Multi-period problems

Multi-period optimisation problems

- **Type of problems**
  - Periodic
    - **same environment, same requirement but independent**
    - **defined by :**
      - conditions(p), requirement(p) for duration(p)
        - » ex. typical days, day/night, summer/winter
    - **optimal operating conditions for each period**
    - **storage is not considered**
  - Operation horizon
    - **defined by :**
      - conditions(t), requirement(t) @ time(t)
        - » ex. daily profiles, batch processes recipe
    - **Scheduling : when to produce what, where**
    - **Optimal management inc. storage**

Problem definition

- **Industrial site**

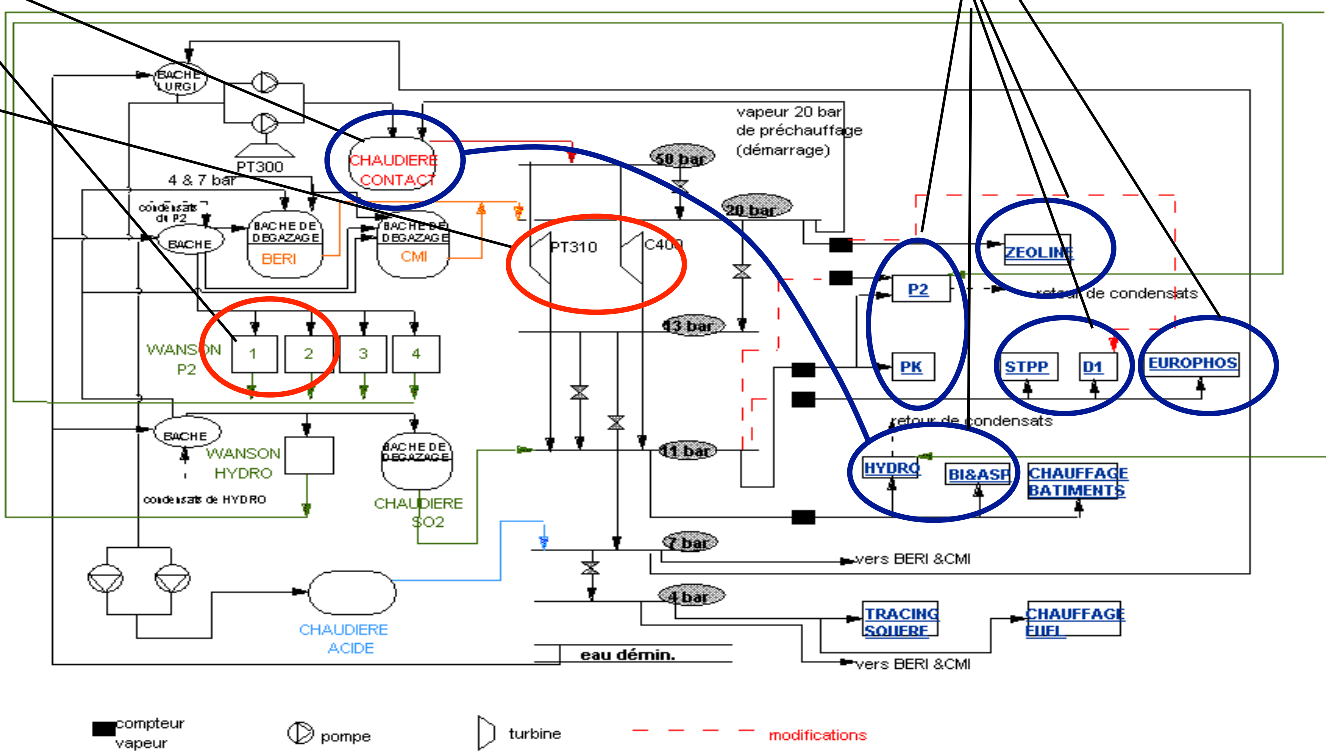
Existing steam network

Import export

Boilers

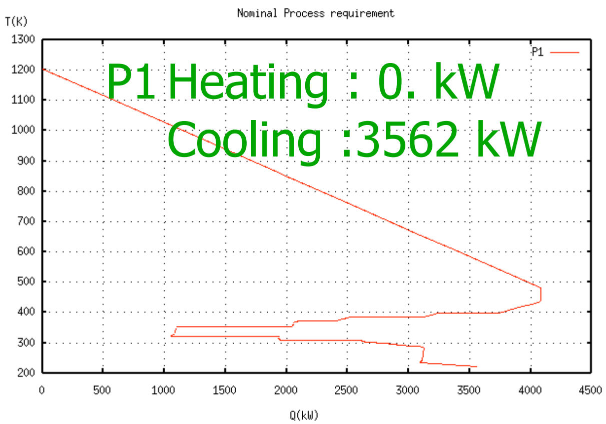
Turbines

5 process sections

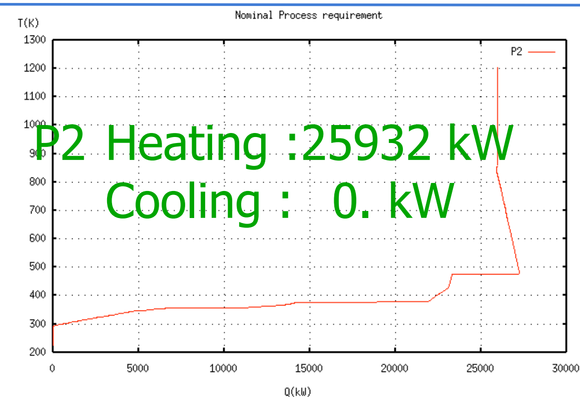


Process Requirements

- **5 processes :**

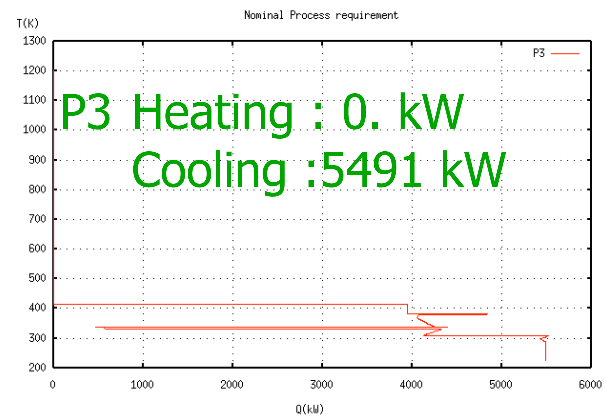


P1 Heating : 0. kW
Cooling :3562 kW

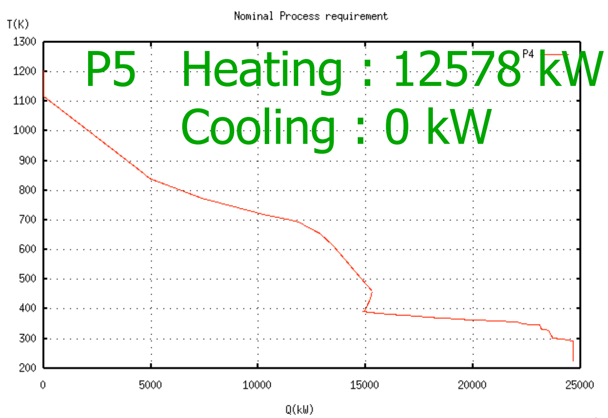


P2 Heating :25932 kW
Cooling : 0. kW

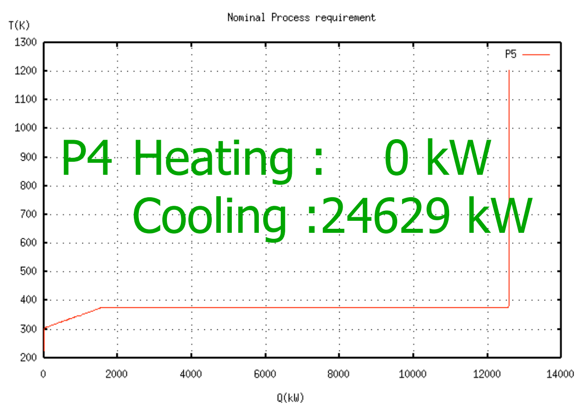
SITE reference
Heating :22560 kW
(38510 kW)
Cooling :17732 kW
(33682 kW)



P3 Heating : 0. kW
Cooling :5491 kW

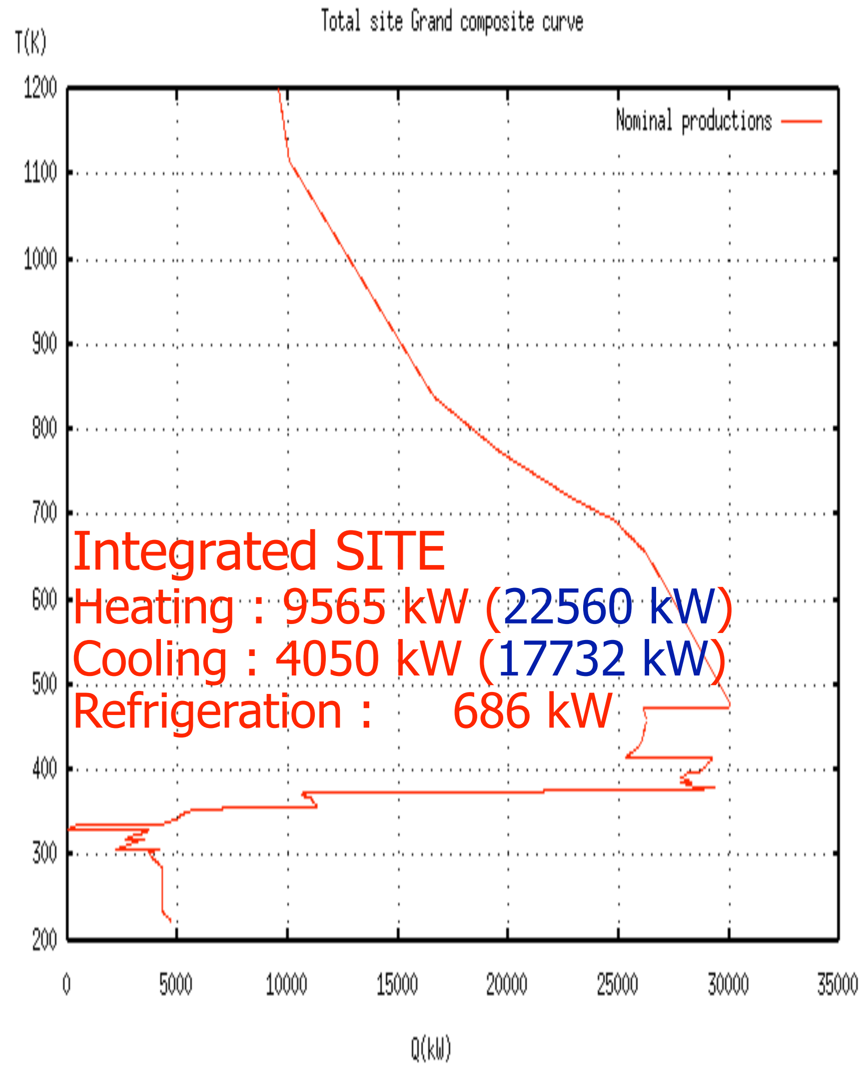


P5 Heating : 12578 kW
Cooling : 0 kW

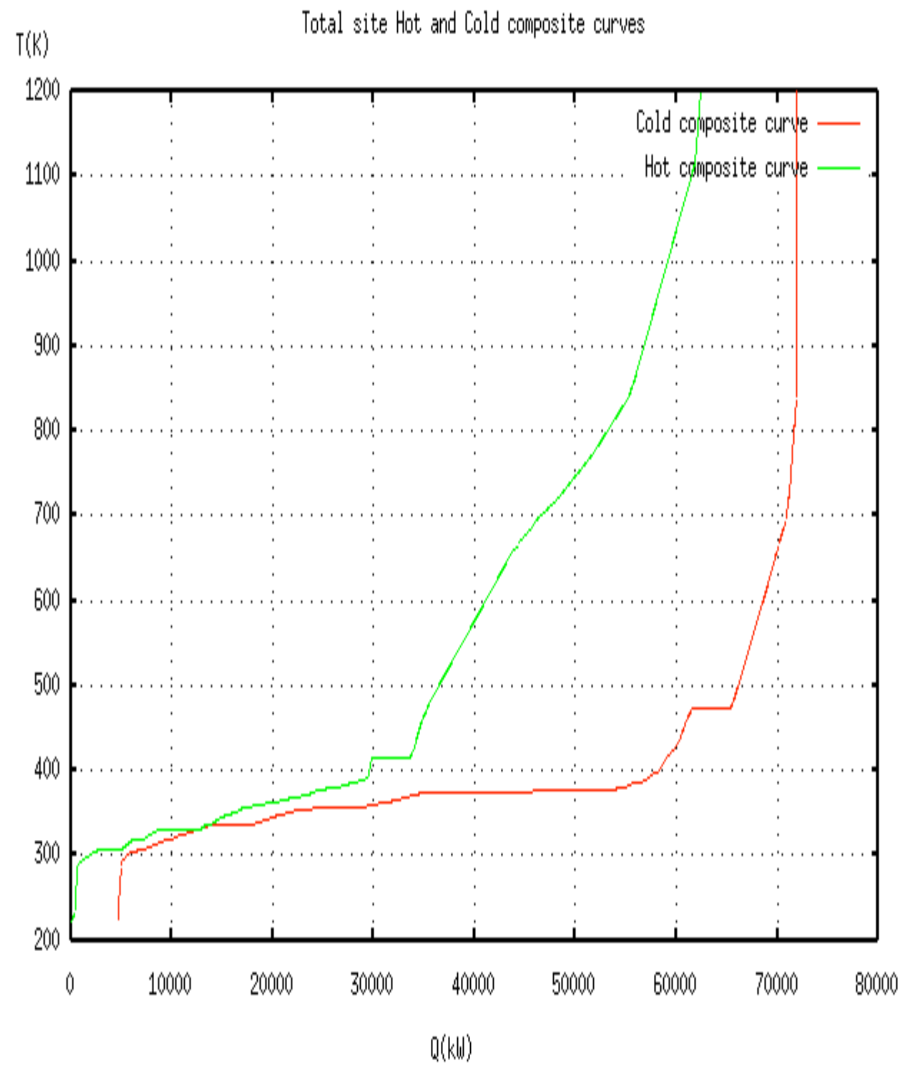


P4 Heating : 0 kW
Cooling :24629 kW

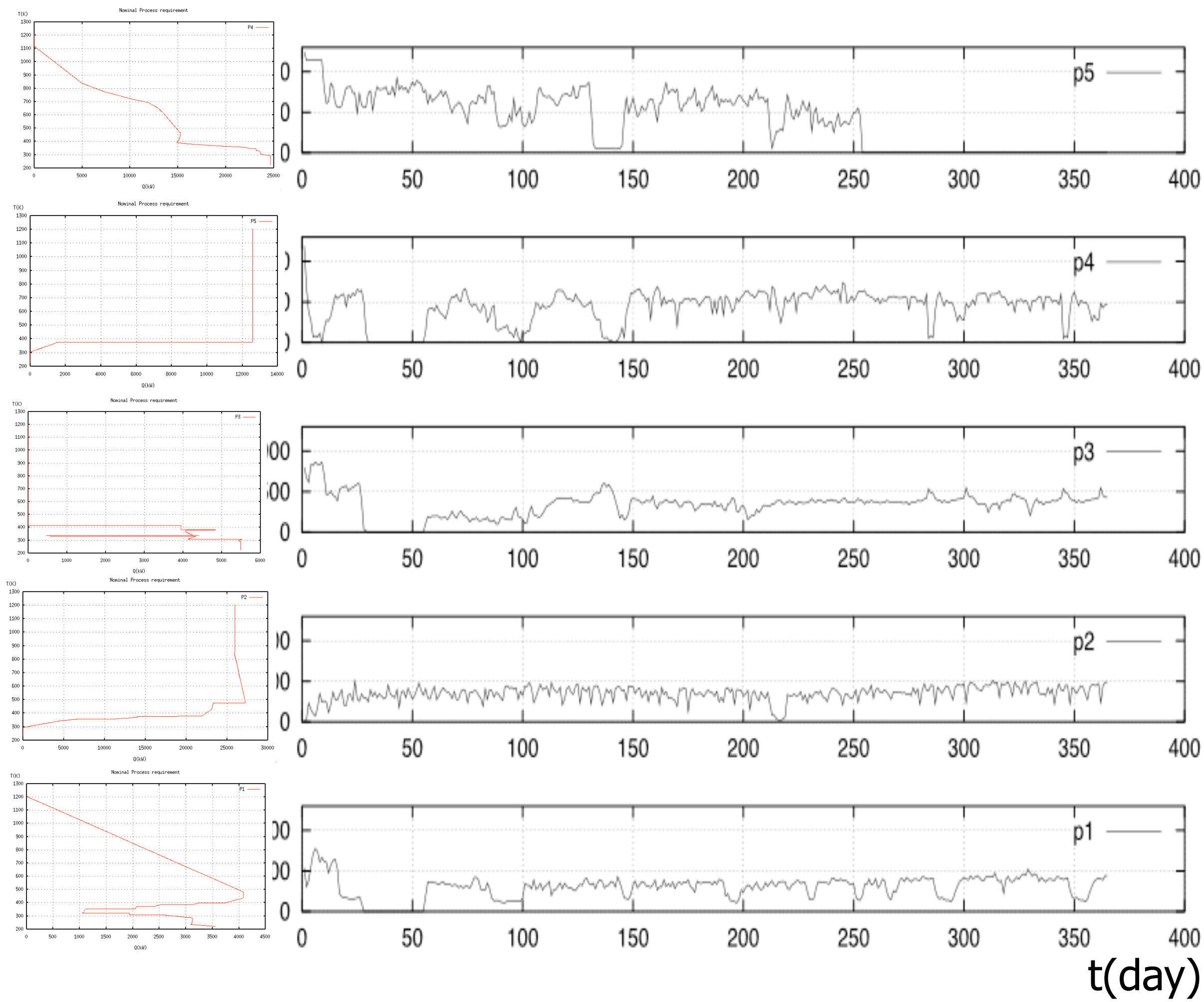
Total site composite



Integrated SITE
Heating : 9565 kW (22560 kW)
Cooling : 4050 kW (17732 kW)
Refrigeration : 686 kW



Production levels variations



Market conditions
Productions shifts
Batch
Efficiency variations
Maintenance
Cleaning procedures

Multi-period principle

- **Process annual operation defined by a set of operating periods**
  - Limited number of sets : n_p
  - For each set we have to define the operating conditions of utility system
- **Definition of operating period p**
  - $L_{j,p}$: production levels or conditions for process j
  - t_p : operating time (h/years) : probability of appearance over the lifetime of the installation
- **Assume no heat storage**
  - Sequence of operations not constraining

Load scenarios calculation

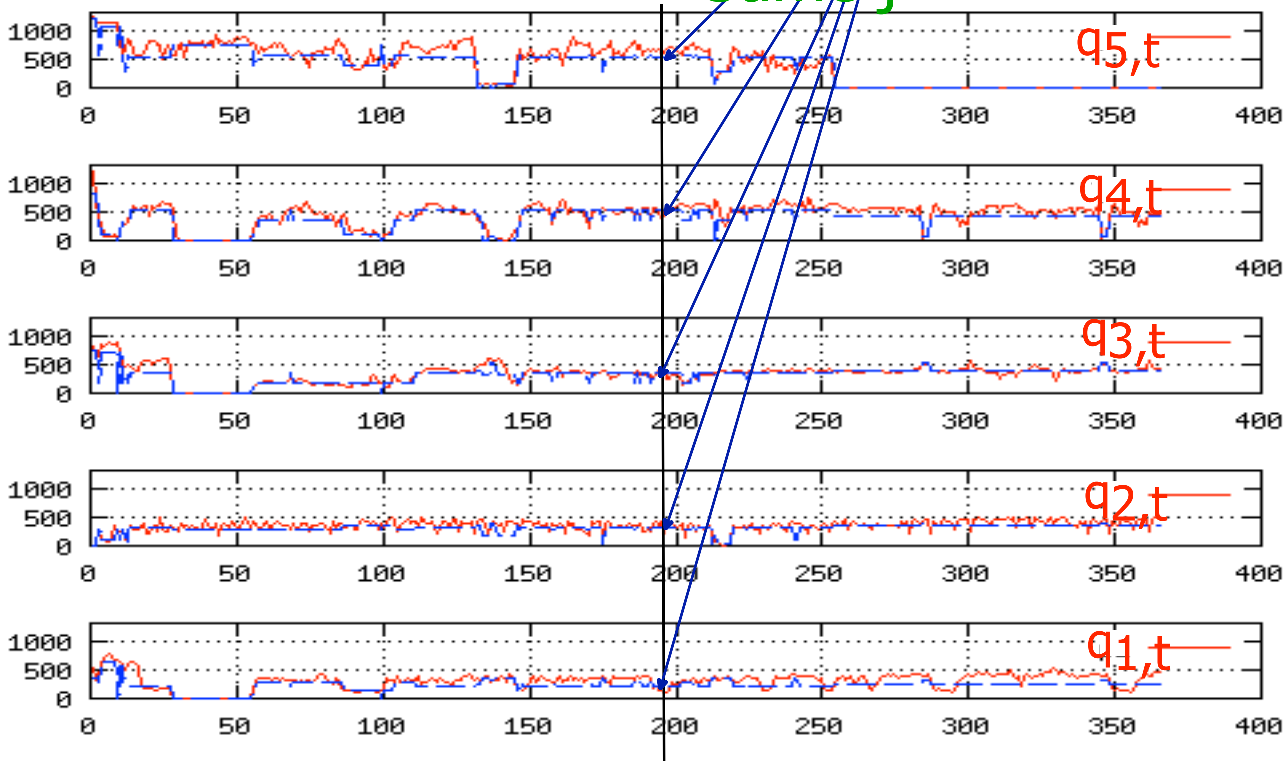
$$min_{L_{j,p}} \sum_{t=1}^{n_t} \left\{ \min_{p=1,\ldots,n_p} \sum_{j=1}^{n_j} \left(\frac{L_{j,p} - q_{j,t}}{\omega_j} \right)^2 \right\}$$

n_p=5

n_t=8760

n_j=10

Same j



Load scenario model

- **Problem characteristics**
  - Discontinuous
  - Non-linear
  - Multi-modal

$$min_{L_{j,p}} \sum_{t=1}^{n_t} \left\{ \min_{p=1,\ldots,n_p} \sum_{j=1}^{n_j} \left(\frac{L_{j,p} - q_{j,t}}{\omega_j} \right)^2 \right\}$$

- **Solved by a Evolutionary Algorithm**
  - Easy implementation
  - Robustness
  - Multi-objectives
    - **Validity for each process**
    - **Trade-off**
      - Number of representing levels
      - Individual errors
      - Overall error
  - ? Computation time

Alternative

Multi-period levels

Minimum energy requirements and levels of operations in the different periods

	MER hot (kW)	MER cold (kW)	MER frg (kW)	time h/year
L1	22581.	0.	155.	432
L2	0.	18119.7	1027.8	48
L3	19814.	987.	347.	192
L4	10146.	1411.	380.	264
L5	27771.	0.	0.	720
L6	13311.	4897.	751.	144
L7	0.	10526.	677.	240
L8	12252.	173.	311.	1224
L9	7061.7	831.	310.	2904
L10	8539.	577.	278.	2592
Average	10270.	385.6	261	8760.

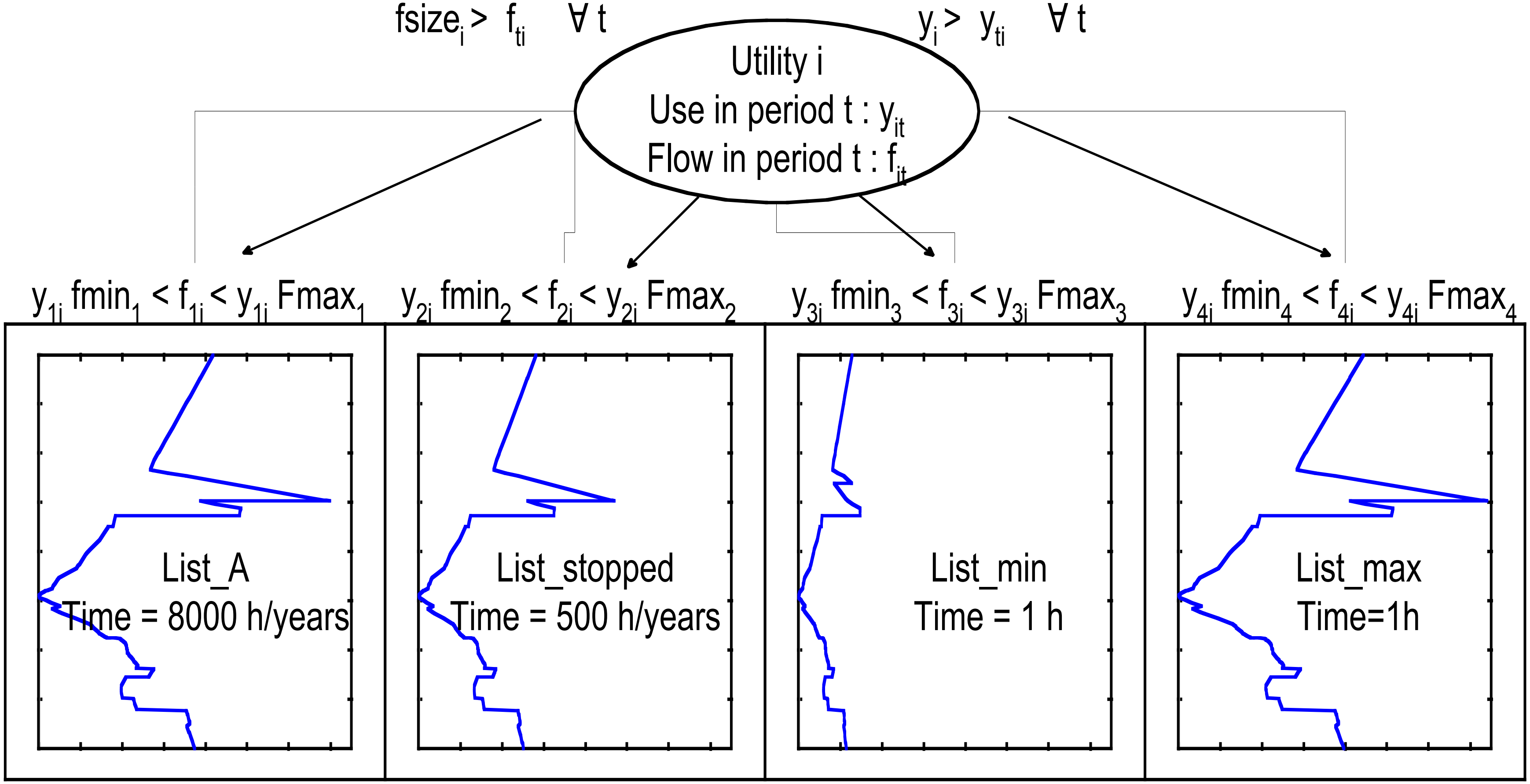
endothermic → L1

exothermic → L7

Multi-period model principle

$$\text{Objective function} = \sum_i y_i \; (Inv_i(fsize_i)) + \sum_i \; \sum_t f_{ti} * C_{t1} * Time_t$$

$fsize_i > f_{ti} \quad \forall t$ $\qquad$ $y_i > y_{ti} \quad \forall t$



Multi-period problem

Generic formulation

$$\underset{y_t, x_t, y, s}{Min} \sum_{t=1}^{n_t} t_t * c(x_t, s) + I(y, s)$$

Submitted to :

$$\begin{aligned}
&h_t(x_t, s) = 0 && \forall t = 1, ..., n_t \\
&g_t(x_t, s) \geq 0 && \forall t = 1, ..., n_t \\
&y_t \leq y \qquad x_t \leq x && \forall t = 1, ..., n_t \\
&y_t, y \epsilon \{0, 1\} && \forall t = 1, ..., n_t
\end{aligned}$$

Only s (size => investment) and y (investment decisions) are shared between periods

Multi-period MILP problem formulation

Operating cost

$$\min_{R_r^p, y_w^p, f_w^p, y_w^{max}, f_w^{max}} \sum_{p=1}^{n_p} \left(\sum_{w=1}^{n_w} (C2_w^p f_w^p) + Cel^p Wel^p - Cel_v^p Wel_s^p \right) * t_p$$

Linearised investment

$$+ \sum_{w=1}^{n_w} (C1_w y_w^{max}) + \frac{1}{\tau} \sum_{w=1}^{n_w} (ICF_w y_w^{max} + ICP_w f_w^{max})$$

$\forall p$ For each period

Heat Cascade

$$\sum_{w=1}^{n_w} f_w^p q_{w,r} + \sum_{i=1}^{n} Q_{i,r} * L_{i,p} + R_{r+1}^p - R_r^p = 0$$

$$R_1^p = 0, R_{n_{r+1}}^p = 0, R_r^p \geq 0$$

Electricity

Consumption

$$\sum_{w=1}^{n_w} f_w^p w_w + Wel^p - L_{c,p} * Wc \geq 0$$

Production

$$\sum_{w=1}^{n_w} f_w^p w_w + Wel^p - Wel_s^p - L_{c,p} * Wc = 0$$

$$Wel^p \geq 0, Wel_s^p \geq 0$$

Gas turbines
Combustion
Steam network …

Part load operation

$$fmin_w y_w^p \leq f_w^p \leq fmax_w y_w^p$$

$$y_w^p \in 0, 1$$

Size $\boxed{f_w^{max} - f_w^p \geq 0}$

Decision $\boxed{y_w^{max} - y_w^p \geq 0}$

Application

- **5 Processes (60 streams)**
- **10 operation levels**
- **4 Gas turbines**
- **Refrigeration system**
- **Steam network**
- **Heat pump**

- **MILP problem characteristics**
  - 1900 constraints
  - 1600 variables
  - 50 decision variables

Part load optimal operation target

Details of the solution computed with $GT1_{int}$

	GT (MWh)	CHP power kW	CHP total (MWh)	Fuel (MWh)	CHP Efficiency (%)	Condensing kmol/s
L1	2382	11493	4965	15867	88.7	0.013
L2	265	13660	656	808	81.1	0.34
L3	1059	10879	2089	6457	85.2	0.0
L4	1456	8407	2219	5618	79.3	0.0
L5	3970	12404	8931	30971	90.0	0.02
L6	794	10775	1552	3900	82.4	0.0
L7	1323	11178	2683	4043	66.3	0.27
L8	6750	10379	12704	30735	85.0	0.0
L9	16015	10158	29499	57981	81.1	0.0
L10	14295	9647,6	25006	54240	80.8	0.0
Total	**48311**		**90303**	**210622**	**82.2**	