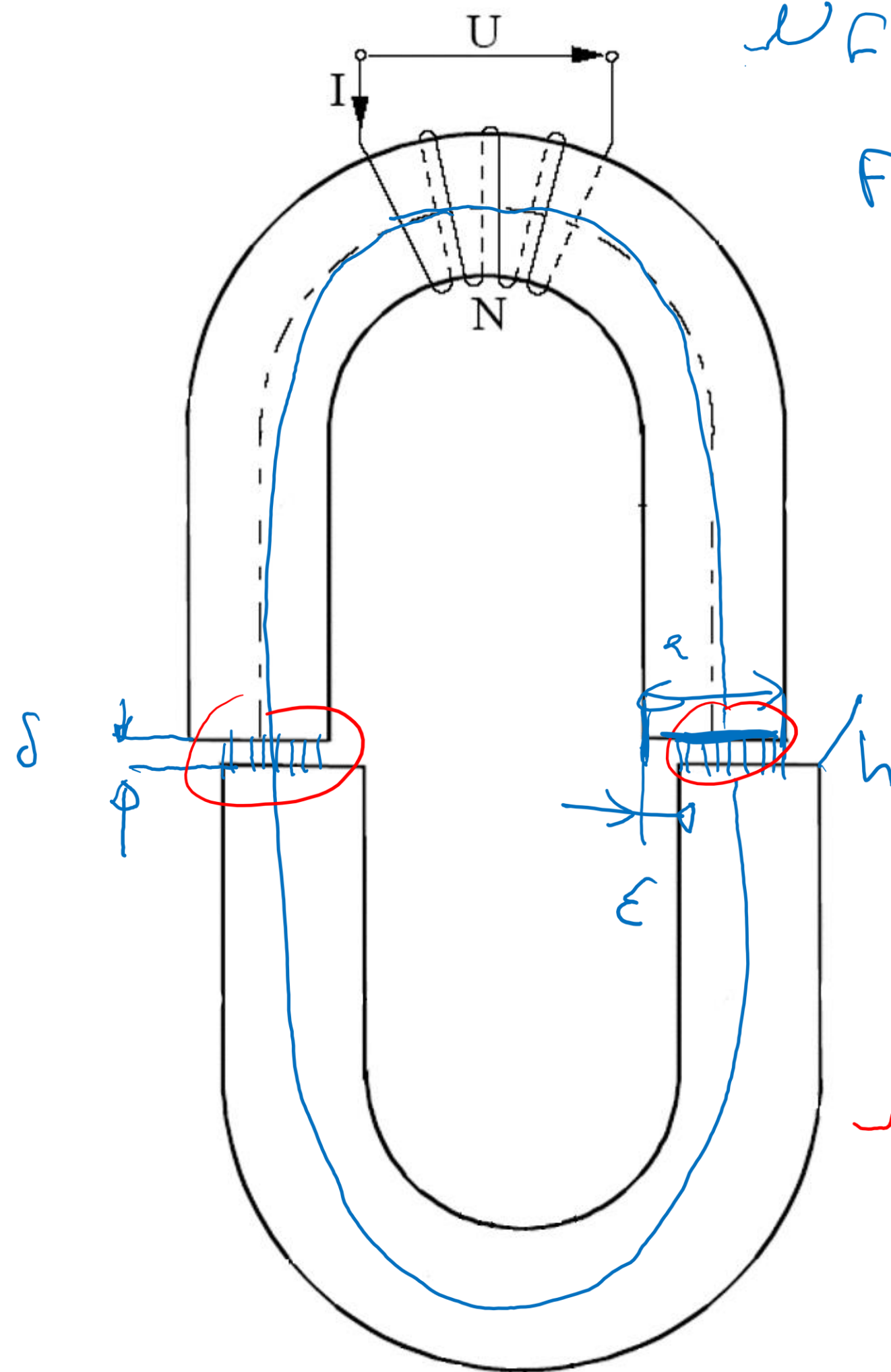
- Force:

$$F_m = \frac{1}{2} \sum_{j=1}^k \sum_{p=1}^k \frac{dL_{jp}}{dx_m} i_j i_p$$
$$F_m = \frac{1}{2} \sum_{j=1}^k \sum_{p=1}^k \frac{d\Lambda_{jp}}{dx_m} \Theta_j \Theta_p$$

- Couple

$$M_m = \frac{1}{2} \sum_{j=1}^k \sum_{p=1}^k \frac{dL_{jp}}{d\alpha_m} i_j i_p$$
$$M_m = \frac{1}{2} \sum_{j=1}^k \sum_{p=1}^k \frac{d\Lambda_{jp}}{d\alpha_m} \Theta_j \Theta_p$$

Exemple: force de centrage



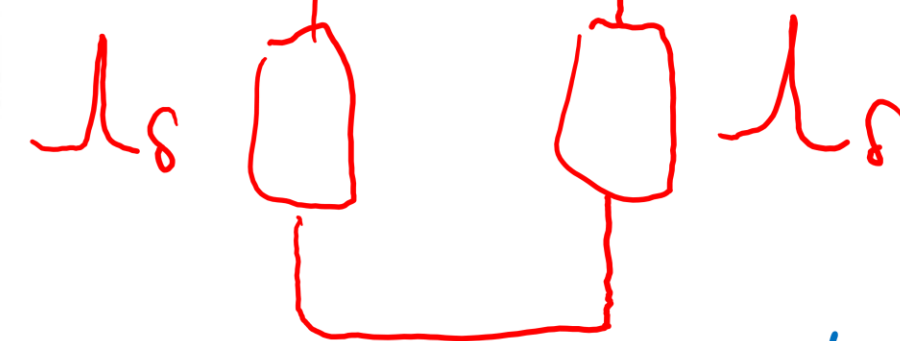
$\mu_{\text{Fer}} = \infty$ pas de fuite, pas de franges

$$F_{\varepsilon} = \frac{1}{2} \sum_{j=1}^K \sum_{p=1}^K \frac{d\mathcal{L}_{jp}}{d\varepsilon} \Theta_j \Theta_p = \frac{1}{2} \frac{d\mathcal{L}_{11}}{d\varepsilon} \Theta_1 \Theta_1$$

$$\Theta_1 = \Theta_b = NI$$

$$\mathcal{L}_{11} = \frac{\Phi_{11}}{\Theta_1} = \frac{\Phi_{bb}}{\Theta_b} \quad \mathcal{L} = \frac{\mathcal{L}_{\delta}}{2}$$

$$\mathcal{L}_{\delta} = \frac{\psi_{\delta}}{I} = \frac{\mu_0 h (a - \varepsilon)}{\delta}$$



$$\Phi_b = \Theta_b \cdot \mathcal{L}_{eq} = \Theta_b \frac{\mathcal{L}_{\delta}}{2}$$

$$F_{\varepsilon} = \frac{1}{2} \frac{d\mathcal{L}_{\delta}}{2d\varepsilon} \cdot N^2 I^2 = \frac{1}{4} \frac{\mu_0 h (-1)}{\delta} N^2 I^2 = - \frac{\mu_0 h N^2 I^2}{4\delta}$$