Mechanics of Earthquake

Advanced contact mechanics

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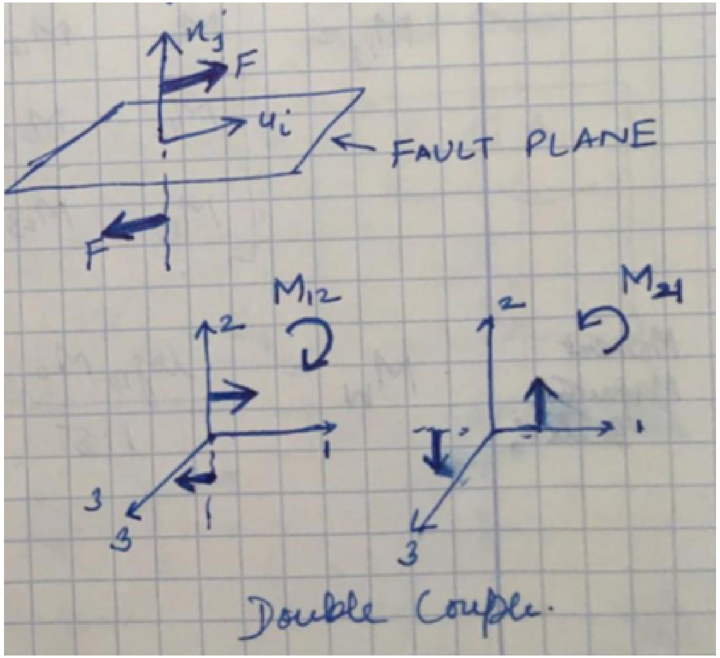
Seismic Moment Tensor

$$M_{ij} = \mu \frac{\Delta u_i n_j + \Delta u_j n_i}{2} A$$

Wrong in book!



Units of M is Nm

μ : *shear modulus*

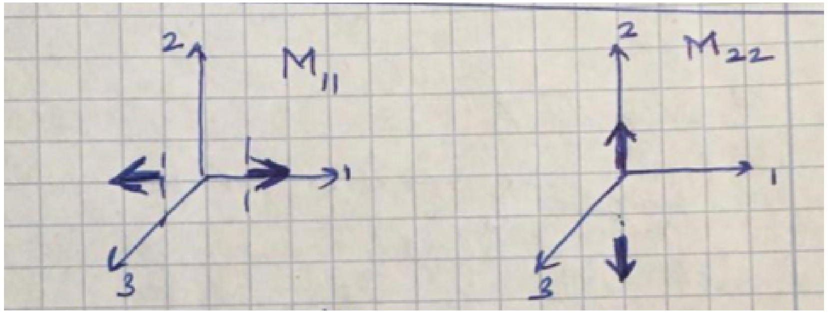
A : *rupture area*

$$M_{ij} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{bmatrix}$$

SYMMETRIC TENSOR

Moment Magnitude Scale:

$$M_W = \frac{\log_{10} M_0}{1.5} - 6.07 \quad (M_0 \text{ in Nm})$$



Moment Magnitude Scale is used now, not Richter Scale to measure earthquakes!

STRESS DROP

$$\Delta\sigma_s = C \mu \frac{\Delta\bar{u}}{\tilde{L}}$$

geometric constant

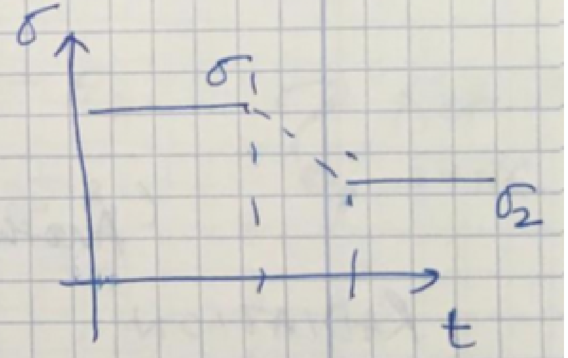
shear Modulus

strain

$$M_0 = \mu \Delta u A$$

can be measured.

Can be measured from SEISMIC DATA.



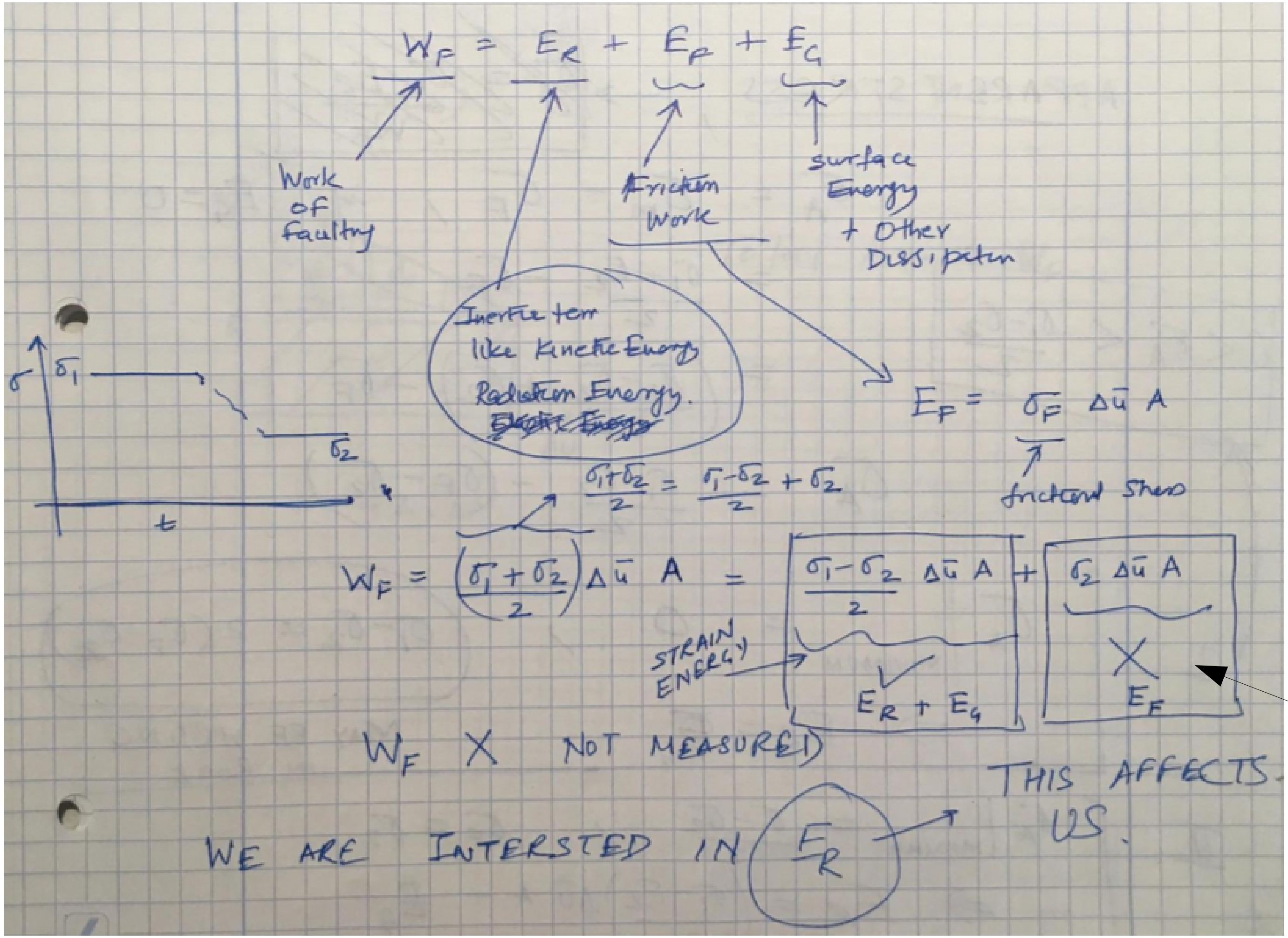
$$\tilde{L} \sim A^{1/2}$$

$$\Delta\sigma_s = C M_0 A^{-3/2} = C M_0 \tilde{L}^{-3}$$

✓ ✓✓✓

we can Measure $\Delta\sigma_s = \sigma_1 - \sigma_2$

Dynamic Energy Balance



Not measurable

Dynamic energy release rate, G is defined using E_G

Some definations ...

Seismic Efficiency,

$$\eta = \frac{E_R}{W_F}$$

$$W_F = \frac{\sigma_1 - \sigma_2}{2}\Delta u A + \sigma_2 \Delta u A$$

Not measurable

Radiation Efficiency,

$$\eta_R = \frac{E_R}{E_R + E_G}$$

Wrong in book!

Apparent Stress

$$\sigma_A = \frac{\sigma_1 + \sigma_2}{2} - \sigma_F$$

$$\sigma_A = \frac{\sigma_1 - \sigma_2}{2} - (\sigma_F - \sigma_2)$$

Wrong in book!

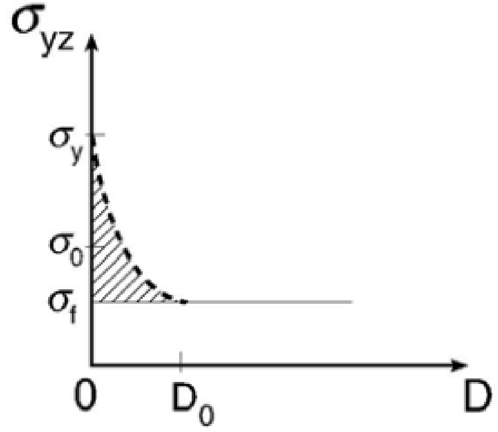
$$\sigma_A = 0,\ if\ \sigma_1 - \sigma_2 = 2(\sigma_F - \sigma_2) => E_R = -E_G$$

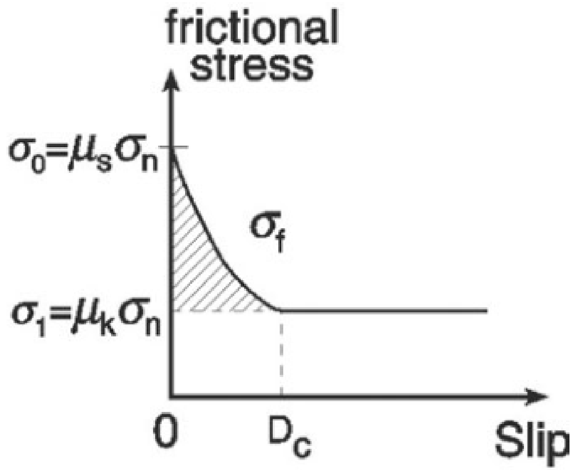
$$\sigma_A = \frac{\sigma_1 - \sigma_2}{2},\ if\ \sigma_F = \sigma_2 => E_R = \frac{\sigma_1 - \sigma_2}{2}\Delta u A - E_G$$

$$\eta_R = \frac{E_R}{E_R + E_G} < \frac{\frac{\sigma_1 - \sigma_2}{2}\Delta u A - E_G}{\frac{\sigma_1 - \sigma_2}{2}\Delta u A}$$

Not sure about inequality!

Crack model vs Friction model for earthquakes





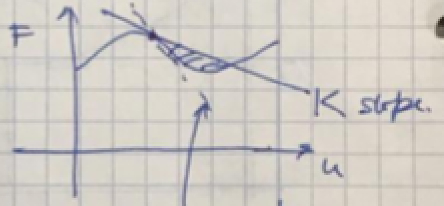
$$D_0 \approx D_c$$

BRITTLE FRACTURE

- $G = G_c$ (CRACK PROPAGATION CRITERIA)
- STRESS AT CRACK TIP $\longrightarrow$ SINGULARITY

STICK-SLIP FRICTION INSTABILITY

- $\left|\frac{\partial F}{\partial u}\right| > K$



$\left|\frac{\partial F}{\partial u}\right|$

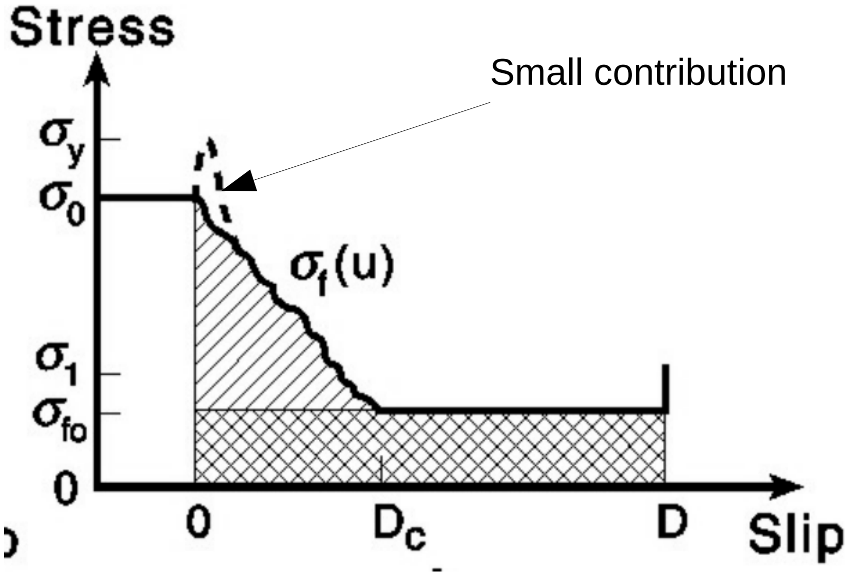
$F = \mu N$

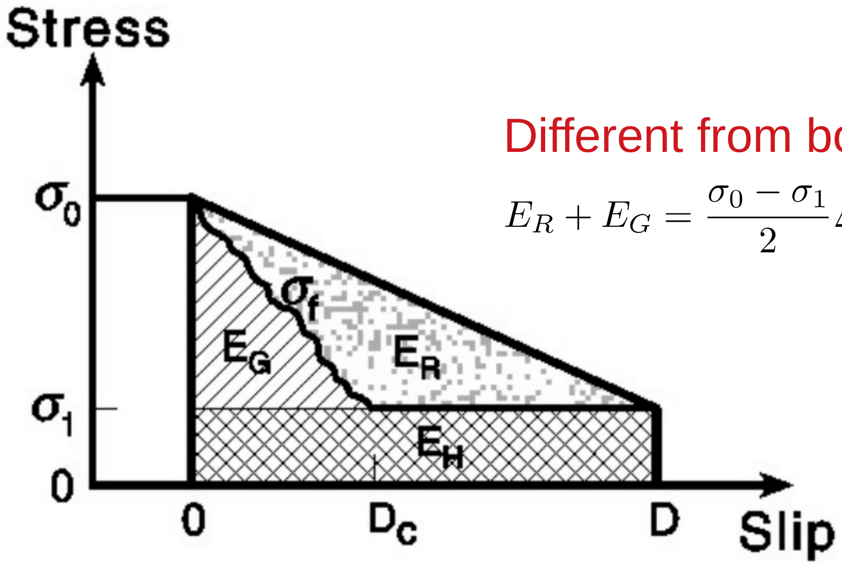
FRICTION COEFF.

- STRESS FINITE

NO ENERGY DISSIPATION

Slip Weakening





Different from book

$$E_R + E_G = \frac{\sigma_0 - \sigma_1}{2} \Delta u A$$

$$W_F = E_R + E_G + E_H$$

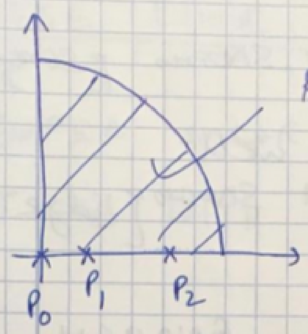
$$\eta_R = \frac{E_R}{E_R + E_G}$$

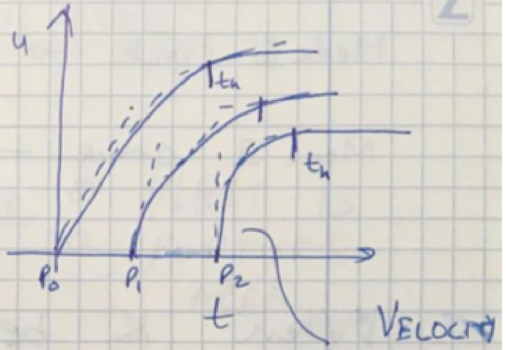
$$\eta_R \approx 1, \quad E_G \approx 0$$

$$\eta_R \ll 1, D_c \ comparable \ to \ D$$

Kanamori, Brodosky 2004

Rupture Propagation

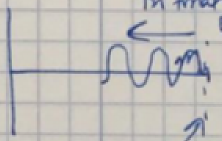




$$K \sim \Delta\sigma_D \, x^{1/2}$$

STRESS INTENSITY FACTOR

crack length



$v_{elocity} \propto K$ (Stress CONCENTRATION)

t_h: REFLECTION OF SELF HEAL PULSE

Mode II Rupture Propagation

When K becomes HIGH ENOUGH,
SO THAT NORMAL STRESS
CAN INTIATE SLIP

Lead to P-waves.

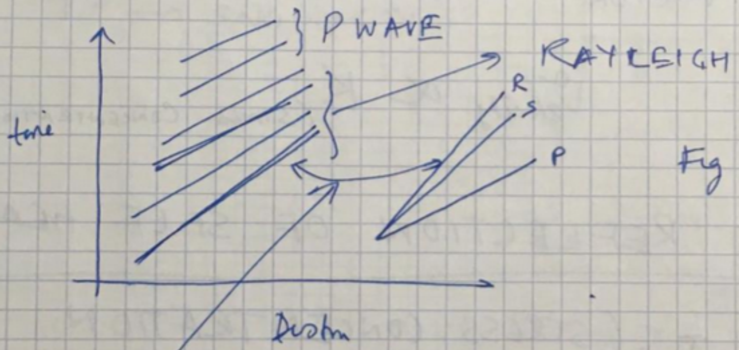


Fig 4.5

PARALLEL TO RAYLEIGH WAVE SPEED

Intially R - wave
AFTER SOME TIME P-WAVE.