

Série 4

Réponses à l'exercice 4.1 : SERIES DE FOURIER COMPLEXES

$T = 3$ donc $\omega_0 = 2\pi/T = 2\pi/3$.

1) $f_1(t) = \cos(3\omega_0 t)$ et $c[n] = \begin{cases} 1/2 & \text{si } n = \pm 3, \\ 0 & \text{sinon.} \end{cases}$

2) $f_2(t) = 2$ et $c[n] = \begin{cases} 2 & \text{si } n = 0, \\ 0 & \text{sinon.} \end{cases}$

3) $f_3(t) = \cos(2\omega_0 t + \phi)$ et $c[n] = \begin{cases} e^{j\phi}/2 & \text{si } n = 2, \\ e^{-j\phi}/2 & \text{si } n = -2, \\ 0 & \text{sinon.} \end{cases}$

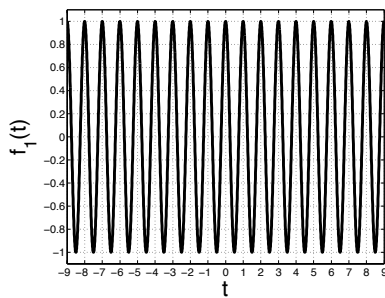
— $\phi = 0$: $f_3(t) = \cos(4\pi t/3)$ et $c[\pm 2] = \frac{1}{2}$,

— $\phi = \pi/2$: $f_3(t) = -\sin(4\pi t/3)$ et $c[\pm 2] = \pm \frac{j}{2}$,

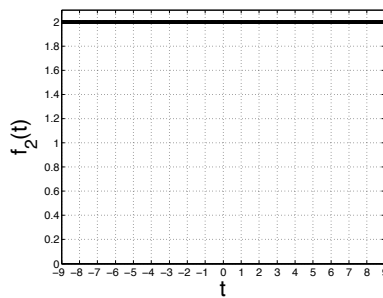
— $\phi = \pi/3$: $f_3(t) = \cos(4\pi t/3)/2 - \sqrt{3}\sin(4\pi t/3)/2$ et $c[\pm 2] = \frac{e^{\pm j\pi/3}}{2}$.

4) $f_4(t) = \cos^2(\omega_0 t)$ et $c[n] = \begin{cases} 1/2 & \text{si } n = 0, \\ 1/4 & \text{si } n = \pm 2, \\ 0 & \text{sinon.} \end{cases}$

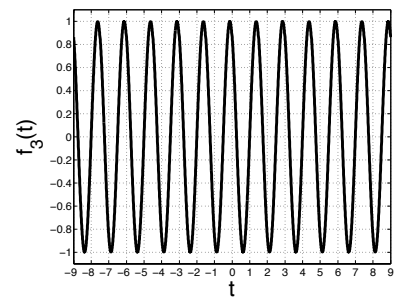
5) Le calcul intégral des coefficients de Fourier donne $c[n] = \frac{\sin(\pi n/3)}{\pi n}$ pour pour $n \neq 0$ et $c[0] = 1/3$.



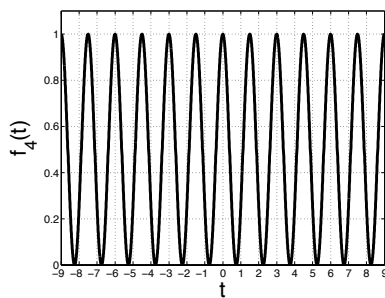
(a)



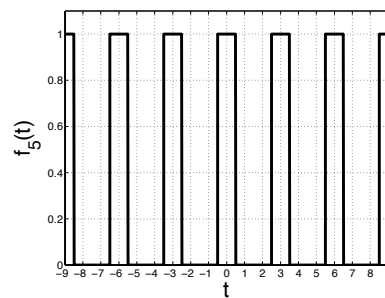
(b)



(c)



(d)



(e)

FIGURE 1 – Graphes des fonctions de l'exercice 4.1.

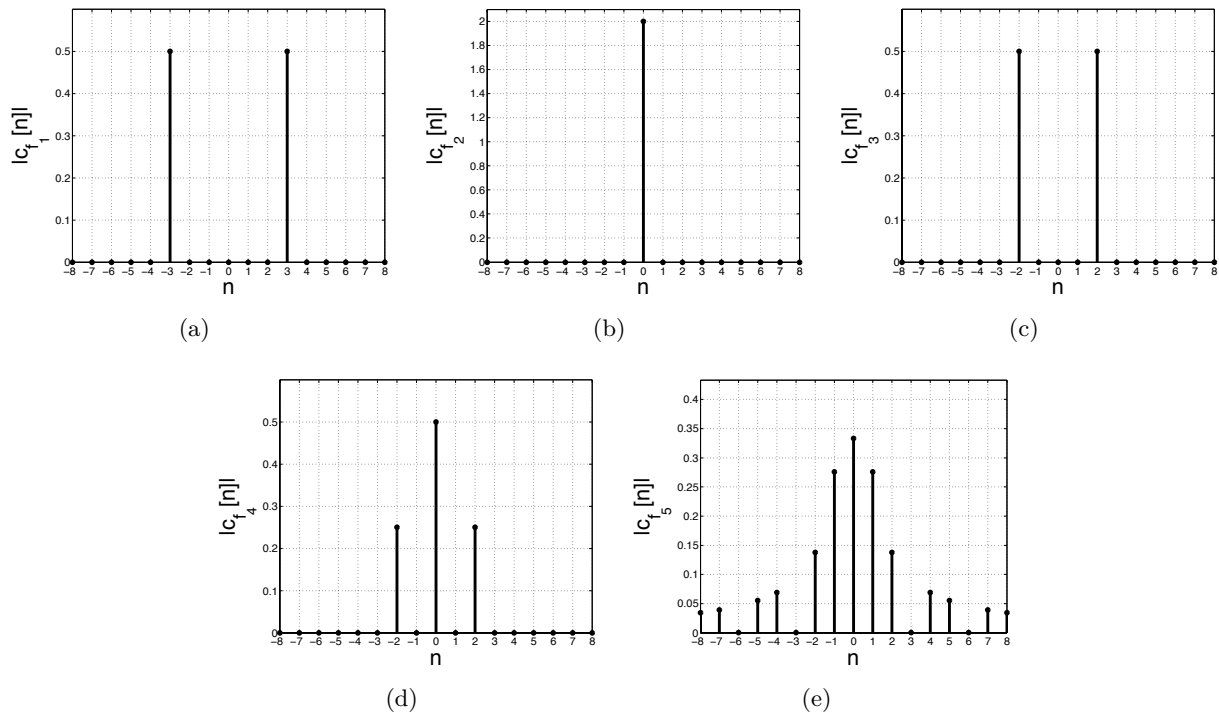


FIGURE 2 – Tracés des modules des coefficients de Fourier des fonctions de l'exercice 4.1.

Réponses à l'exercice 4.2 : CORRÉLATION ET CONVOLUTION

1) $c_{xy}(\tau) = u(\tau)e^{-\tau}(1 - e^{-\tau}).$

2) $c_{xy}(\tau) = \frac{1}{\omega}u(\tau - 3)\sin(\omega(\tau - 3)).$

3) $c_{xy}(\tau) = \left(\frac{\tau^2}{12} + \frac{\tau}{3} + \frac{1}{2}\right)\tau_+^2.$

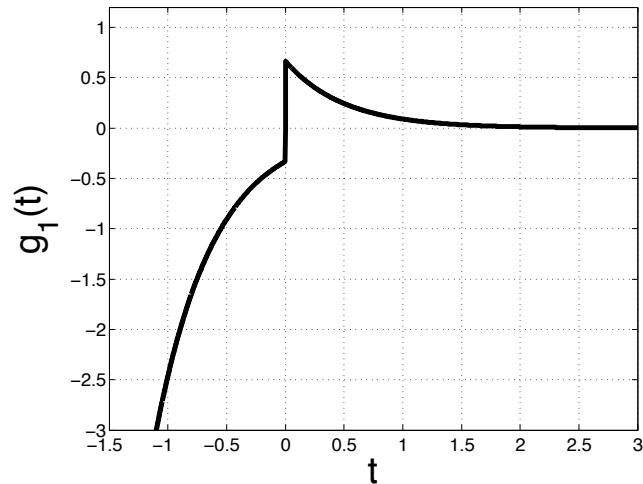
4)

$$c_{xy}(\tau) = \begin{cases} 2\tau - \frac{\tau^2}{2} & \text{si } 0 \leq \tau \leq 1, \\ \frac{3}{2} - 2(\tau - 1) + \frac{1}{2}(\tau - 1)^2 & \text{si } 1 < \tau \leq 2, \\ 0 & \text{sinon.} \end{cases}$$

Réponses à l'exercice 4.3 : FONCTIONS DE GREEN

1) $(D + 2I)\{g\}(t) = \delta(t).$

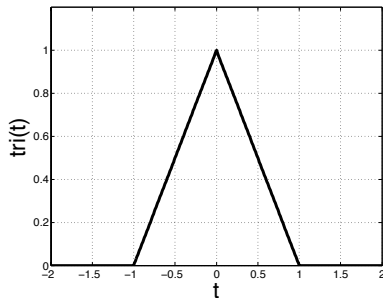
2) $\frac{dg_1}{dt}(t) + 2g_1(t) = \delta(t).$ La fonction g_1 est non-causale.



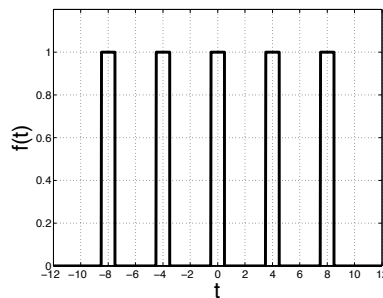
- 3) $T\{f\}(t) = 0$ donc il existe $C \in \mathbb{R}$ tel que $f(t) = Ce^{-2t}$. Comme $f(0) = \frac{1}{3}$, $C = \frac{1}{3}$.
- 4) $g_0(t) = u(t)e^{-2t}$.
 $T\{g_0\}(t) = T\{g_1\}(t) + T\{f\}(t) = \delta(t) + 0 = \delta(t)$ donc g_0 est une fonction de Green de T .
 $g_0(t) = 0$ pour $t < 0$: g_0 est causale.

Réponses à l'exercice 4.4 : CORRÉLATION ET CONVOLUTION

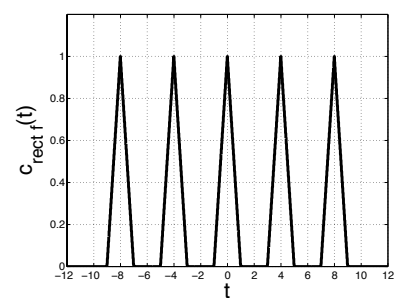
- 1) La fonction $\text{tri}(t)$ est définie à la page 4-52 du cours et est représentée sur la Figure 3(a).
- 2) $f(t) = \sum_{k=-2}^2 \text{rect}(t - 4k)$, voir Figure 3(b).
- 3) $c_{\text{rect},f}(t) = \sum_{k=-2}^2 \text{tri}(t - 4k)$, voir Figure 3(c).
- 4) Le signal de sortie a un support de longueur $16 + L$.



(a)



(b)



(c)

FIGURE 3 – Graphes des fonctions de l'exercice 4.4.