Process integration

Heat exchanger network design

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Heat exchangers network synthesis

Find a heat exchangers network that satisfies:
- MER
- Minimum number of units
- Minimum investment
- Other criteria

- Which hot stream with which cold stream ?

- What is the heat exchanged ?

- What is the structure : serial or //, ...

Above pinch point

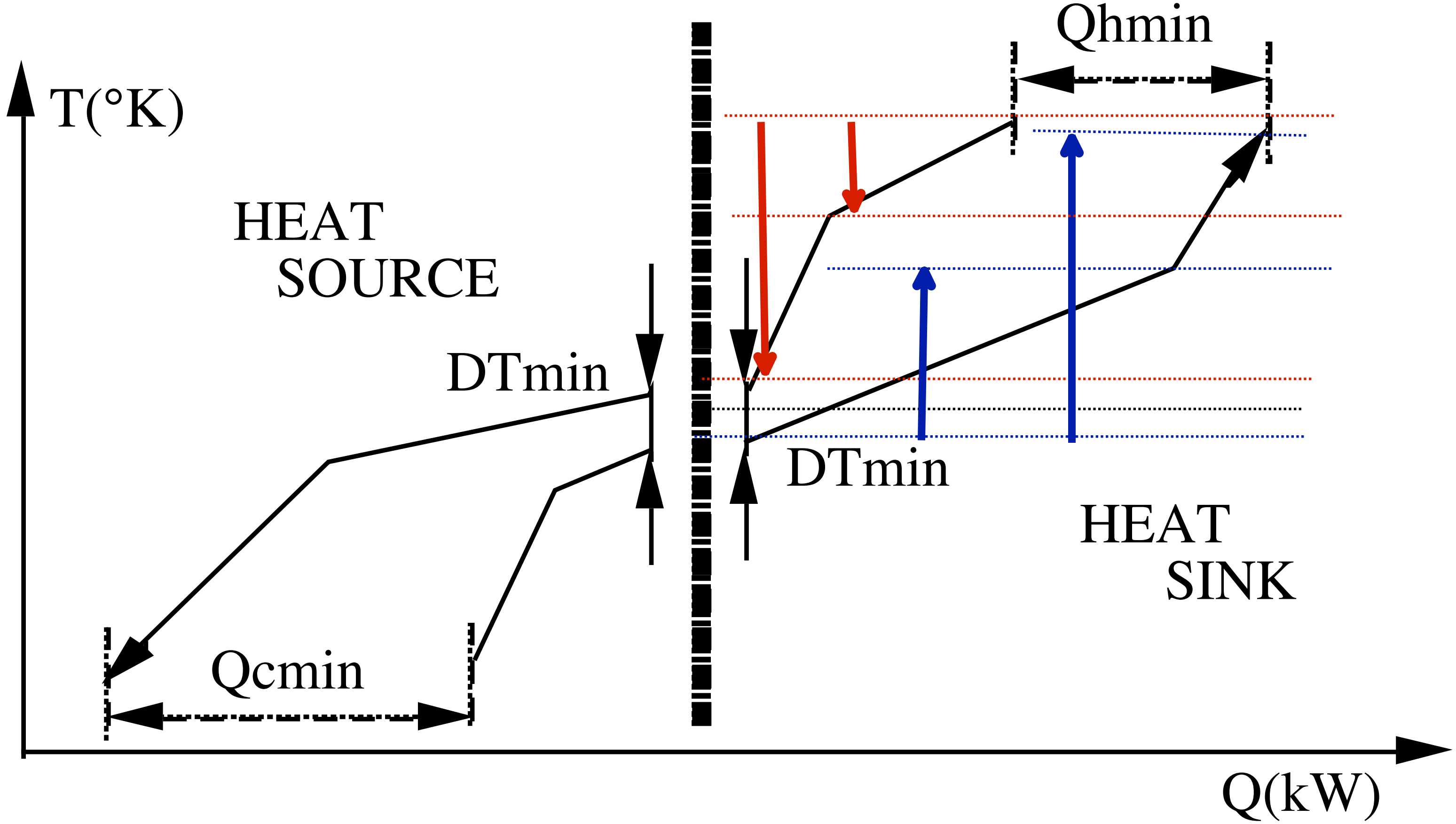
Drive hot streams to the pinch point without cold utility

Below pinch point

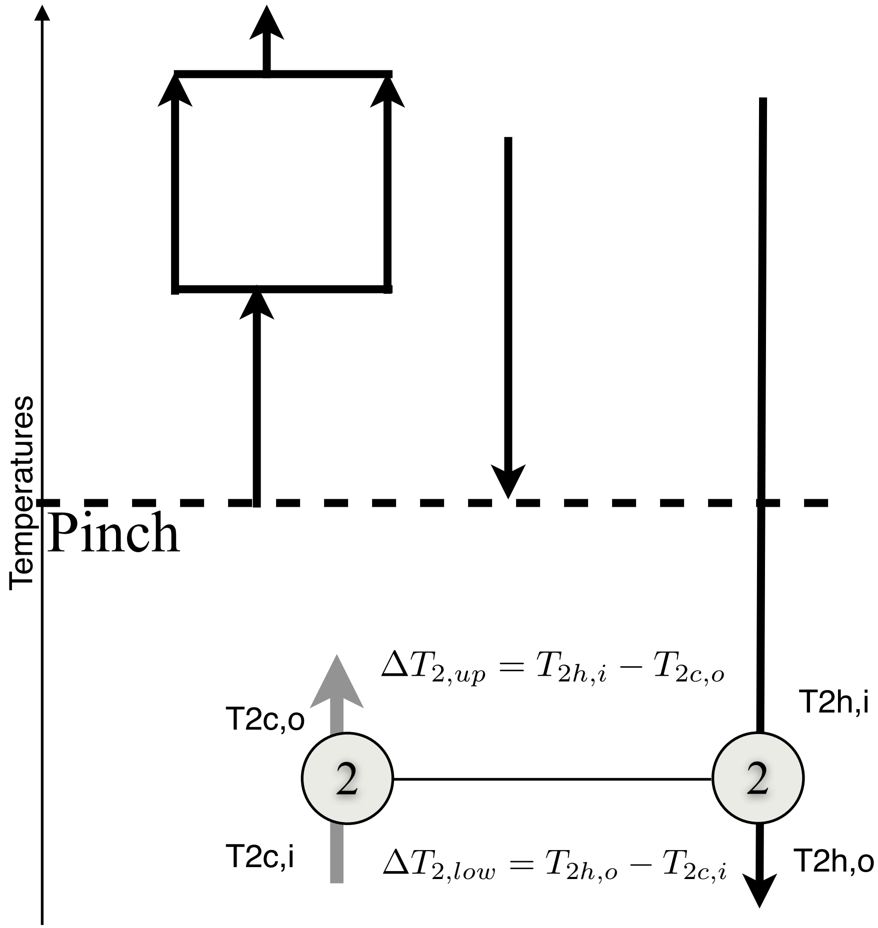
Drive the cold streams to the pinch point without hot utility

Pinch point

Two independent sub-systems



Grid representation of HEN



$$\dot{Q}_2 = \dot{M}_{2h} cp_{2h} (T_{2h,i} - T_{2h,o})$$

$$\dot{Q}_2 = \dot{M}_{2c} cp_{2c} (T_{2c,o} - T_{2c,i})$$

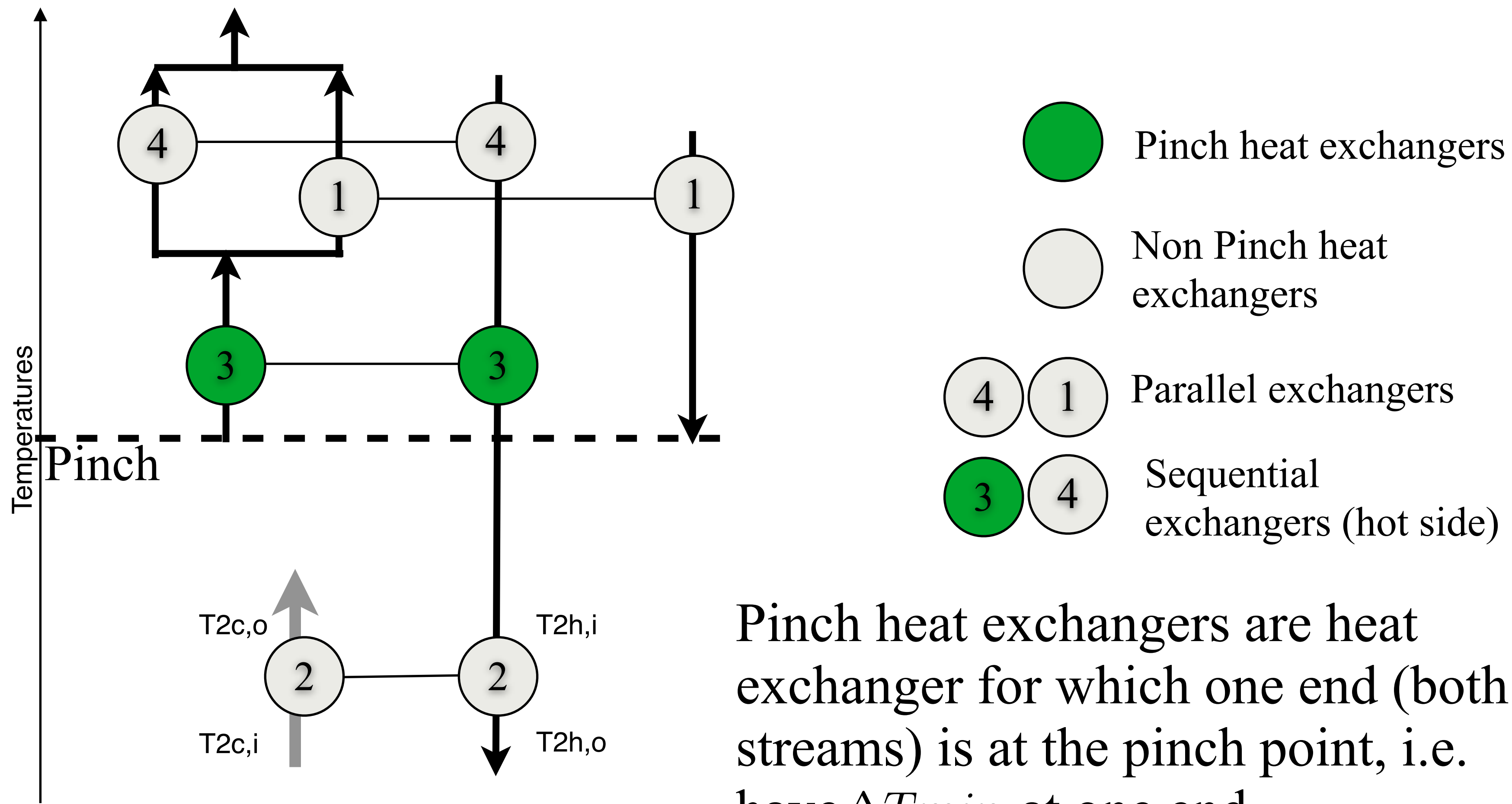
$$\Delta T_{lm,2} = \frac{\Delta T_{2,up} - \Delta T_{2,low}}{ln(\frac{\Delta T_{2,up}}{\Delta T_{2,low}})}$$

$$A_2 = \frac{\dot{Q}_2 \cdot (\frac{1}{\alpha_{2h}} + \frac{1}{\alpha_{2c}})}{\Delta T_{lm,2}}$$

Streams are vertical lines with a clear location of the pinch point (do not use real temperature scales as streams might have infinite cp when phase change occur).
Pinch streams are streams that are start from or cross the pinch
Heat exchangers are represented by horizontal lines with 2 circles identifying the connected streams. Counter current heat exchangers have therefore the hot end at the top and the cold end at the bottom, logmean temperature difference are easy to identify

Grid representation of HEN

Streams with inlet and outlet temperatures



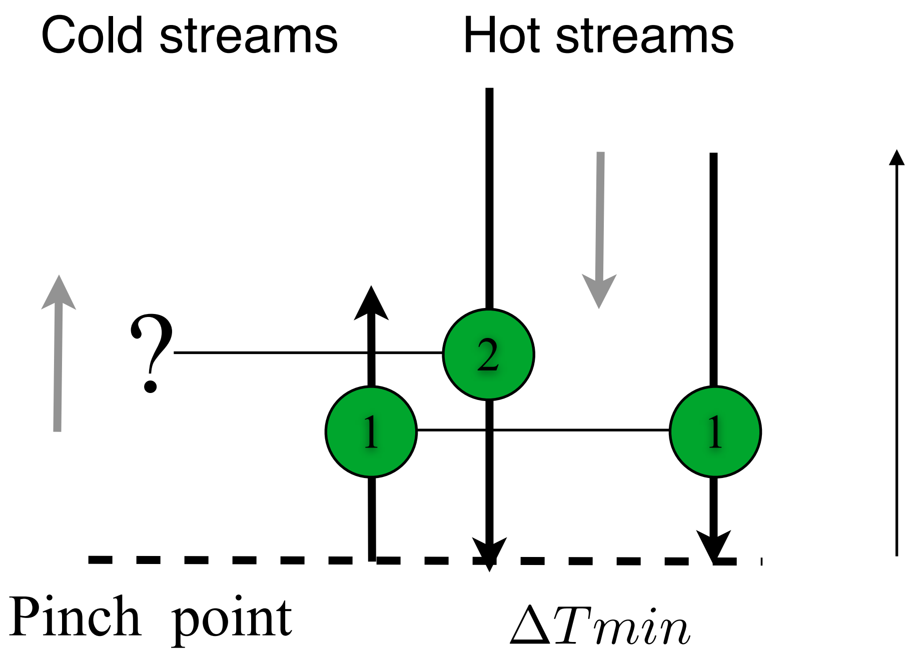
Pinch heat exchangers are heat exchanger for which one end (both streams) is at the pinch point, i.e. have $\Delta Tmin$ at one end

Feasibility rules for heat exchanger placement

- **At the pinch point : i.e. for pinch streams**
  - Temperature difference is known : DTmin
  - Above (or Below) the goal is known
    - above : cool down to pinch without cold utility
    - below : heat up to pinch without hot utility
- **=> Feasibility rules for pinch heat exchangers**

Number of streams rule

Above the pinch :
Start from the pinch point and go towards increasing temperatures
The goal is to cool down hot streams to the pinch without cold utility



$$N_{pinch,c} \geq N_{pinch,h}$$

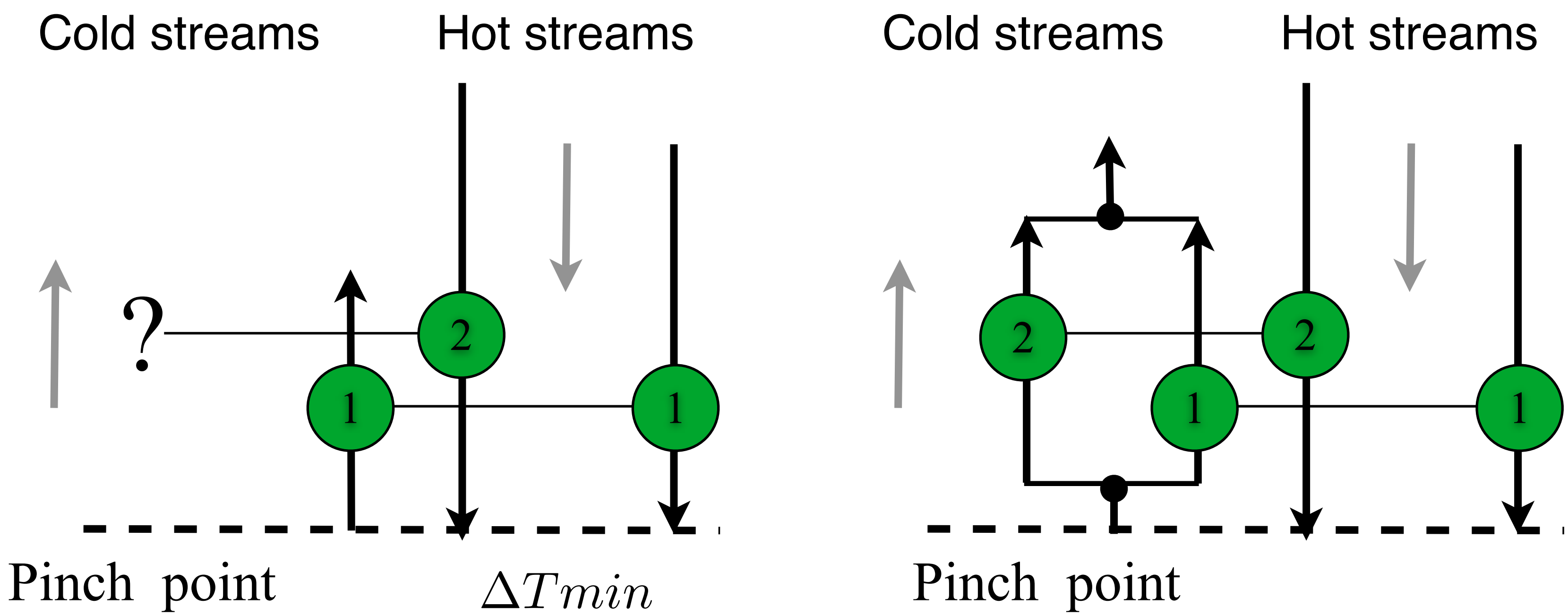
Direction of calculation : from the pinch to the highest temperatures
Note for the key streams (hot streams) we start from the target temperature , i.e. reach the pinch and go to highest temperatures, i.e. to the start temperature. the reverse for the cold streams, we start from th epinch or start temperature and go to the target (finishing with the hot utility).

Number of streams solution : splitting cold streams

Above the pinch :

Start from the pinch point and go towards increasing temperatures

The goal is to cool down hot streams to the pinch without cold utility



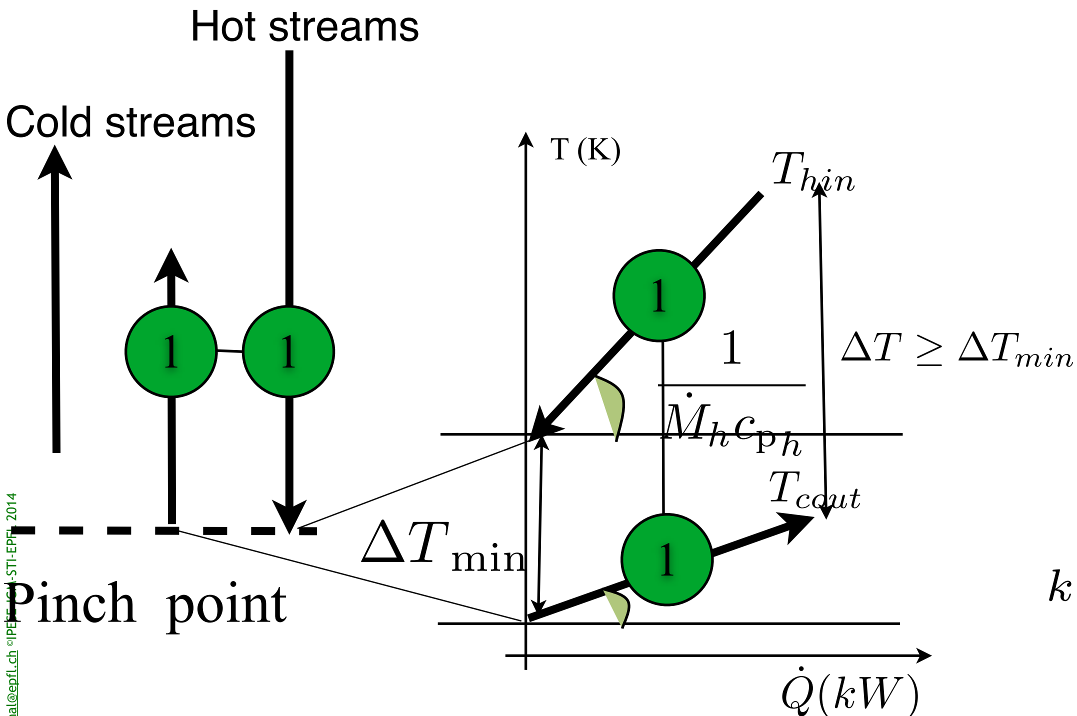
$$N_{pinch,c} \geq N_{pinch,h}$$ otherwise split 1 cold stream

At the pinch we know that the temperature difference is the ΔT_{min} , it can therefore be used as the starting point for the calculation in order to calculate the other side of the counter current heat exchanger

CP Rule : for pinch exchangers above the pinch

Above the pinch :
Start from the pinch point and go towards increasing temperatures
The goal is to cool down hot streams to the pinch without cold utility

Connexion feasibility between c and h



$$\dot{M}c_{\mathrm{p}_h} \leq \dot{M}c_{\mathrm{p}_c}$$

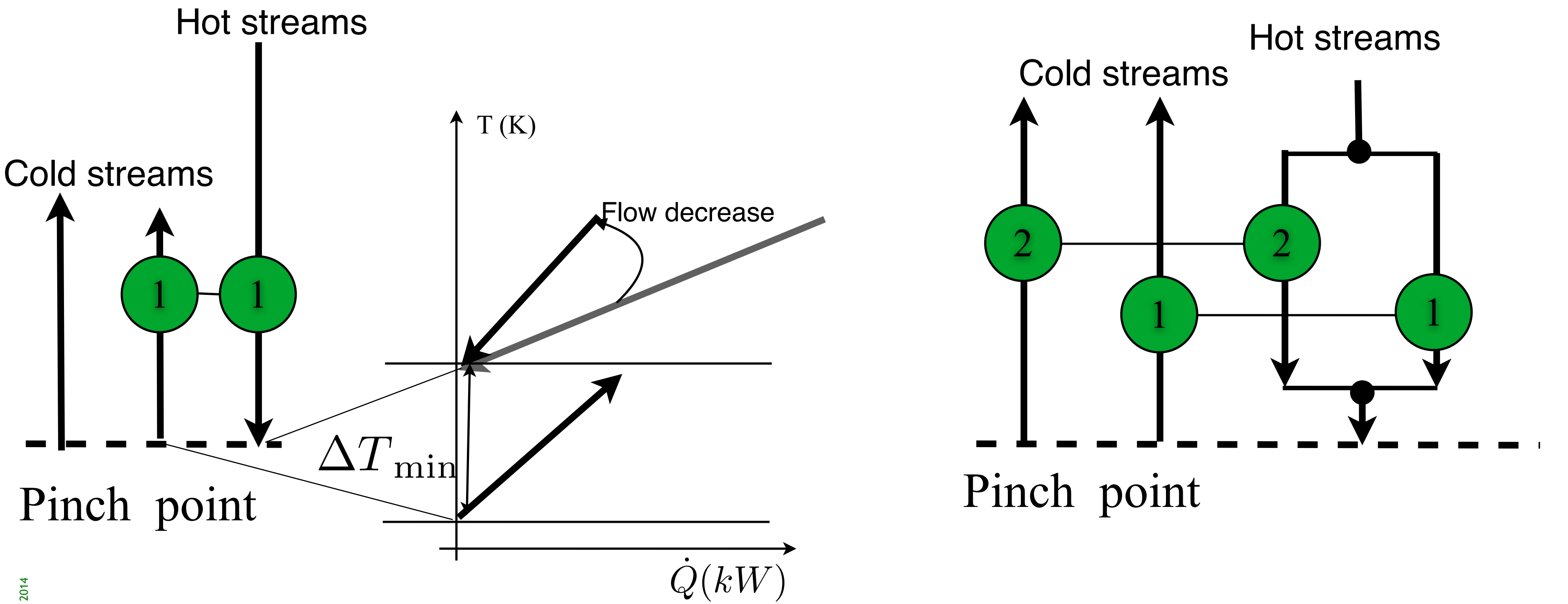
AND

$$\sum_{k=1,k\neq h}^{n_k} \dot{M}c_{\mathrm{p}_k} \leq \sum_{j=1,j\neq c}^{n_{k-1}} \dot{M}c_{\mathrm{p}_j}$$

At the pinch we know that the temperature difference is the ΔT_{min} , it can therefore be used as the starting point for the calculation in order to calculate the other side of the counter current heat exchanger

CP Rule : for pinch exchangers above the pinch

If Cp rule not satisfied split 1 hot stream



This is reducing the Mcp of hot stream in heat exchanger
CP rule is satisfied
Is Number of streams rule still valid ?

Feasibility rules for heat exchanger placement

- **At the pinch point : i.e. for pinch streams**
  - Temperature difference is known : DTmin
  - Above (or Below) the goal is known
    - above : cool down to pinch without cold utility
    - below : heat up to pinch without hot utility
- **=> Feasibility rules for pinch heat exchangers**
  - Number of streams
  - Cp rule

"this allows to select in the list of streams the one that could be potentially connected"

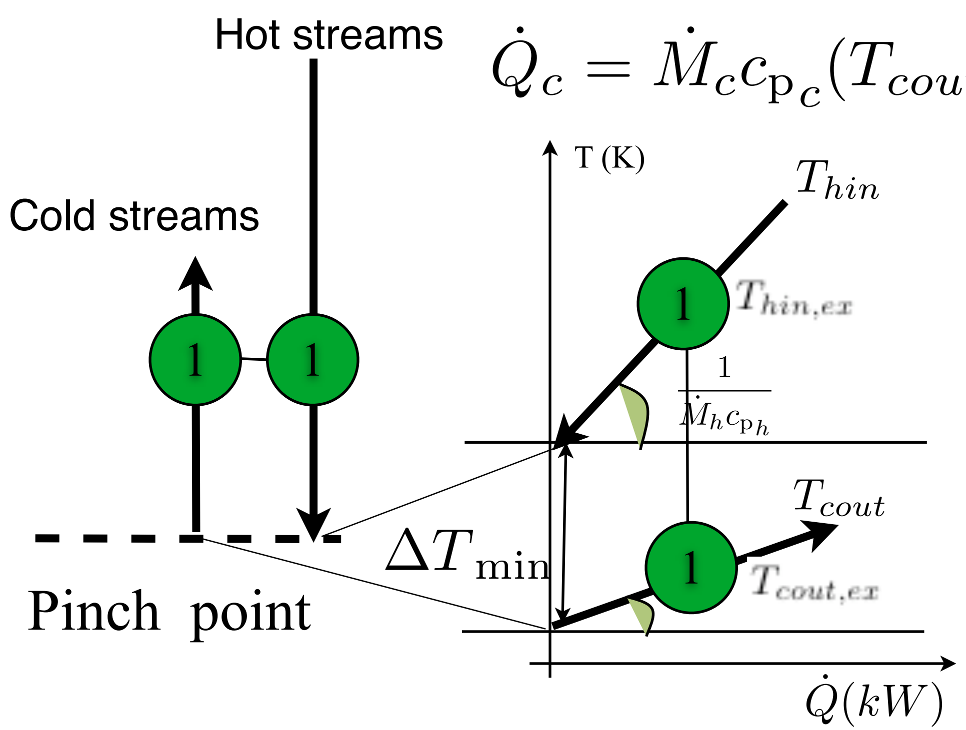
  - What is the heat load ?

Tick off rule : satisfy the heat load of one stream

In order to satisfy the minimum number of units rule

$$\dot{Q}_h = \dot{M}_h c_{p_h}(T_{hin} - (T_p^* + \Delta T_{\min}/2))$$

$$\dot{Q}_c = \dot{M}_c c_{p_c}(T_{cout} - (T_p^* - \Delta T_{\min}/2))$$



Above the pinch
Calculation starts from the pinch and goes to higher temperatures

Both side !!!

We calculate the inlet conditions of the hot stream and the outlet conditions of the cold stream

$$\dot{Q} = min(\dot{M}_c c_{p_c}(T_{cout} - (T_p^* - \Delta T_{\min}/2)), \dot{M}_h c_{p_h}(T_{hin} - (T_p^* + \Delta T_{\min}/2)))$$

$$\Rightarrow T_{hin,ex}, T_{cout,ex}$$

$$T_{hin,ex} = T_p^* + \Delta T_{\min}/2 + \frac{\dot{Q}}{\dot{M}_h c_{p_h}} \qquad T_{cout,ex} = T_p^* - \Delta T_{\min}/2 + \frac{\dot{Q}}{\dot{M}_c c_{p_c}}$$

Heuristic rules

1 - Order the streams by decreasing Cp

Goals :

Above the pinch point: cool down the hot streams without cold utilities.

Below the pinch point : heat up the cold streams without hot utilities.

Start with pinch exchangers

2 - verify feasibility rules and split if no connection found

3 - The heat load is calculated to satisfy the heat load of one of the two stream involved : “tick-off”

- work from the pinch

4 - Place the utilities at the end of the streams (control purposes)

Remaining problem analysis

Remaining problem

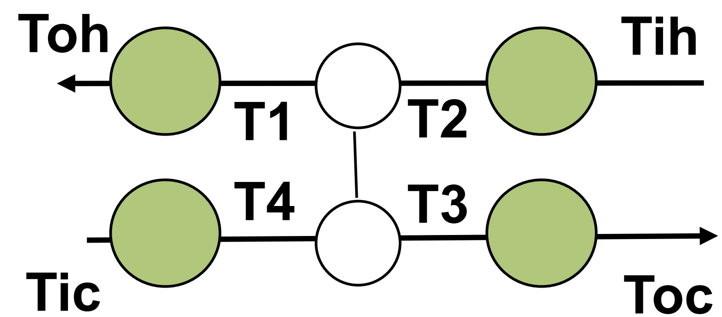
Initial problem:

Hot stream : Tih -> Toh

Cold stream: Tic -> Toc

=> MER0

Place a heat exchanger



New Hot streams:

Tih -> T2

T1 -> Toh

New Cold streams:

Tic -> T4

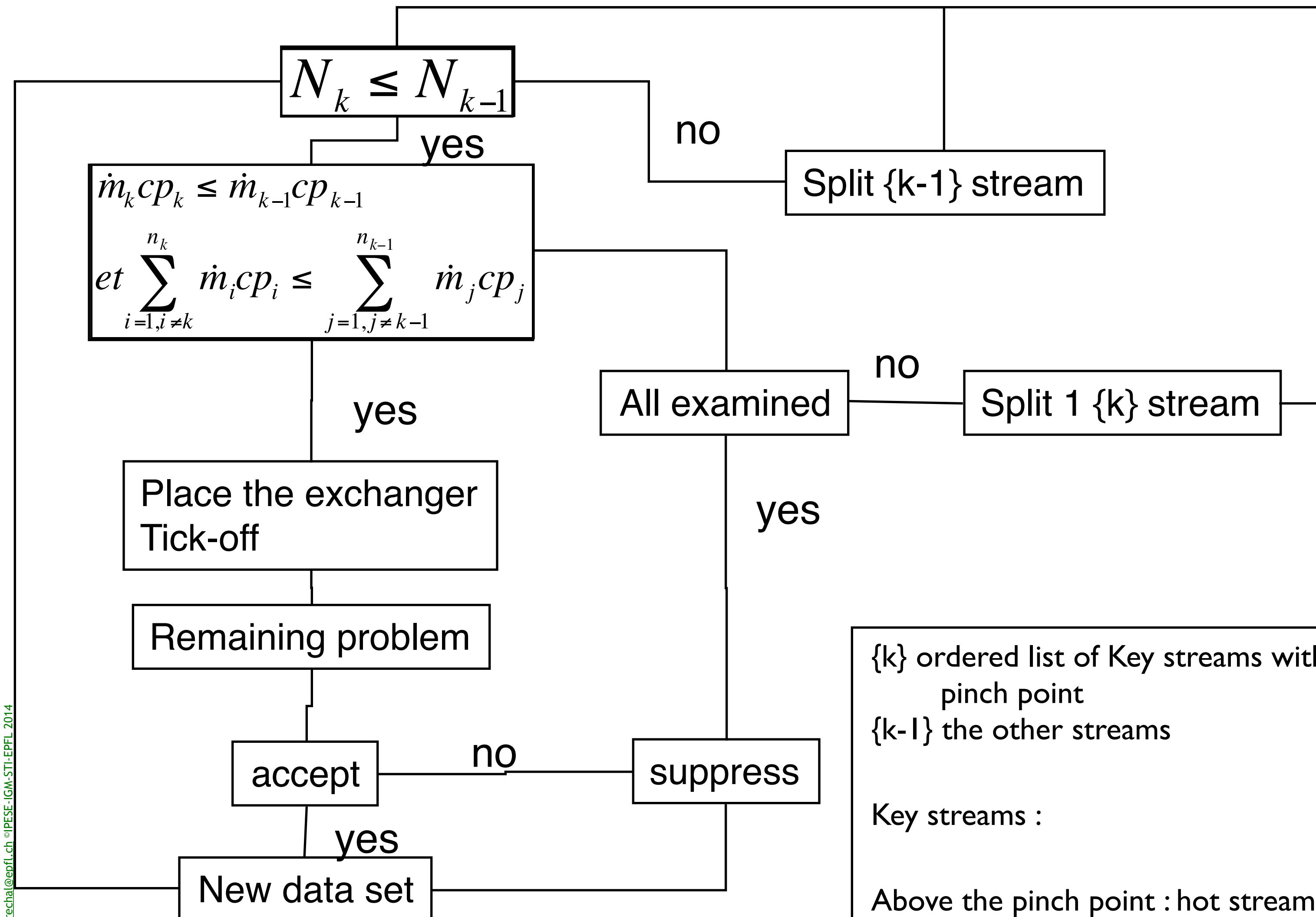
T3 -> Toc

=>MER1

if MER1=MER0

=> Exchanger is well placed

The synthesis of the HEN synthesis algorithm



{k} ordered list of Key streams with decreasing Cp at the pinch point

{k-1} the other streams

Key streams :

Above the pinch point : hot streams

Below the pinch point : cold streams

Optimal DTMIN for heat exchangers

- Two different values of DTmin
  - HRAT (Heat Recovery Approach Temperature)
    - Used to compute energy targets
    - 1 single value for the system level
    - independent of the HEN structure
    - Used to identify pinch point
  - $EMAT_i$ (Exchanger Minimum Approach Temperature)
    - Used for the optimal use of the heat exchangers
    - for each heat exchanger $EMAT_i \geq \mathbf{EMAT} \quad \forall i \in \{\text{Heat Exchangers}\}$
    - HEN structure dependent
    - Optimal Heat exchanger area usage

Calculating of the optimal HEX areas

NLP(Non Linear Programming) optimisation problem

$$\underset{\dot{M}_u,\ A_{ex}}{Minimise} = \left(\sum_{u=1}^{N_u} c_u^+ \cdot \dot{M}_u\right) \cdot time_{year} + \frac{1}{\tau} \sum_{ex=1}^{N_{ex}} \left(a_{ex} (A_{ex})^{b_{ex}}\right)$$

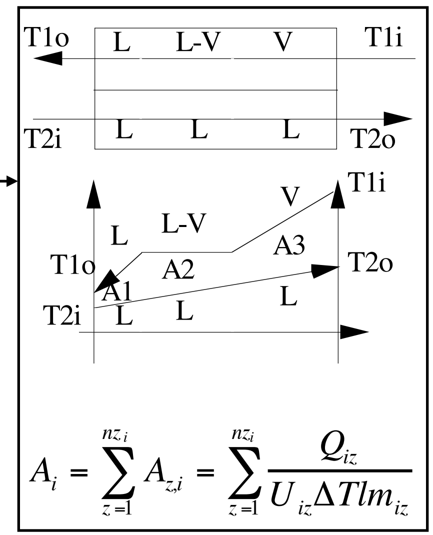
Constraints

Heat and mass balances
Rating equations
Specifications :

F(X) = 0

Bounds and limits

G(X) ≤ 0



X : flows (split factors, utility flows), pressure, temperature, area, heat exchanged, ...

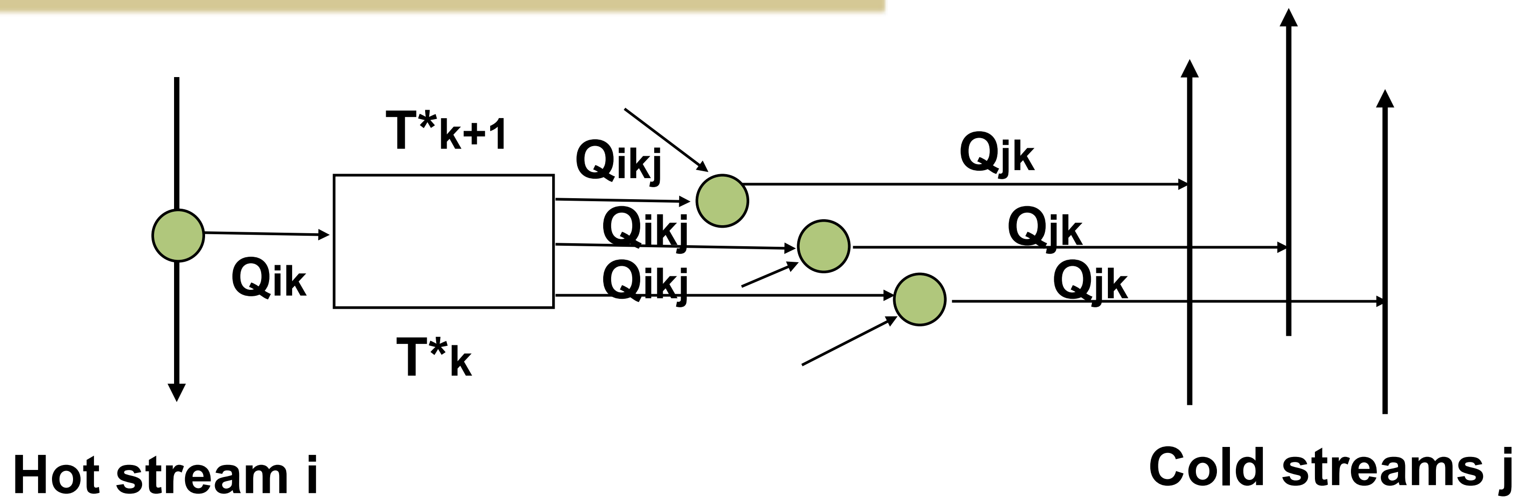
Optimising the flows (e.g. in splitters and utility streams) and temperatures therefore changing the heat exchange areas allows to calculate the best ΔTmin value for each of the heat exchangers (EMAT). The calculation is a non linear programming problem that can be quite complex to solve, due to the interrelations between the heat exchangers and the difficulty of the infeasible heat exchanges (ΔT < 0).
Non linear programming : class of optimisation that involves non linear equality (heat balances, heat transfer) and inequality constraints (flows >0, ΔT>0).

Difficulty of the method

- **Sequential approach**
  - interdependent decisions
- **Multiple solutions**
- **Do we reach the minimum number of units ?**
- **Is the network optimal ?**

- **Reducing the number of units**
  - save the (fixed part) of investment
  - first m2 costs more
- **Optimisation is needed**
  - $EMAT_i$ optimisation

Heat load distribution



Hot stream i in temperature interval k

$$\sum_{j=1}^{nc} Q_{ikj} = Q_{ik} \qquad \forall i = 1,\ldots,nh; \forall k = k_1,\ldots,k_2$$

Cold stream j in and above temperature interval k

$$\sum_{i=1}^{nh}\sum_{r=k}^{k_2} Q_{irj} - \sum_{r=k}^{k_2} Q_{jr} \leq 0 \qquad \forall j = 1,\ldots,nc; \forall k = k_1,\ldots,k_2$$

connection between i et j (integer variable)

$$\sum_{r=k}^{k_2} Q_{irj} - y_{ij} Q\max_{ij} \leq 0 \qquad \forall j = 1,\ldots,nc; \forall i = 1,\ldots,nh$$

Heat load distribution

MILP formulation

Minimize the number of connections

$$\min_{y_{ij},Q_{ikj}} \sum_{i=1}^{nh} \sum_{j=1}^{nc} y_{ij} \qquad y_{ij} \in \{0,1\}$$

$$\sum_{j=1}^{nc} Q_{ikj} = Q_{ik} \qquad \forall i = 1,\dots,nh; \forall k = k_1,\dots,k_2$$

$$\sum_{i=1}^{nh} \sum_{r=k}^{k_2} Q_{irj} - \sum_{r=k}^{k_2} Q_{jr} \leq 0 \qquad \forall j = 1,\dots,nc; \forall k = k_1,\dots,k_2$$

$$\sum_{r=k}^{k_2} Q_{irj} - y_{ij} Q\max_{ij} \leq 0 \qquad \forall j = 1,\dots,nc; \forall i = 1,\dots,nh$$

Multiple solutions

- **Add heuristic rules**
  - favor the connexion with utility streams
  - favor close connexions
  - favor connexion in closer sub-systems
- **A heuristic rule is applied only if it does not penalize the minimum number of solution target**

Introduce heuristic rules in MILP programs

- **The weight of priority rule k is given by :**
  - the number of possible connexions satisfying rule k

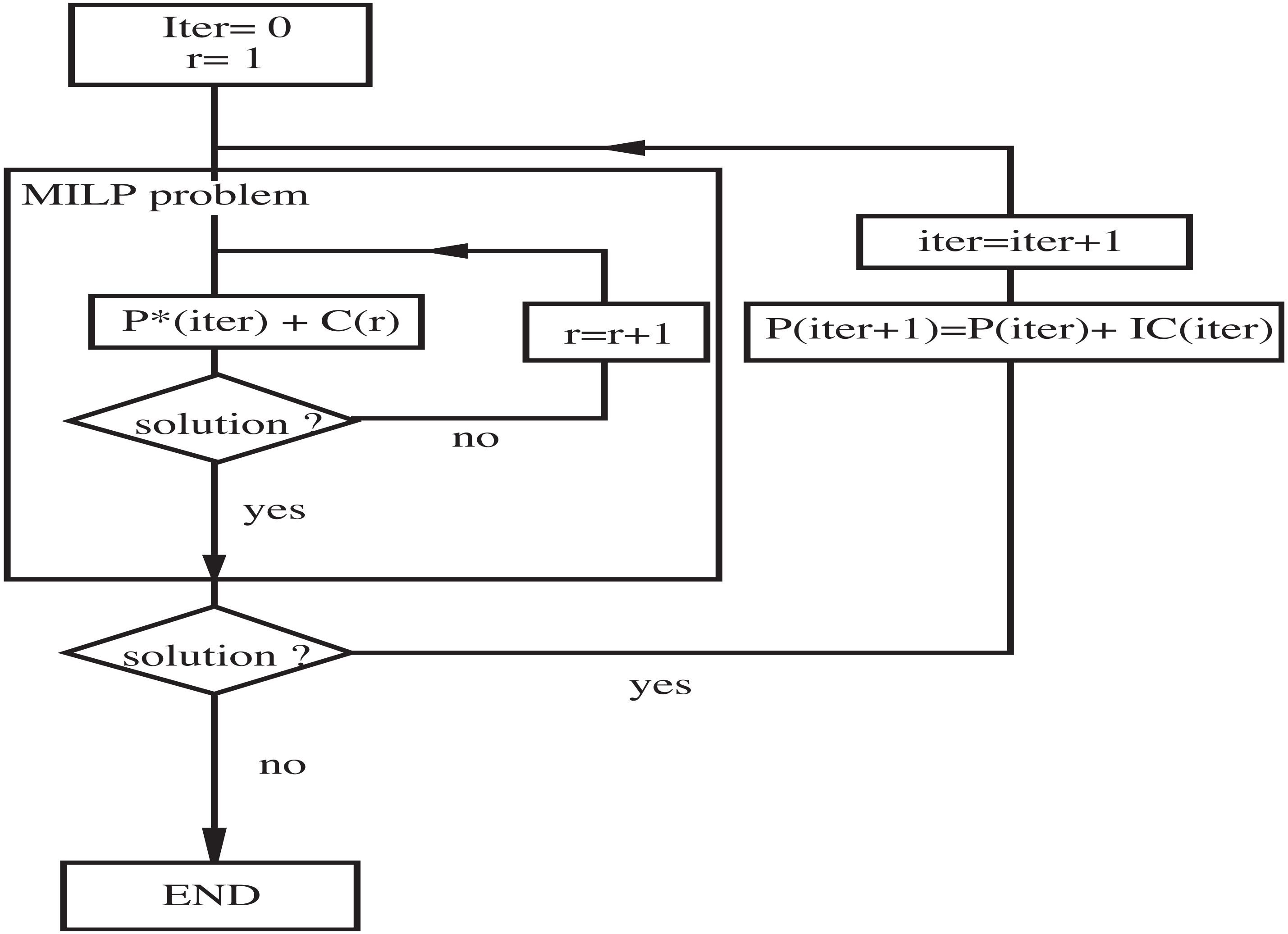
$$P_k = \sum_{j=1}^{n_c} \sum_{i=1}^{n_h} (p_{kij})$$

  - an improved objective function :

$$\min_{y_{ij} Q_{ikj}} NT = \sum_{j=1}^{n_c} \sum_{i=1}^{n_h} \left(\prod_{k=1}^{p_{ij}-1} (P_k + 1)\, y_{ij} \right)$$

$$\frac{\prod_{k=1}^{r} (P_k + 1)}{\prod_{k=1}^{r-1} (P_k + 1)} = P_r + 1 > \sum_{i=1}^{n_h} \sum_{j=1}^{n_c} P_{rij}$$

Improving the speed of convergence



Generating multiple solutions

- **Integer cut constraint**
  - assuming that we know k solutions
  - problem k + 1 is defined by adding to the previous MILP problem the integer cut constraint
-

$$Problem^{k+1} =$$

$$Problem^{k} +$$

$$\sum_{p=1}^{n_p}\sum_{c=1}^{n_c}(2 * y_{p,c}^{k} - 1) * y_{p,c} \leqslant \sum_{p=1}^{n_p}\sum_{c=1}^{n_c} y_{p,c}^{k}$$

The synthesis method

Calculate the heat load distribution for each section

Multiple solutions using integer cuts

Heuristic rules or user

$$\sum_{p=1}^{n_p}\sum_{c=1}^{n_c}(2*y_{p,c}^k-1)*y_{p,c} \leqslant \sum_{p=1}^{n_p}\sum_{c=1}^{n_c} y_{p,c}^k$$

-> screening and choice of the appropriate solution

Define the HEN structure

Apply feasibility rules and heuristics

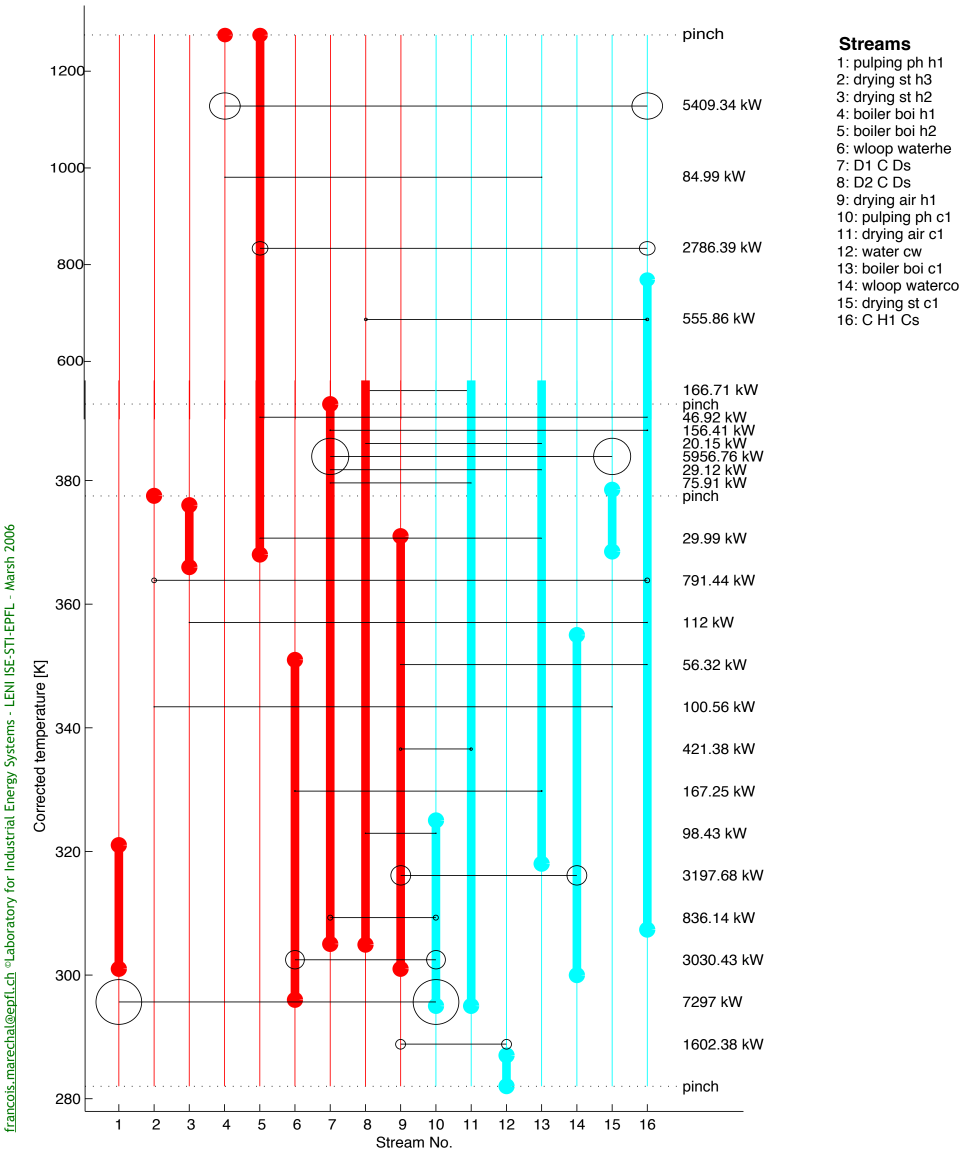
Splits and serial exchanges

Optimize the HEN

Total cost criteria

no DTmin nor MER fixed

Heat Load Distribution

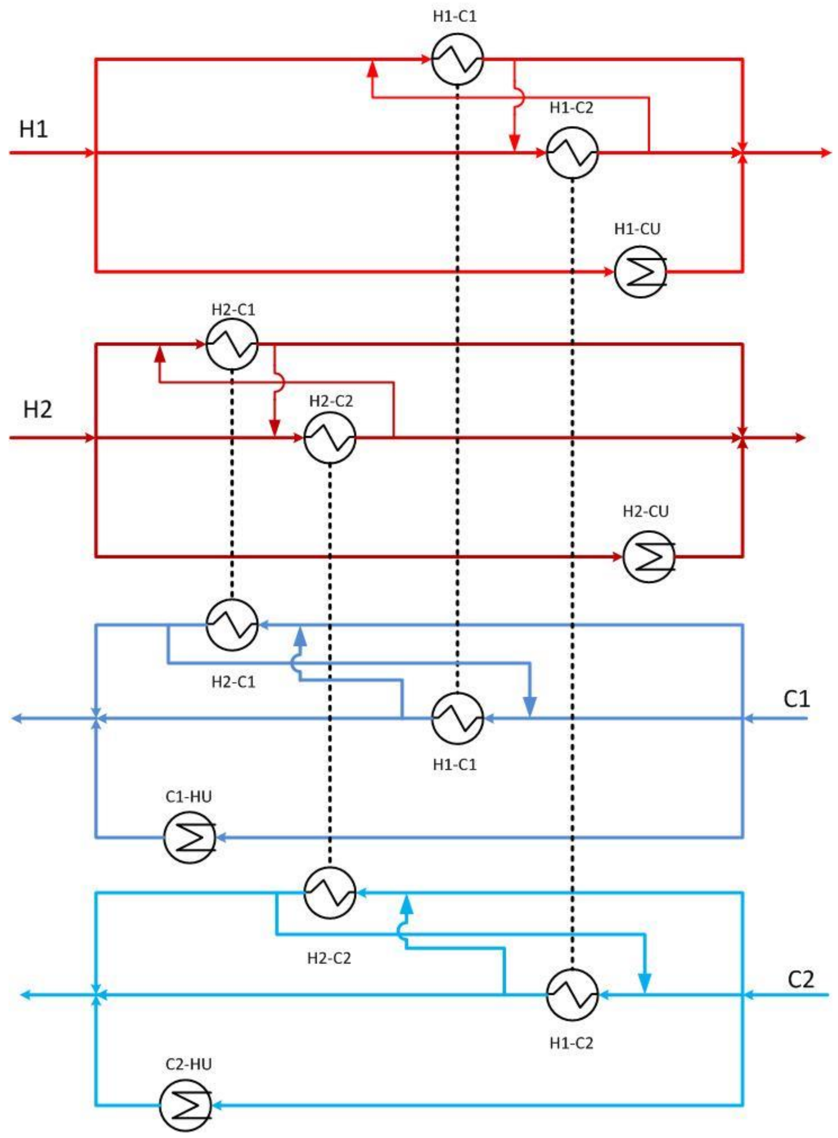


Example zone 1

Hot stream	Cold stream	Heat load [kW]
pulping_ph_h1	pulping_ph_c1	7297.0
wloop_eauhe	pulping_ph_c1	3030.4
D1_C_Ds	pulping_ph_c1	836.1
D2_C_Ds	pulping_ph_c1	98.4
drying_air_h1	drying_air_c1	421.4
drying_air_h1	water_cw	1602.4
boiler_boi_h2	boiler_boi_c1	30.0
wloop_waterhe	boiler_boi_c1	167.3
drying_air_h1	wloop_waterco	3197.7
drying_st_h3	drying_st_c1	100.6
drying_st_h3	C_H1_Cs	791.4
drying_st_h2	C_H1_Cs	112.0
drying_air_h1	C_H1_Cs	56.3

Non linear optimisation : superstructure

- **From the results of the heat load distribution**
  - **super-structure**
  - **from $y_{i,j}$ to exchangers**



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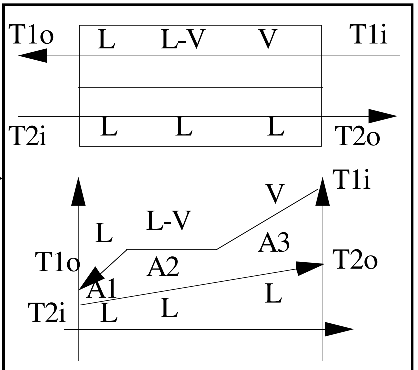

$$A_i = \sum_{z=1}^{nz_i} A_{z,i} = \sum_{z=1}^{nz_i} \frac{Q_{iz}}{U_{iz} \Delta Tlm_{iz}}$$

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Mixed Integer Non linear optimisation

$$
\min \sum_i \sum_j \sum_k c_f z_{ijk} + \sum_i c_f z_{cui} + \sum_j c_f z_{huj} + \sum_i c_{cu} q_{cui} + \sum_j c_{hu} q_{huj} +
$$

$$
\sum_i \sum_j \sum_k c \left(\frac{q_{ijk}}{U_{ij} LMTD_{ijk}} \right)^{\beta} + \sum_i c \left(\frac{q_{cui}}{U_{cui} LMTD_{cui}} \right)^{\beta} + \sum_j c \left(\frac{q_{huj}}{U_{huj} LMTD_{huj}} \right)^{\beta}
$$

{Minimum total annual cost **SYNHEAT Model**}

subject to

$$
\begin{aligned}
\sum_i \sum_k q_{ijk} + q_{cui} &= F_i \left(T_i^{in} - T_i^{out} \right), i \in HP \\
\sum_j \sum_k q_{ijk} + q_{huj} &= F_j \left(T_j^{out} - T_j^{in} \right), j \in CP
\end{aligned}
$$

{Overall heat balance for each stream}

$$
\begin{aligned}
\sum_j q_{ijk} &= F_i \left(t_i^k - t_i^{k+1} \right), i \in HP \\
\sum_i q_{ijk} &= F_j \left(t_j^k - t_j^{k+1} \right), j \in CP
\end{aligned}
$$

{Heat balance at each stage}

$$
t_i^{k=1} = T_i^{in}, t_j^{k=NOK} = T_j^{in}
$$

{Assignment of superstructure inlet temperatures}

$$
\begin{aligned}
& t_i^k \geq t_i^{k+1}, t_j^k \geq t_j^{k+1} \\
& t_i^{k=NOK+1} \geq T_i^{out}, t_j^{k=1} \geq T_j^{out}
\end{aligned}
$$

{Monotonic decrease in temperatures}

$$
\begin{aligned}
q_{cui} &= F_i (t_i^{NOK+1} - T_i^{out}) \\
q_{huj} &= F_j \left(T_j^{out} - t_j^{i} \right)
\end{aligned}
$$

{Hot and cold utility loads}

$$
dt_{ijk} \geq \Delta T_{\min}, dt_{cui} \geq \Delta T_{\min}, dt_{huj} \geq \Delta T_{\min}
$$

{Minimum approach temperature constraints}

$$
\begin{aligned}
& q_{ijk} \geq \Omega z_{ijk}, q_{cui} \geq \Omega z_{cui}, q_{huj} \geq \Omega z_{huj} \\
& dt_{ijk} \geq t_i^k - t_j^k + \Gamma \left(1 - z_{ijk} \right) \\
& dt_{ijk} \geq t_i^{k+1} - t_j^{k+1} + \Gamma \left(1 - z_{ijk} \right) \\
& dt_{cui} \geq t_i^{NOK} - t_{cu}^{out} + \Gamma (1 - z_{cui}) \\
& dt_{cui} \geq T_i^{out} - t_{cu}^{in} + \Gamma (1 - z_{cui}) \\
& dt_{huj} \geq t_{hu}^{out} - t_j^1 + \Gamma \left(1 - z_{huj} \right) \\
& dt_{huj} \geq t_{hu}^{in} - T_j^{out} + \Gamma \left(1 - z_{huj} \right)
\end{aligned}
$$

{Logical constraints}

$$
LMTD_{ijk} = \frac{dt_{ijk} - dt_{ijk+1}}{\ln \left(\frac{dt_{ijk}}{dt_{ijk+1}} \right)}
$$

{LMTD definition for heat exchangers}

$$
LMTD_{cui} = \frac{dt_{ijk} - dt_{cui}}{\ln \left(\frac{dt_{ijk}}{dt_{cui}} \right)}, LMTD_{huj} = \frac{dt_{ijk} - dt_{huj}}{\ln \left(\frac{dt_{ijk}}{dt_{huj}} \right)}
$$

{LMTD definition for utility exchangers}

$$
T_i^{out} \leq t_i^k \leq T_i^{in}, \quad T_j^{in} \leq t_j^k \leq T_j^{out}
$$

{Bounds}

$$
q_{ijk}, q_{cui}, q_{huj}
$$

{Nonnegativity constraints}

$$
z_{ijk}, z_{cui}, z_{huj} = 0 - 1
$$

{Integrality conditions}

IPES INDUSTRIAL PROCESS

MINLP : outer approximation

- Decomposition theorem
  - Partition variables in 2 sets
    - complicating set (integer)
    - continuous variables
  - Solve 2 problems
    - NLP with fixed integer (lower bound)
    - MILP : outer-approximate the objective function (upper bound)
  - Lower = upper => convergence
    - integer cut to avoid looping
- Problems :
  - NLP converge ?
  - Calculation time ?
  - Initial set feasible ?
  - Derivatives for MILP
    - outer approximation