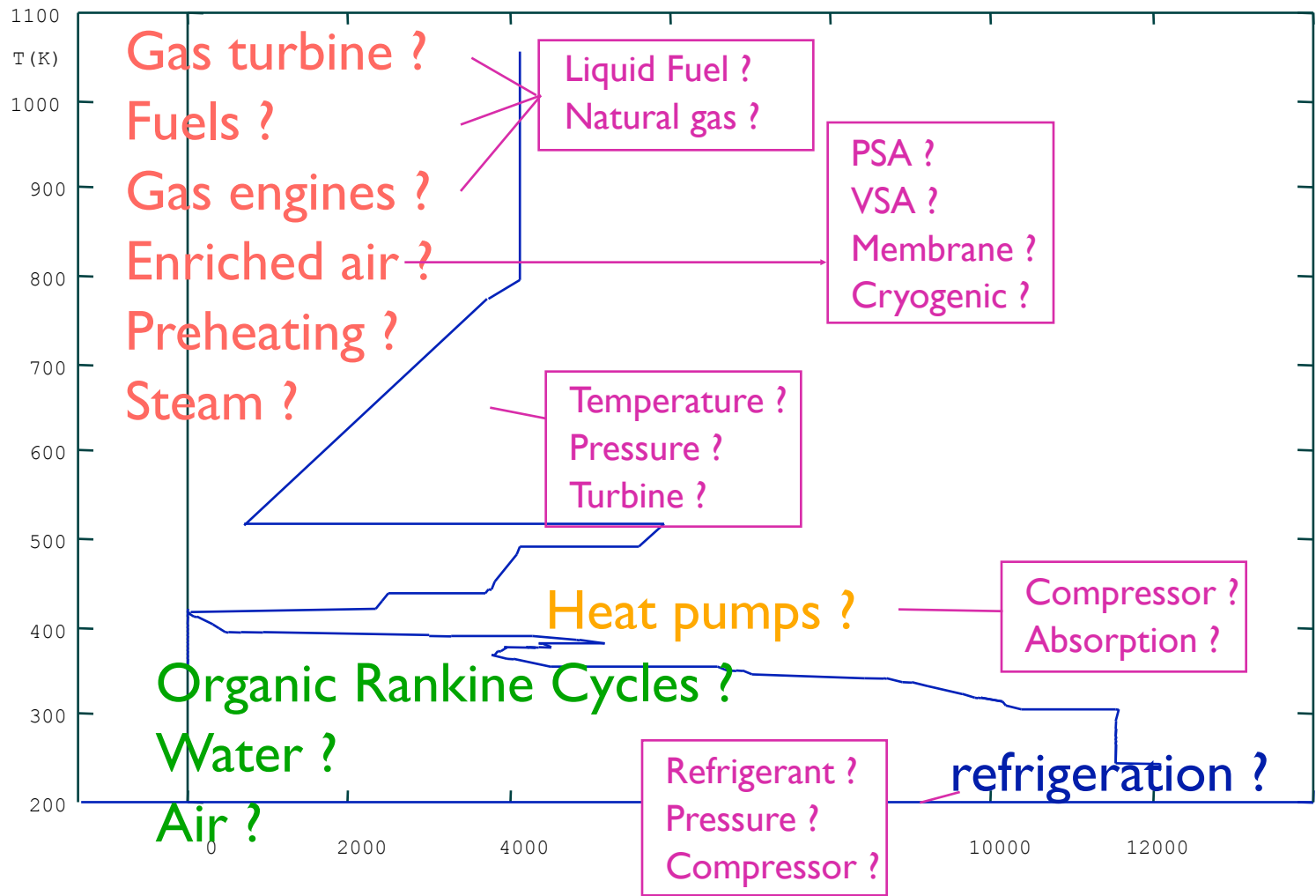

Utility Integration by optimisation

François Marechal

Utilities



Integration of the energy conversion system

Technology w with nominal flow

Hot/cold streams

$$T_{w,i}^{in}, P_{w,i}^{in}, \dot{m}_{w,i}, x_{w,i}$$

$$\downarrow q_w = \dot{m}_{w,i}(h_{w,i}^{in} - h_{w,i}^{out})$$

$$T_{w,i}^{out}, P_{w,i}^{out}, \dot{m}_{w,i}, x_{w,i}$$

Mechanical power/electricity

e_w

Cost

$\$C1_w, C2_w, CI1_w, CI2_w$

Decision variables

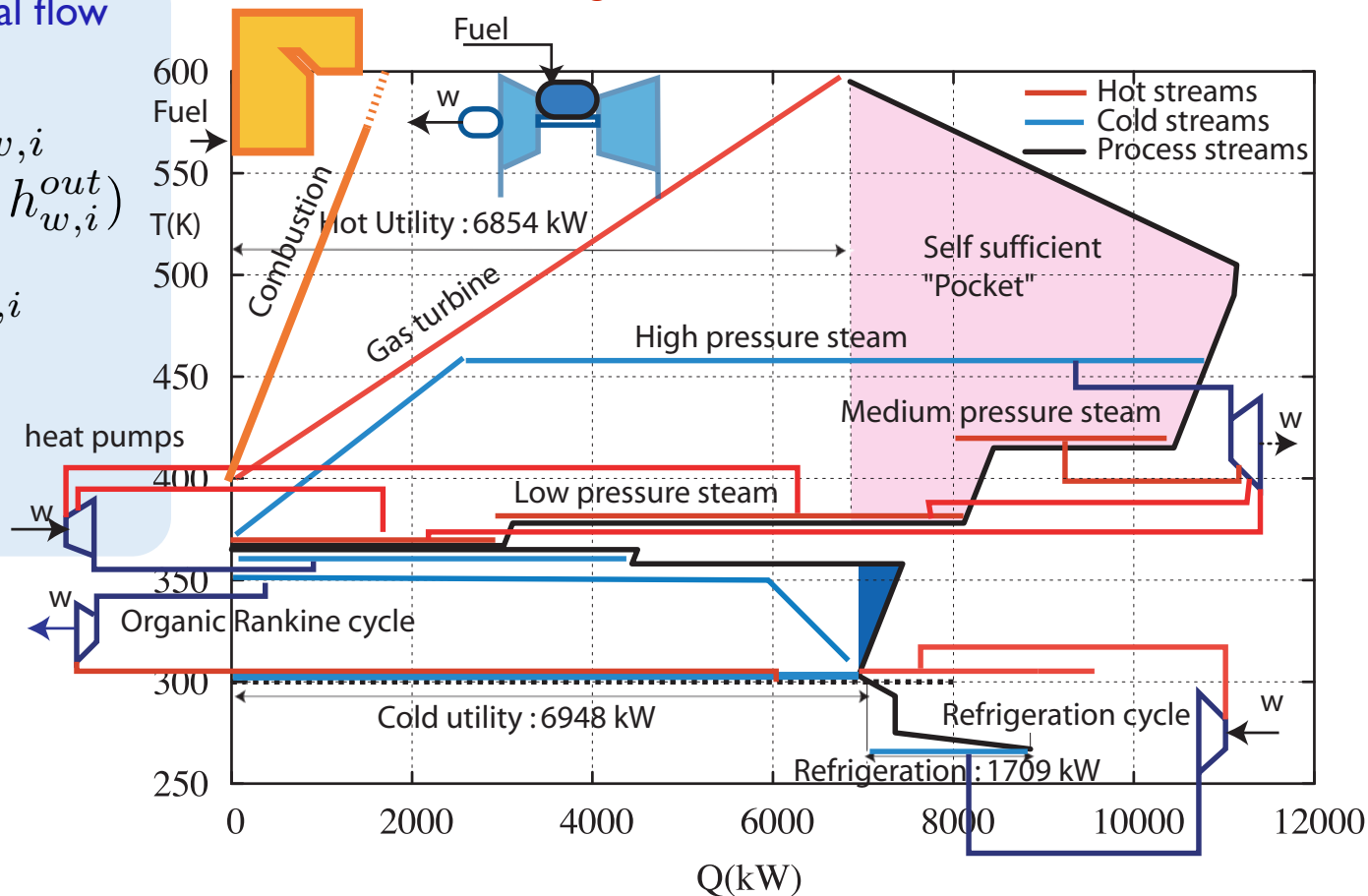
Level of usage of w

f_w

Buy/use technology w ?

y_w

Creative engineers ?



MILP formulation

$$\min_{R_r, y_w, f_w, E^+, E^-} \left(\sum_{w=1}^{n_w} C2_w f_w + C_{el+} E^+ - C_{el-} E^- \right) * t \quad \text{Operating cost}$$

$$+ \sum_{w=1}^{n_w} C1_w y_w + \frac{1}{\tau} \left(\sum_{w=1}^{n_w} (CI1_w y_w + CI2_w f_w) \right) \quad \text{Investment}$$

Fixed maintenance

Subject to : Heat cascade constraints

$$\sum_{w=1}^{n_w} f_w q_{w,r} + \sum_{s=1}^{n_s} Q_{s,r} + R_{r+1} - R_r = 0 \quad \forall r = 1, \dots, n_r$$

Feasibility

$$R_r \geq 0 \quad \forall r = 1, \dots, n_r; R_{n_r+1} = 0; R_1 = 0 \quad E^+ \geq 0; E^- \geq 0$$

Electricity consumption

$$\sum_{w=1}^{n_w} f_w e_w + E^+ - E_c \geq 0$$

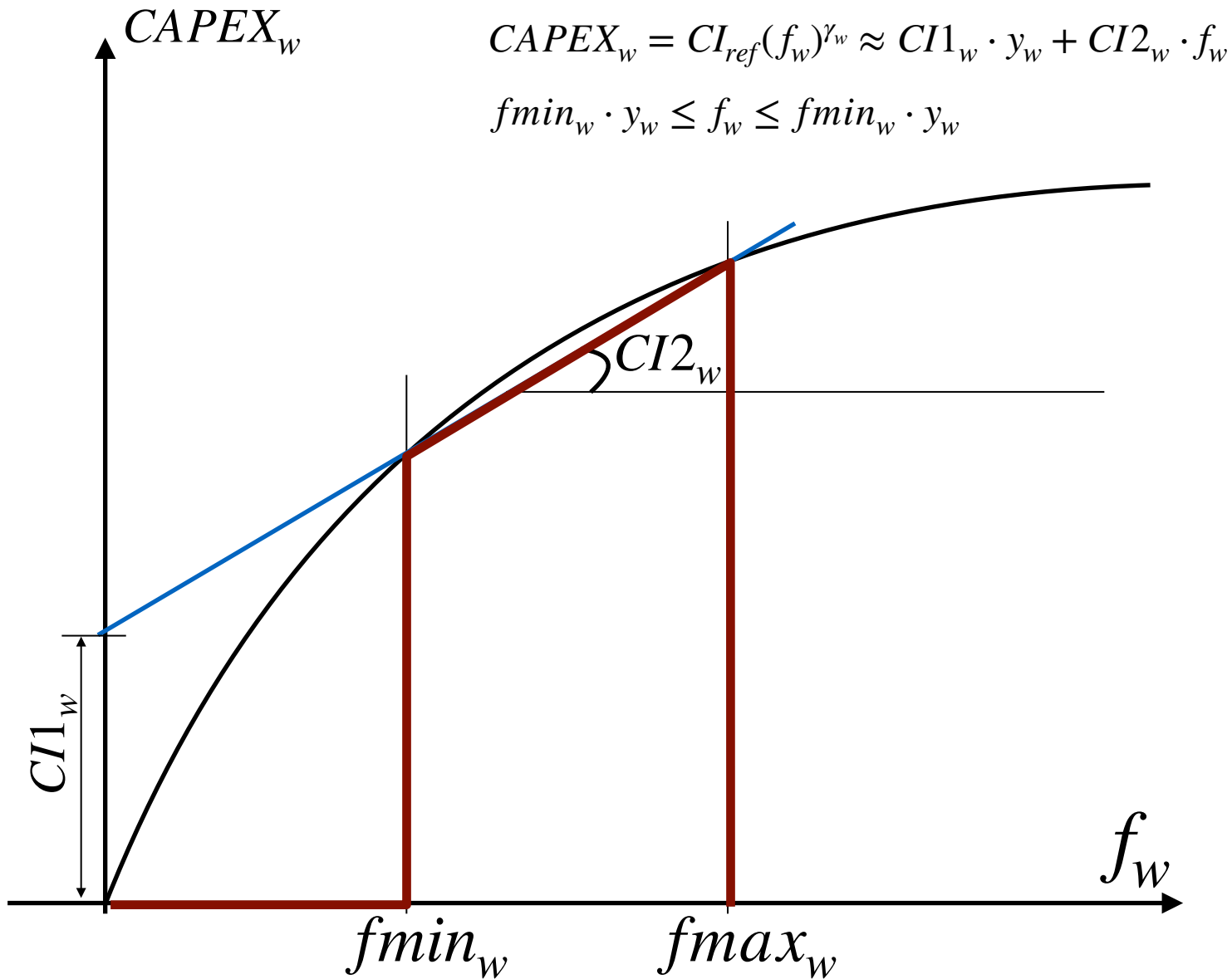
Electricity production

$$\sum_{w=1}^{n_w} f_w e_w + E^+ - E_c - E^- = 0$$

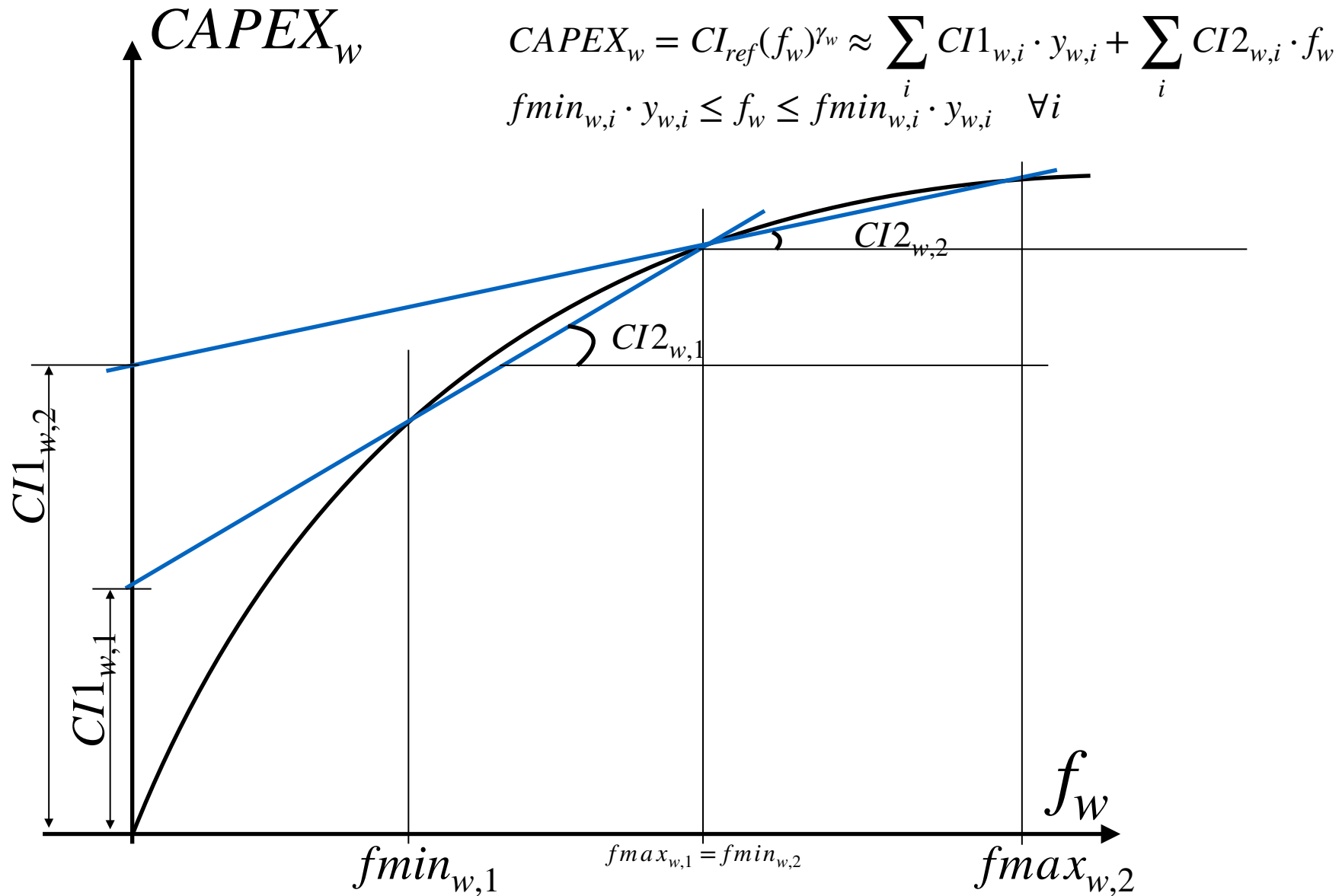
Energy conversion Technology selection

$$fmin_w y_w \leq f_w \leq fmax_w y_w \quad y_w \in \{0, 1\}$$

Linearisation of non linear function



Piecewise linearisation of non linear function



MILP problems

Mixed Integer Problems

- **MILP**

- Mixed Integer Linear Programming problems
 - Linear constraints & objective
 - continuous and integer variables

- **MINLP**

- Mixed Integer Non Linear Programming problems
 - Non linear constraints & objective
 - continuous and integer variables

MILP : Branch & Bound Method

- Solve LP at each node
 - start with integer variables = continuous

$$0 \leq z_i \leq 1 \quad \forall i = 1, \dots, 3$$

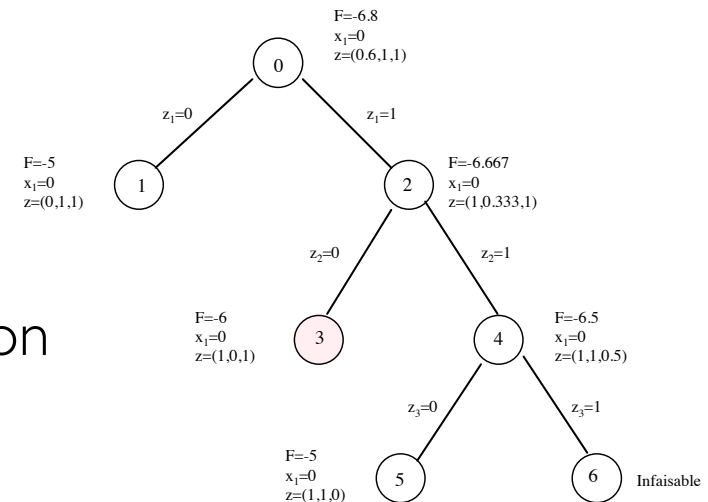
- Progressively “integerify” by systematically adding constraints (Branch)

$$z_i = 1 \text{ or } 0 \text{ for } i \text{ in Nodes} \quad 0 \leq z_j \leq 1 \quad \forall j \neq i$$

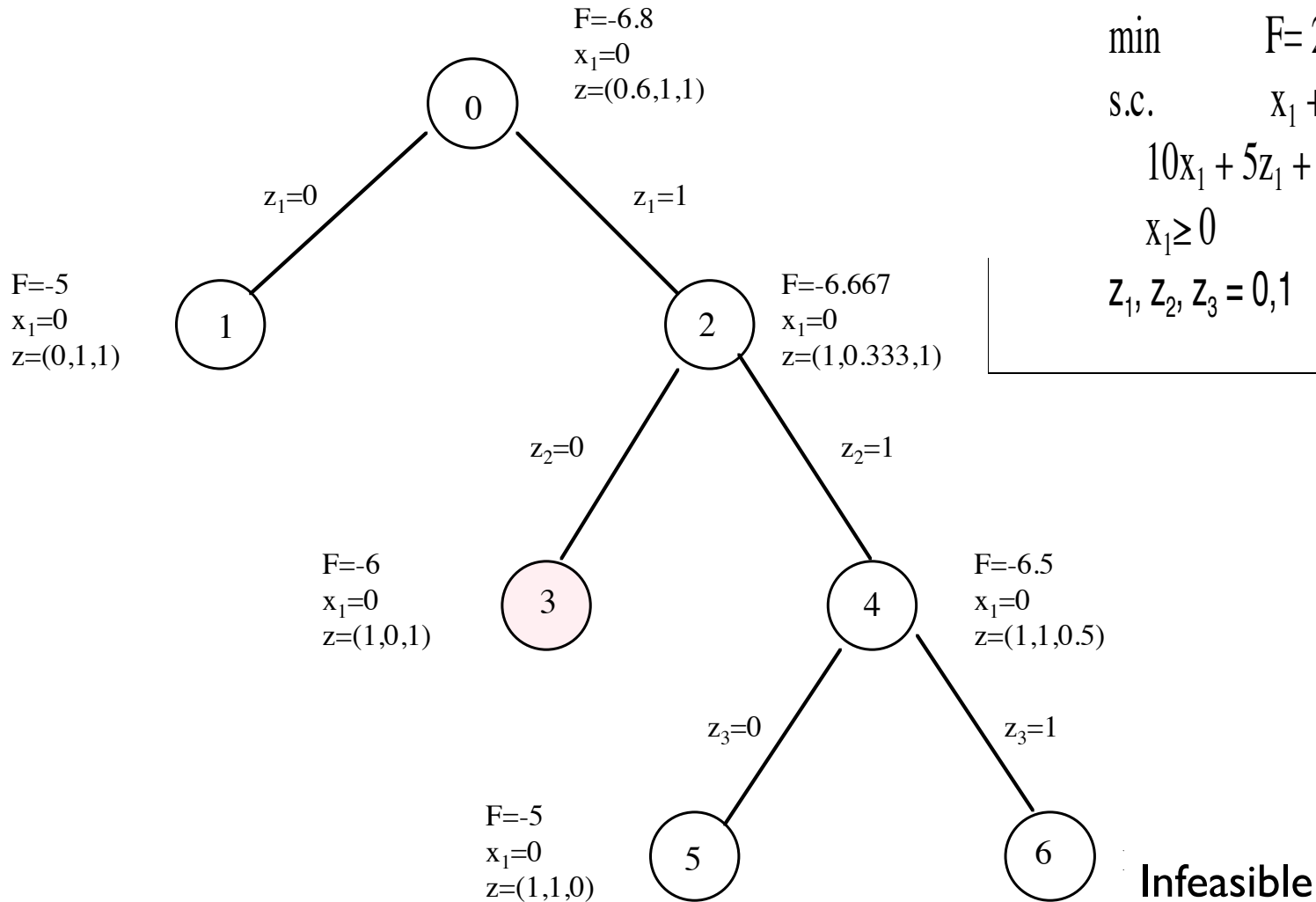
➡ the objective is worsening

- when a set where all z_i is integer
 - define the bound (the objective function will never be worse than this value)
- re-explore the branches
 - cut the three exploration when the objective function reaches the bound

$$\begin{aligned} \min \quad & F = 2x_1 - 3z_1 - 2z_2 - 3z_3 \\ \text{s.t.} \quad & x_1 + z_1 + z_2 + z_3 \geq 2 \\ & 10x_1 + 5z_1 + 3z_2 + 4z_3 \leq 10 \\ & x_1 \geq 0 \\ & z_1, z_2, z_3 = 0, 1 \end{aligned}$$



MILP : Branch & Bound Method



MILP optimization

- **Linear programming**

- optimum defined by constraints
 - max/min
 - Pinch points
- Cost may create strange results
 - if electricity is cheaper than the fuel, a heat pump becomes an electrical heater
- Integer variables for technology selection
 - Can be used to select among options

- **Heat balance constraints**

- if the hot and cold utility have not the appropriate levels no solution is found
- max flows may prevent to close the balance
- max flows may prevent convergence

$$y_i \cdot f_{min} \leq f \leq y_i \cdot f_{max}$$

$$1. \leq 0.000001 \cdot 1'000'000$$

$$\text{is } y_i = 0.000001 =? 0 \text{ or } 1$$

- **Additional constraints**

- have to be satisfied

- **Need to analyze solutions**

Logical constraints

At least 1 of 4 $\sum_{i=1}^4 y_i \geq 1$

At most 1 of 4 $\sum_{i=1}^4 y_i \leq 1$

y_1 or y_2 $y_1 + y_2 = 1$

if y_1 then y_2 $y_2 \geq y_1$

if y_1 then not y_2 $y_b \leq (1 - y_a)$

Generating order list of Integer Sets Solutions

- Integer cut constraint

- assuming that we know already k solutions
- problem k + 1 is defined by adding to the previous MILP problem the integer cut constraint

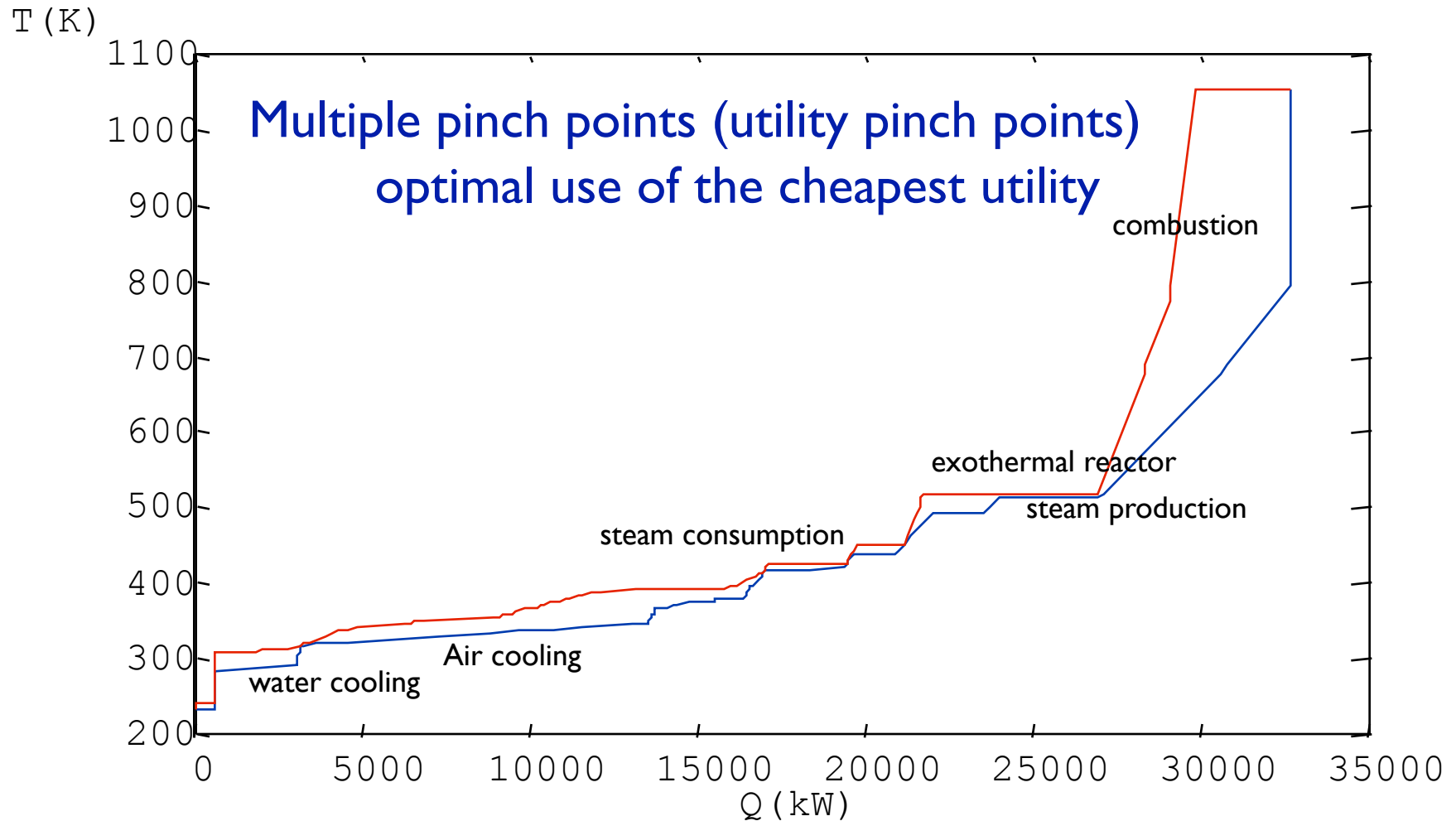
Problem^{k+1} :

Problem^k

$$\sum_{i=1}^{n_y} (2y_i^k - 1) * y_i \leq \sum_{i=1}^{n_y} y_i^k$$

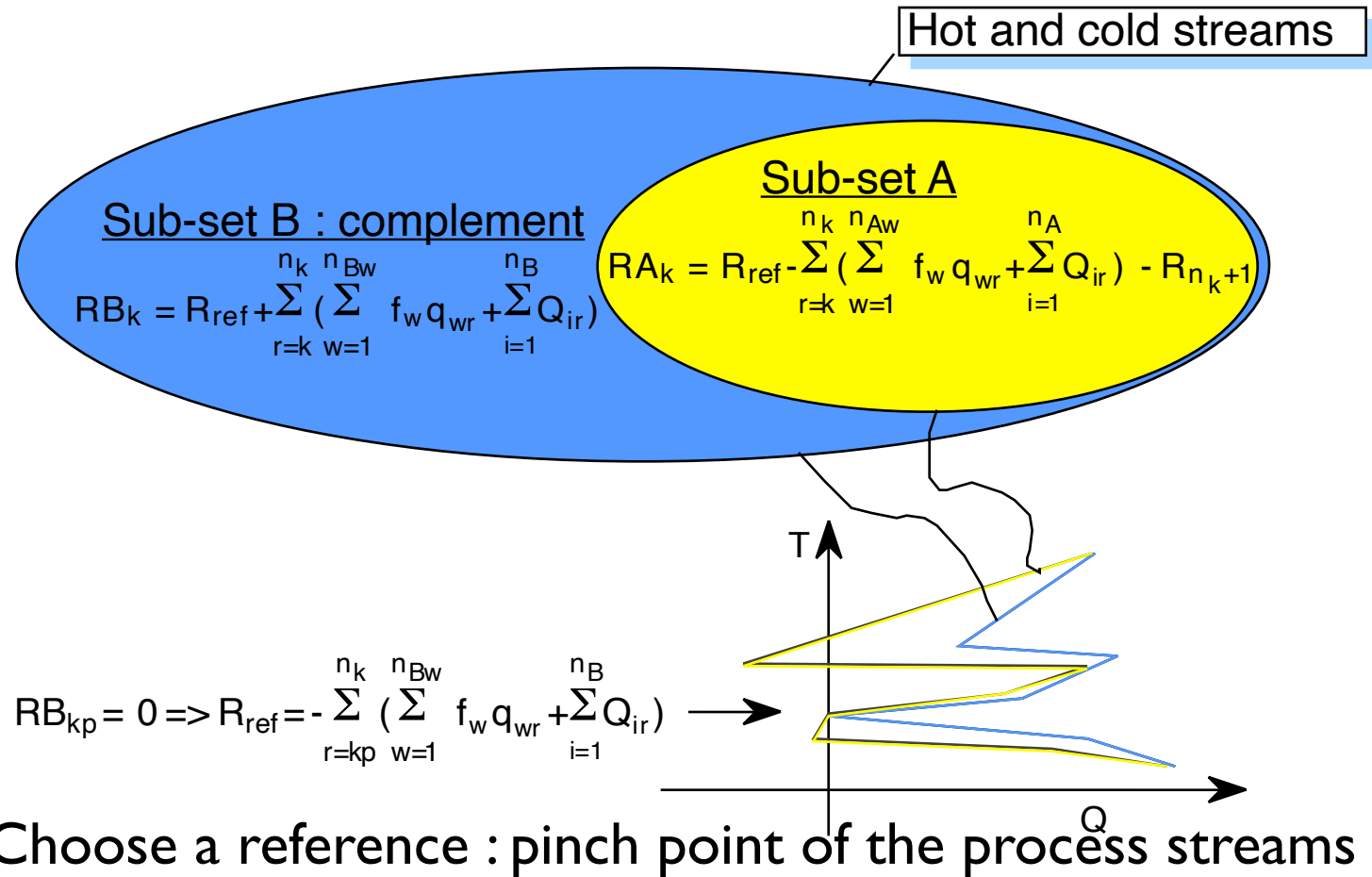
where y_i^k value of y_i in solution of problem k

Results : Balanced composite curves

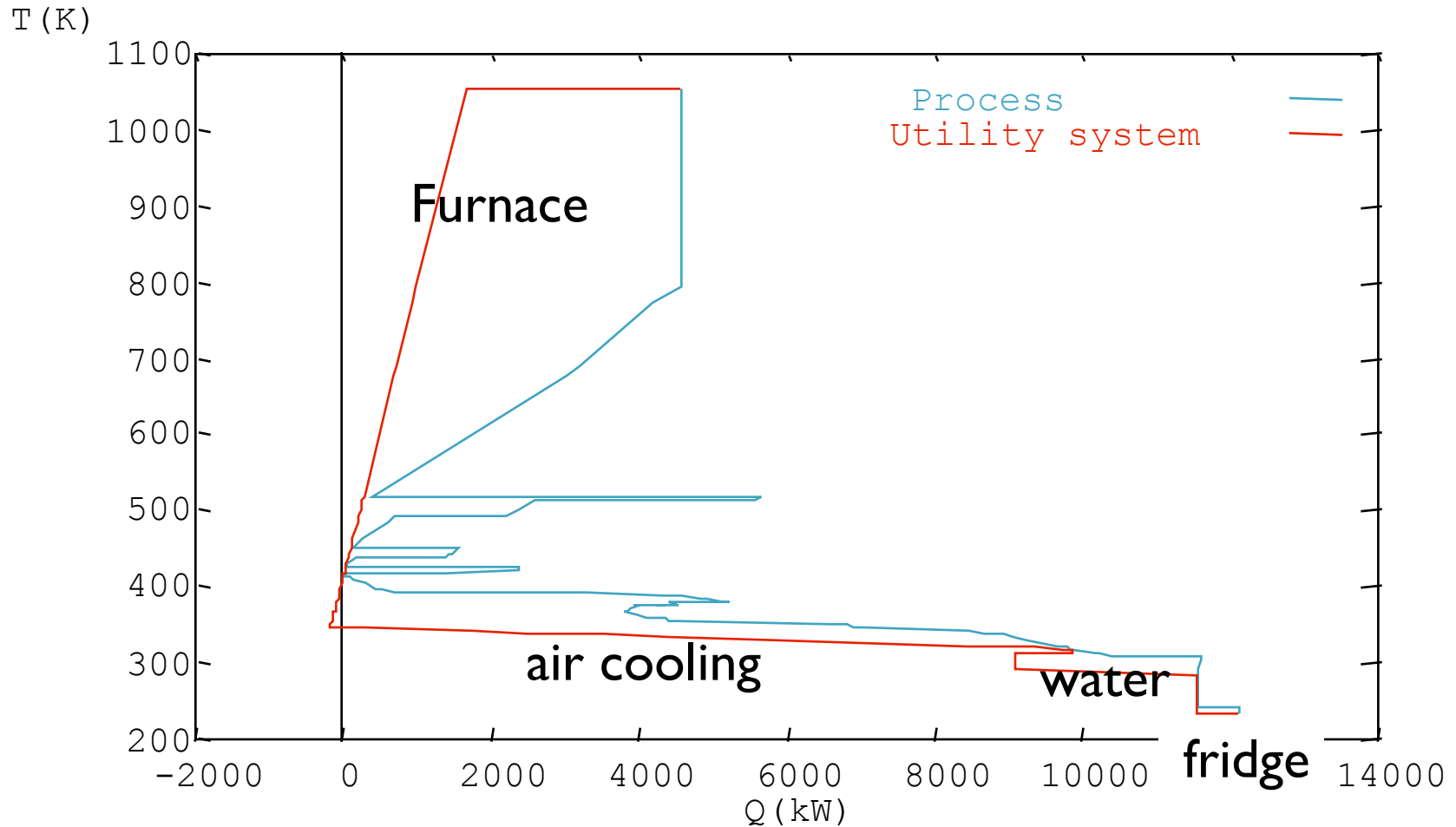


Evaluate : the Integrated Composite Curves

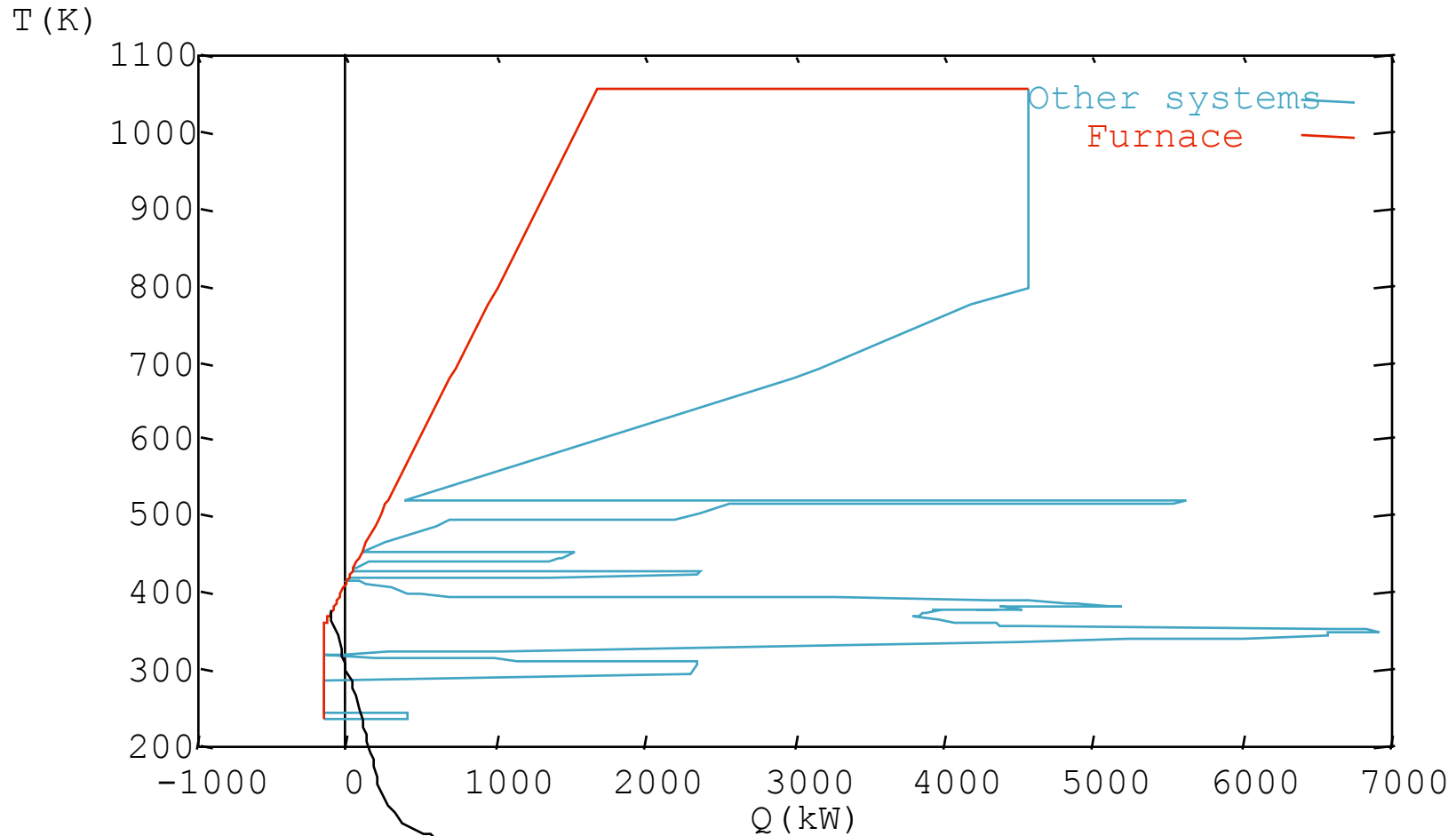
The goal is to understand the solutions



ICC for utility system integration

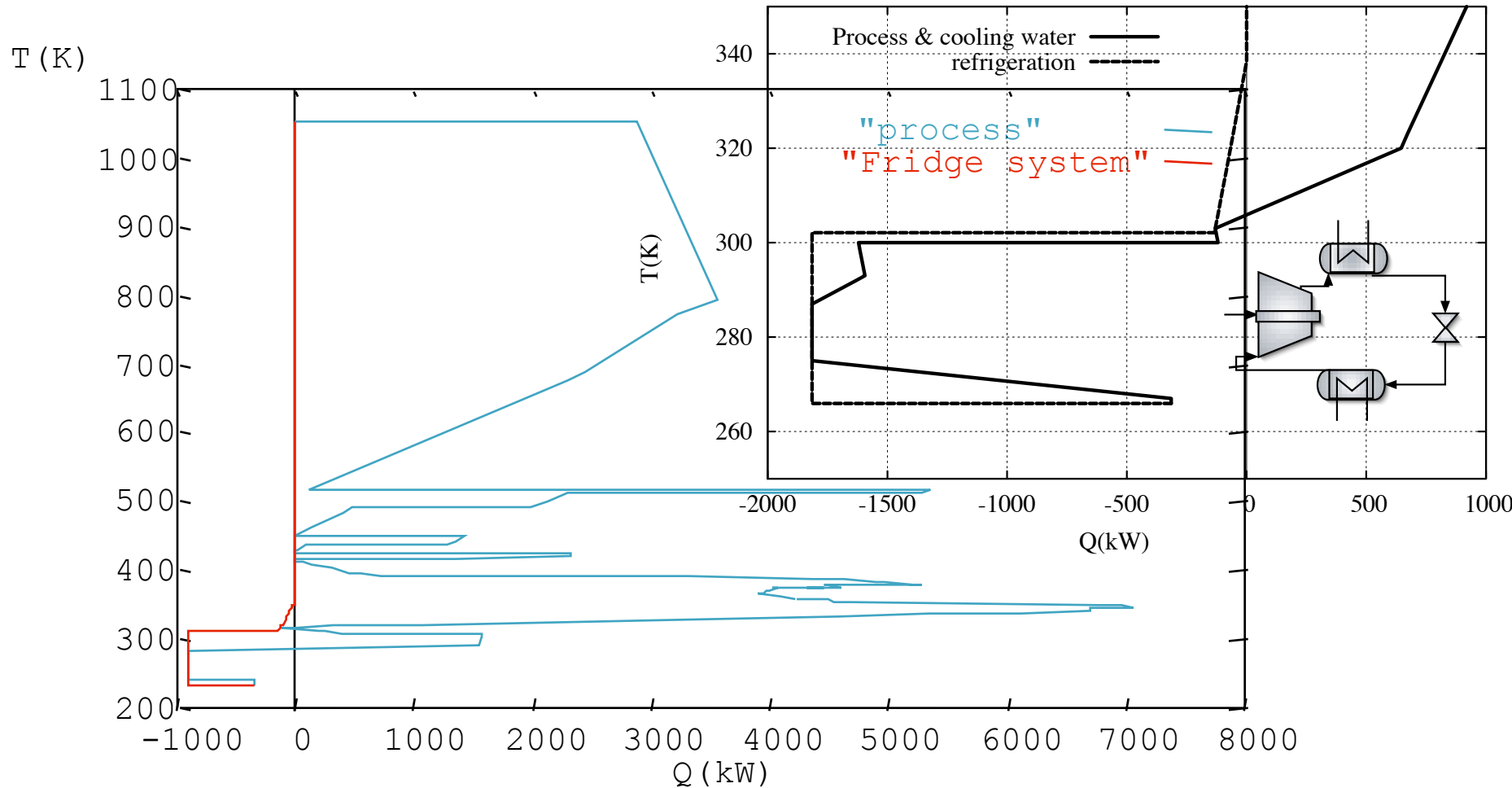


ICC for the integration of the fumes

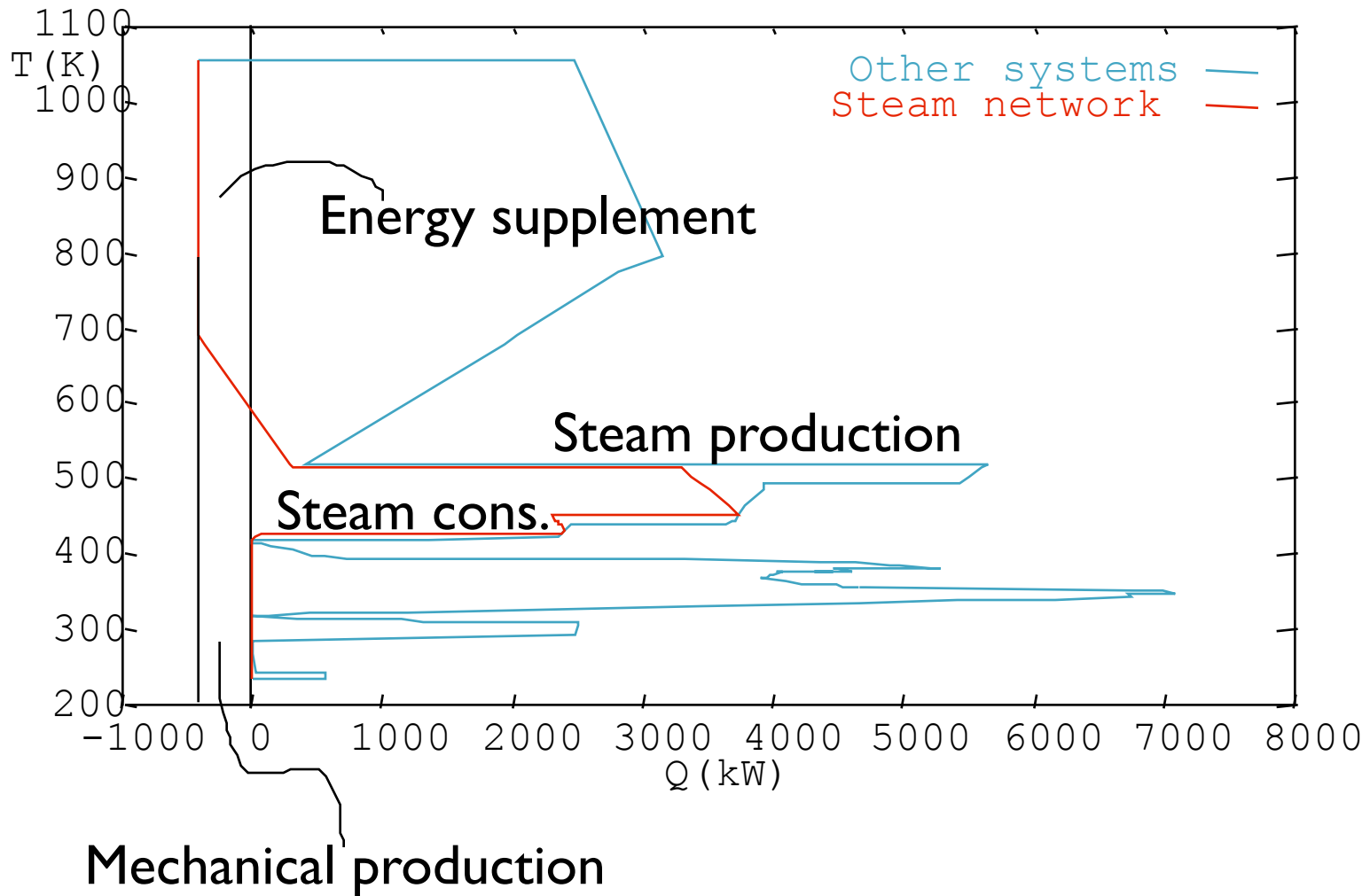


Excess of heat in the fumes

ICC for refrigeration cycle integration



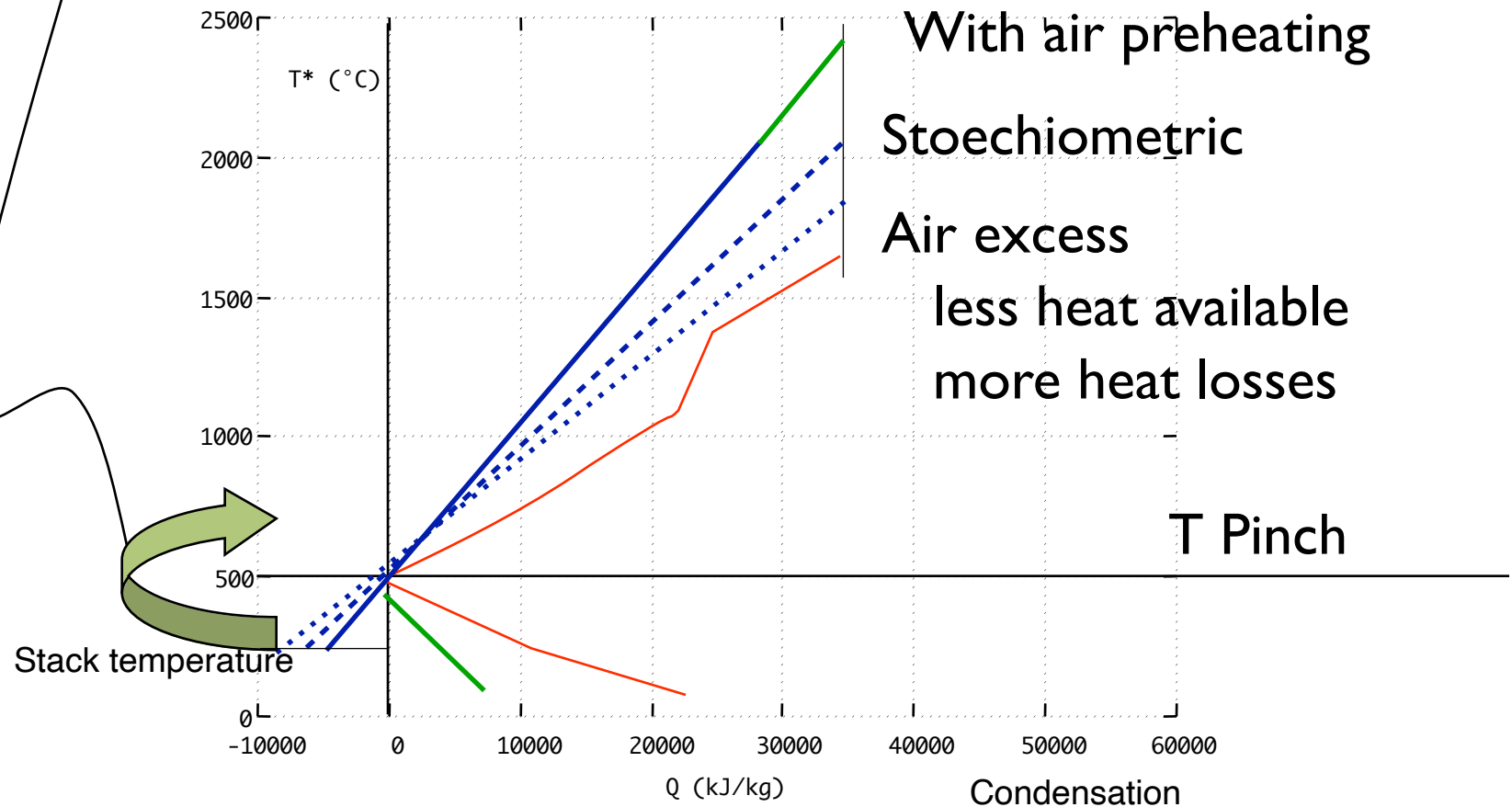
ICC of the steam network



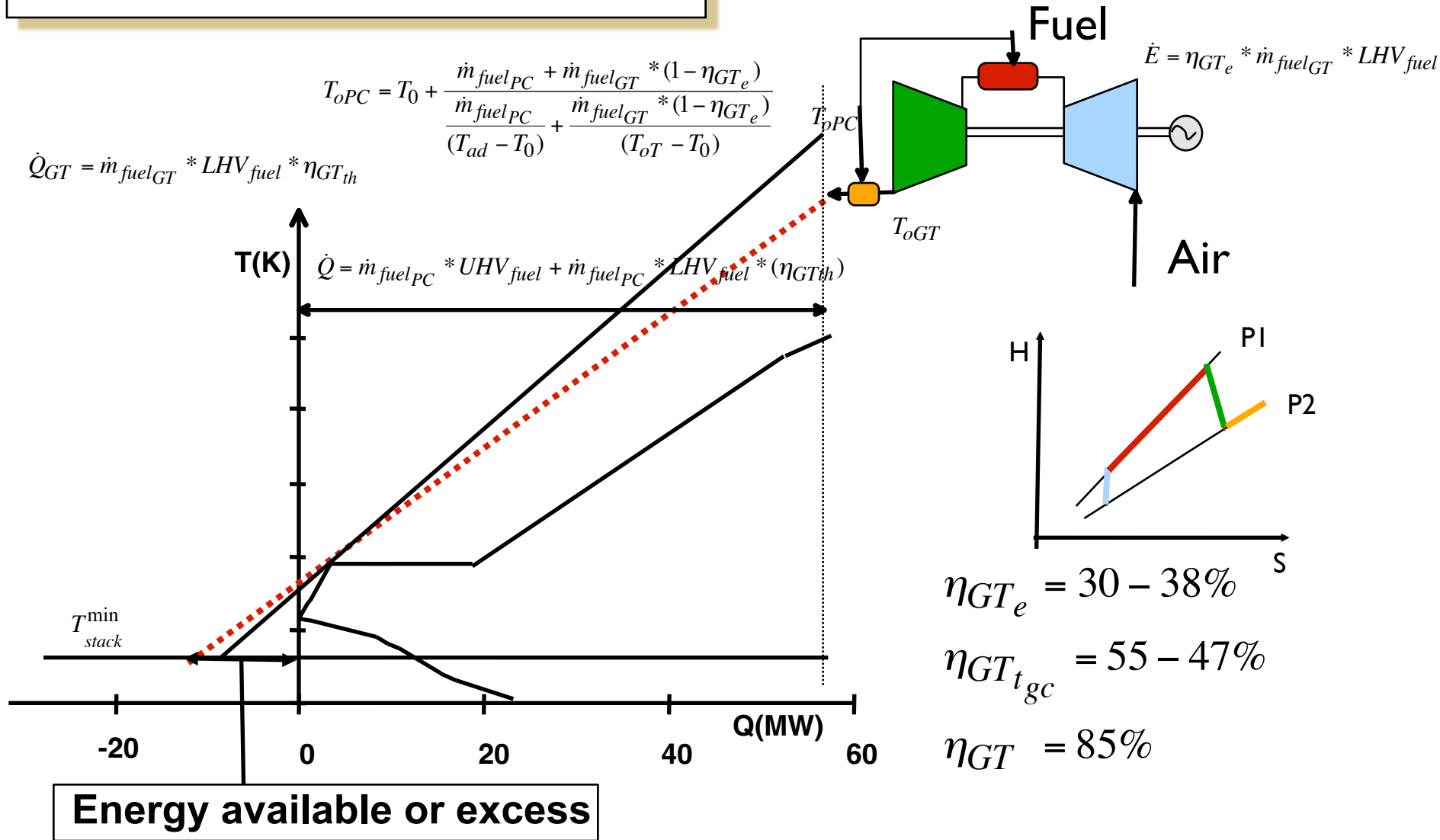
COMBUSTION INTEGRATION : Plus-Minus

AIR PREHEATING

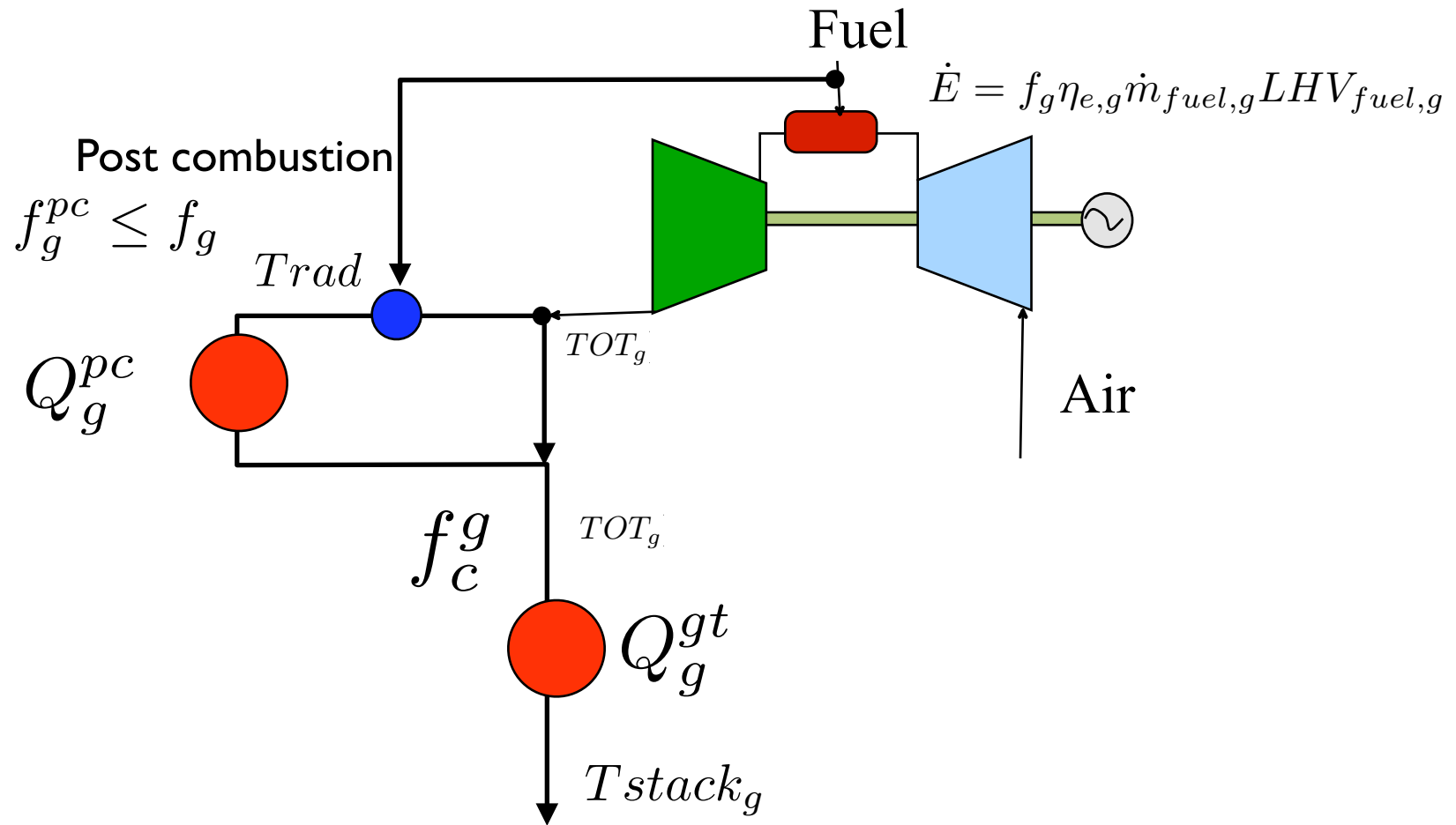
More heat available above the pinch point (plus-minus)



Gas Turbine : fixed size



Targeting model : combustion and gas turbine



Targeting the optimal integration : model

- MILP formulation

Gas turbine g : hot stream from T_{oT} to T_{stack}

unknown $Q_g^{gt} = f_g * \dot{m}_g * c_p f_g * (T_{oT_g} - T_{stack_g})$

Fuel balance
$$\sum_{c=1}^{n_{cgt}} f_c^g * LHV_c - \sum_{g=1}^{n_g} y_g * FCI_g + f_g * FCP_g = 0$$

Electricity
$$W_{gt} - \sum_{g=1}^{n_g} y_g * WI_g + f_g * WP_g = 0$$

Operating cost
$$\sum_{g=1}^{n_g} (y_g * OCI_g + f_g * OCP_g) - OC_{gt} = 0$$

Investments
$$\sum_{g=1}^{n_g} y_g * ICI_g - IC_{gt} = 0$$

Part load efficiency $f_g^{min} * y_g \leq f_g \leq y_g * f_g^{max}$

Combustion model

Post combustion Hot stream from Trad to T_{OT}

$$Q_g^{pc} = f_g^{pc} * \dot{m}_g * cp f_g * (Trad - TOT_g) \quad f_g^{pc} \leq f_g$$

O2 balance

$$\sum_{g=1}^{n_g} f_g^{pc} * \dot{m}_g * x_g^{O_2} + f_{air} * \dot{m}_{air} * x_{air}^{O_2} - \sum_{a=1}^{n_a} f_a * \dot{m}_a * x_a^{O_2} - \sum_{c=1}^{n_c} f_c^c * \kappa_c^{O_2} \geq 0$$

Heat above Trad

$$\begin{aligned} & \sum_{c=1}^{n_c} (f_c * (LHV_c + (cp_{air} * \frac{\kappa_c^{O_2}}{x_{air}^{O_2}} * (Trad - T_0)))) - f_{air} * cp_{air} * (Trad - T_0) \\ & - \sum_{a=1}^{n_a} f_a * \dot{m}_a * cp_a * (Trad - TO_a) - \sum_{g=1}^{n_g} Q_g^{pc} + Q_{prh} - Q_{rad} = 0 \end{aligned}$$

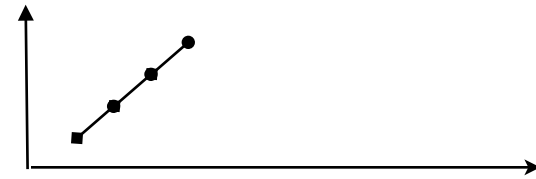
Heat from Trad to stack

$$\begin{aligned} & f_{air} * cp_{air} * (Trad - T_{stack}) + \sum_{c=1}^{n_c} (f_c * (\dot{m}_c^f * cp f_c - cp_{air} * \frac{\kappa_c^{O_2}}{x_{air}^{O_2}}) * (Trad - T_{stack})) \\ & + \sum_{a=1}^{n_a} f_a * \dot{m}_a * cp_a * (Trad - T_{stack}) - Q_{env} = 0 \quad (11) \end{aligned}$$

Outlet temperature calculation

Define n streams as segments

Stream i : from T_i^{air} to $T_{i+1}^{air} = T_i^{air} + \Delta T$



Add linear constraints

$$f_{air} \geq f_{a_i} \quad \forall i = 1, \dots, n_i \quad (19a)$$

$$Q_{prh} = \sum_{i=1}^{n_i} f_{a_i} c_{p_{air,i}} (T_{i+1} - T_i) \quad (19b)$$

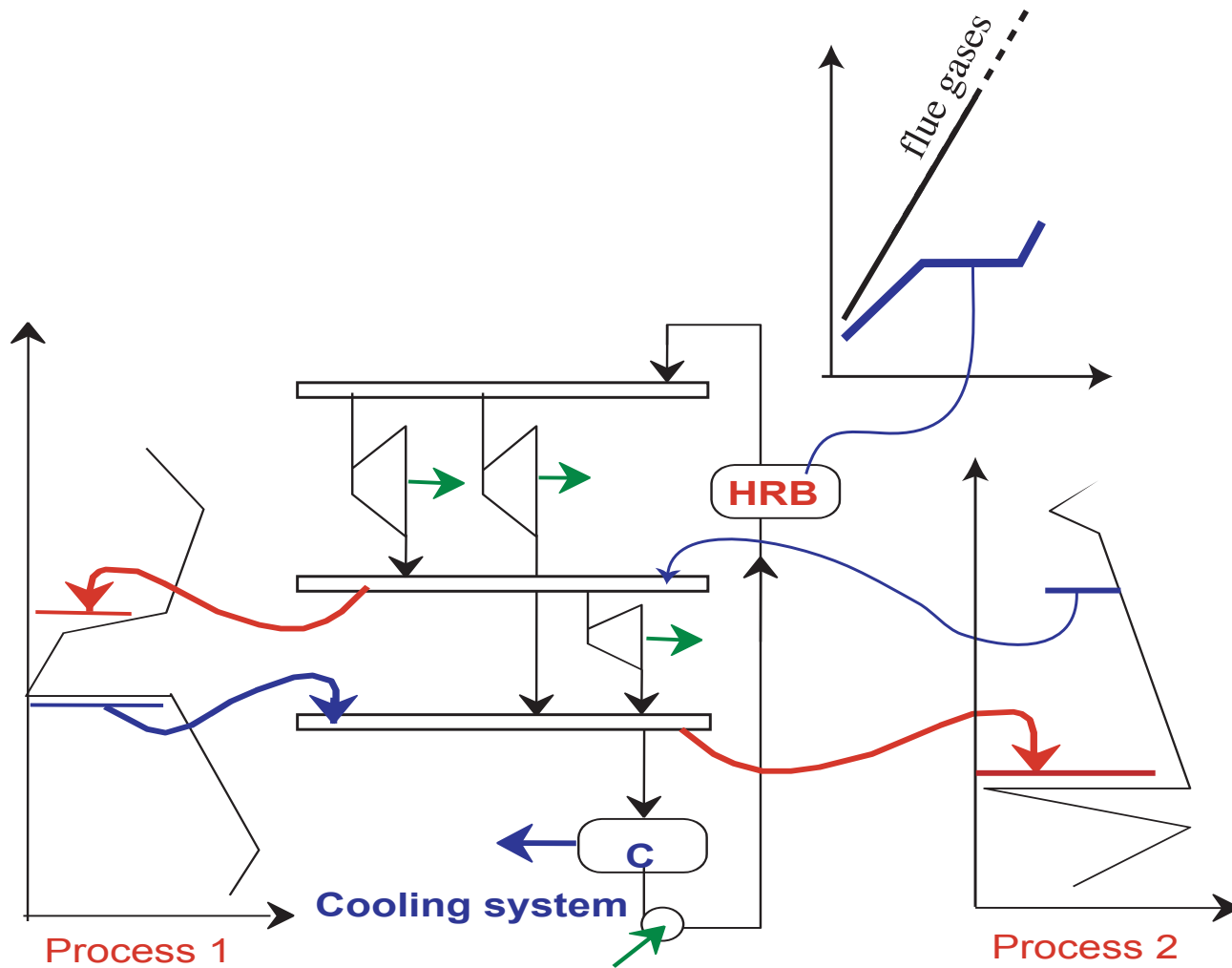
f_{a_i} the flowrate of air preheated from T_i to T_{i+1} .

$c_{p_{air,i}}$ the specific heat capacity of the air flowrate between T_i to T_{i+1} .

Compute the temperature a posteriori

- (1) solve the model and compute the optimal flowrates in each interval (f_{a_i});
- (2) compute the resulting temperature To_{n_i} by solving from $i = 1$ to n_i ,
 $To_i = \frac{(f_{a_{i-1}} - f_{a_i})To_{i-1} + f_{a_i}T_{i+1}}{f_{a_{i-1}}}$ with $To_0 = Ta_{in}$ the inlet temperature of the stream a .

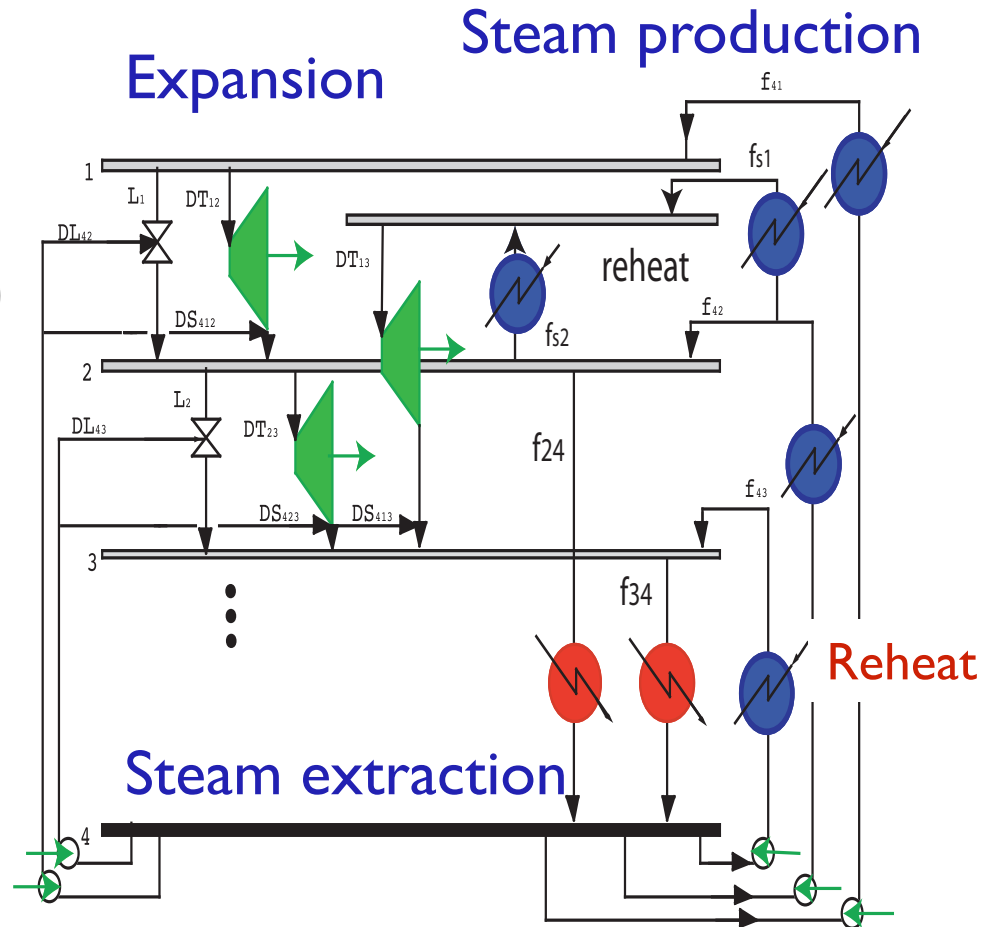
Steam network integration



Steam cycle superstructure

- Systematic generation
- Only T,P for each level
 - Mechanical power
- $$w_{i,j} = \min((h_i - h_j), \eta_{is}(h_i - h_{is}(P_i, T_i, P_j)))$$
- Hot & cold streams
- DTmin/2
- Level heat&mass balance

[1] F. Marechal and B. Kalitventzeff. Targeting the optimal integration of steam networks. *Computers and Chemical Engineering*, 23:s133–s136, 1999.



MILP model with the steam cycle

Heat cascade equation

$$i = 1, \dots, n_v; j = 1, \dots, n_u; k = 1, \dots, n_k$$

$$\sum_{w \in \{(j,k);(k,i);c\}} f_w q_{w,r} + \sum_{s=1}^{n_s} Q_{s,r} + R_{r+1} - R_r = 0 \quad \forall r = 1, \dots, n_r$$

$$R_r \leq 0 \quad \forall r = 1, \dots, n_r; R_{n_r+1} = 0; R_1 = 0$$

Pressure level balance

$$\sum_{i=1}^{n_v} \overset{\text{IN}}{k_{i,w}} f_{i,w}^t + \sum_{k=1}^{n_k} f_{k,w} - \sum_{k=1}^{n_k} f_{w,k} - \sum_{i=1}^{n_v} \overset{\text{OUT}}{f_{w,i}} - \sum_{i=1}^{n_u} f_{w,i}^t = 0 \quad \forall w$$

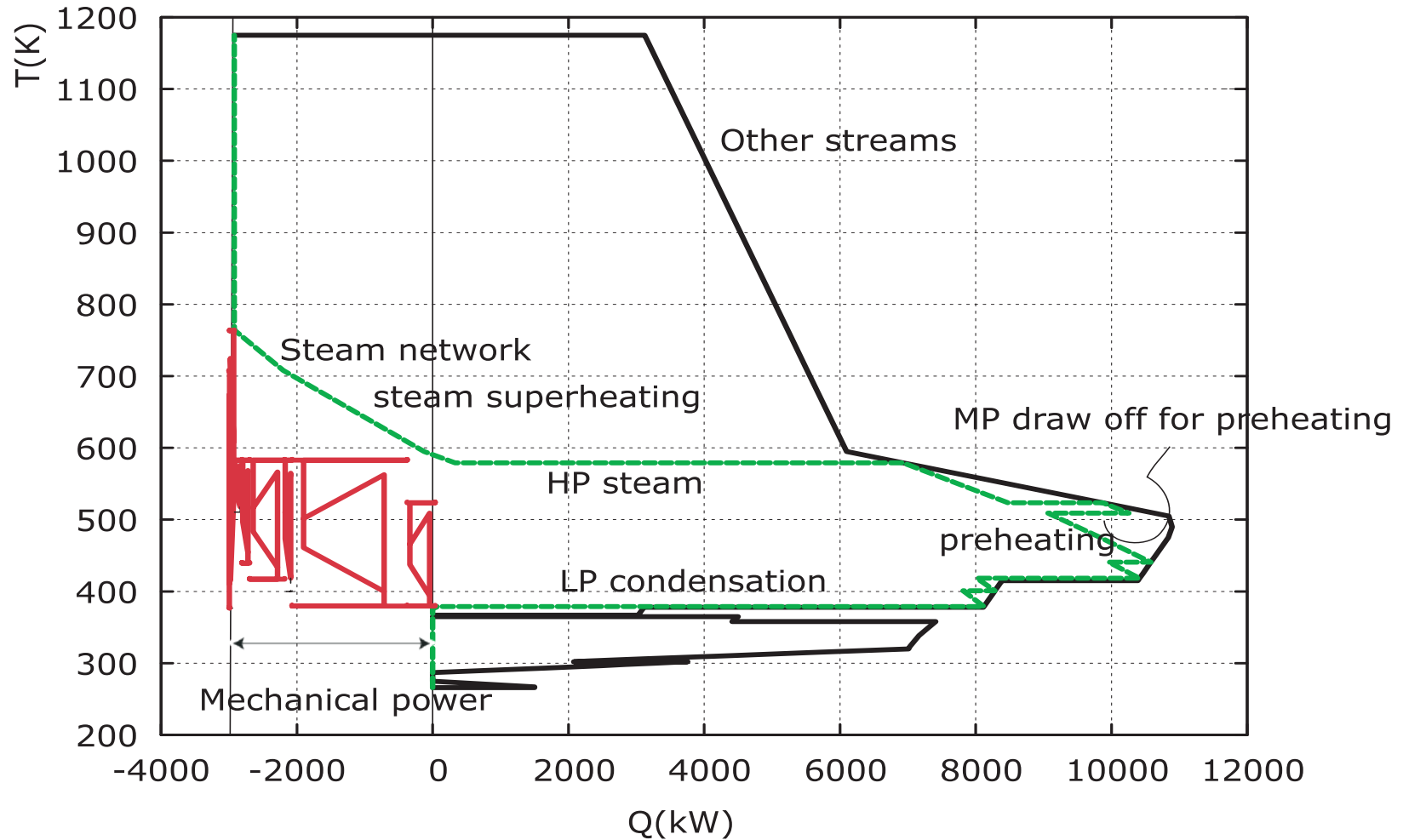
Mechanical power balance

$$\sum_{w \in \{(i,j)\}} f_w^t w_w - \sum_{w \in \{(k,i)\}} f_w w p_w = W$$

Existence of heat exchange or expansion

$$fmin_w y_w \leq f_w \leq fmax_w y_w \quad y_w \in \{0, 1\}$$

Integrated composite curve : steam network

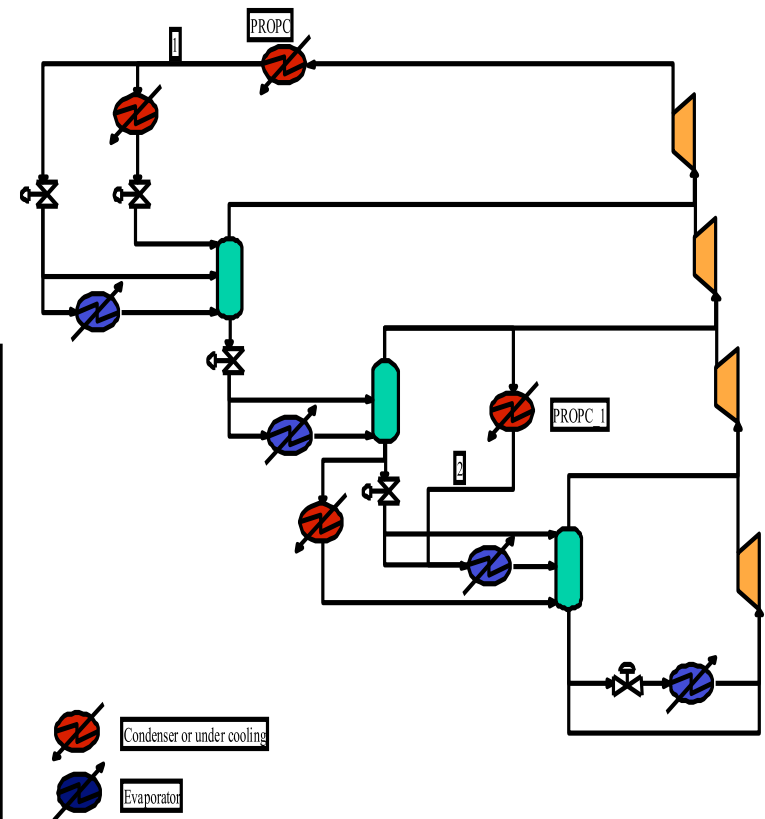


The refrigeration system

- Multi components
- Multi pressure levels
- Methane : 1 level
- Ethylene : 3 levels
- Propylene : 4 levels

Cycle	T evaporator (K)
METH1	136
ETH1	171,6
ETH2	199,25
ETH3	212,51
PROP1	233
PROP2	248
PROP3	277
PROP4	291

Complex systems



Refrigeration effect : reference flow = flow in the condenser

Condenser : hot stream

$$q_{cond}^{c,r} = (h_{out_comp}^r - h_{out_cond}^r)$$

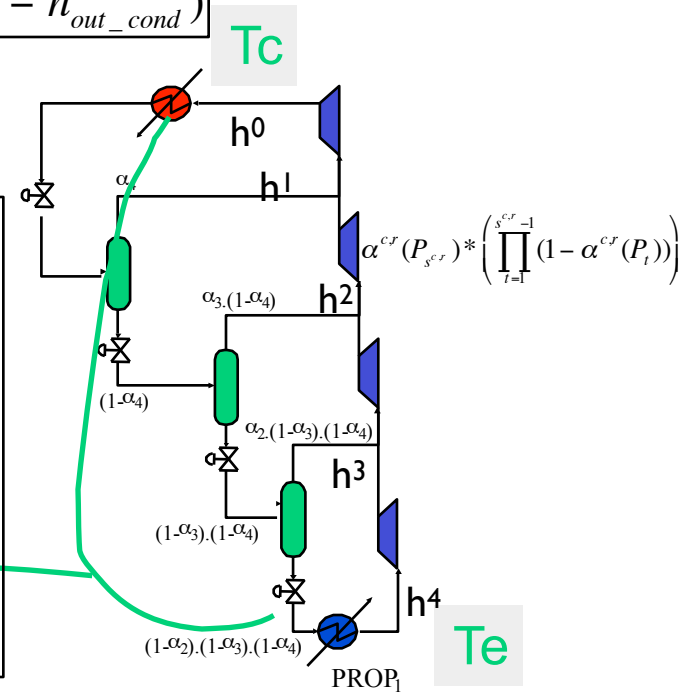
Compression : mechanical power

$$w^{c,r} = \left\{ \prod_{t=1}^{n_s^{c,r}-1} ((1 - \alpha^{c,r}(P_t))) \right\} * (h_{out_comp}^r - h_{sat_vap}^{c,r}(P_{n_s^{c,r}})))$$

$$+ \sum_{s=1}^{n_s^{c,r}} \alpha^{c,r}(P_s) * \left\{ \prod_{t=1}^{s-1} ((1 - \alpha^{c,r}(P_t))) \right\} * (h_{out_comp}^r - h_{sat_vap}^{c,r}(P_s)))$$

$$w_{PROP_1} = (1 - \alpha_1) * (1 - \alpha_2) * (1 - \alpha_3) * (h^4 - h^0) + \alpha_3(1 - \alpha_1) * (1 - \alpha_2) * (h^3 - h^0)$$

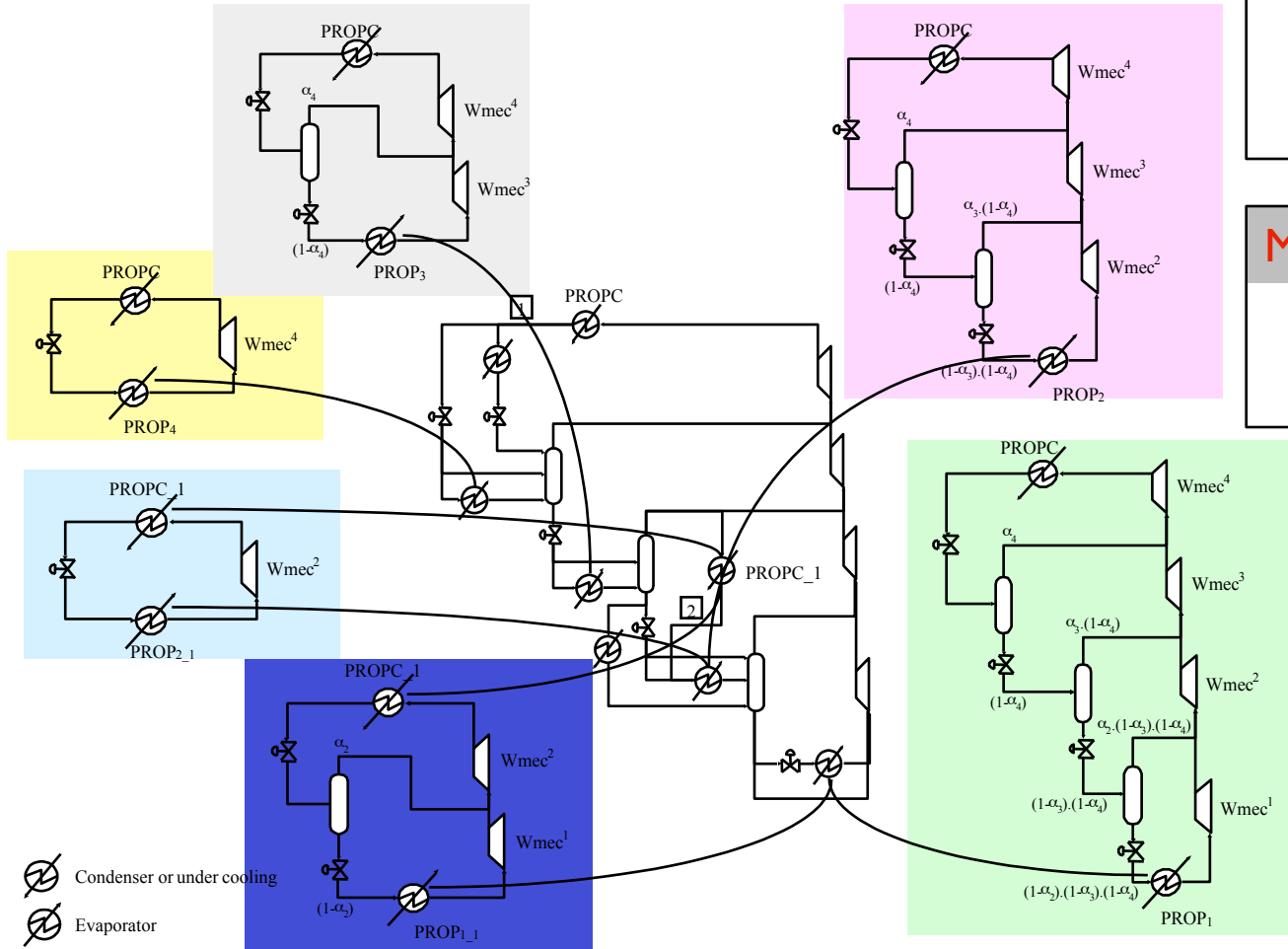
$$+ \alpha_2(1 - \alpha_1) * (h^2 - h^0) + \alpha_1(h^1 - h^0)$$



Evaporation : cold stream

$$q_{eva}^{c,r} = (h_{out_cond}^r - h_{sat_vap}^{c,r}(P_{n_s^{c,r}})) = \left\{ \prod_{t=1}^{n_s^{c,r}} ((1 - \alpha^{c,r}(P_t))) \right\} * (h_{sat_liq}^{c,r}(P_{n_s^{c,r}}) - h_{sat_vap}^{c,r}(P_{n_s^{c,r}}))$$

Propylene cycle model



Heat load in the condenser

$$Q_{cond}^r = \sum_{c=1}^{n_c^r} f^{c,r} * q_{cond}^{c,r}$$

Mechanical power

$$W^r = \sum_{c=1}^{n_c^r} f^{c,r} * w^{c,r}$$

Optimisation model

- Goal : to compute the optimal flow-rate in each effect

$$\text{Minimise}_{R_k, y_w, f_w} \sum_{w=1}^{n_w} (y_w C1_w + f_w C2_w) + Cel * EL_i - Cel_o * EL_o$$

subject to :

heat balance of the temperature interval k

$$\sum_{w=1}^{n_w} f_w q_{wk} + \sum_{r=1}^{n_r} \sum_{c=1}^{n_c^r} f^{c,r} q_{cond,k}^{c,r} - \sum_{r=1}^{n_r} \sum_{c=1}^{n_c^r} f^{c,r} q_{eva,k}^{c,r} + \sum_{i=1}^n Q_{ik} + R_{k+1} - R_k = 0$$

$$WC_r - \sum_{c=1}^{n_c^r} f^{c,r} * w^{c,r} = 0$$

$$f \min^{c,r} y_w \leq f^{c,r} \leq f \max^{c,r} y_w, y_w \in \{0,1\}$$

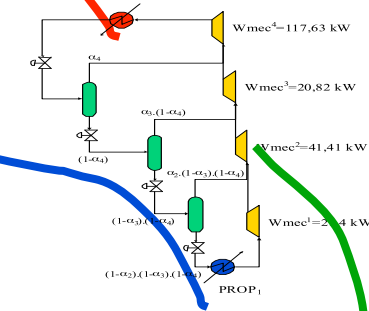
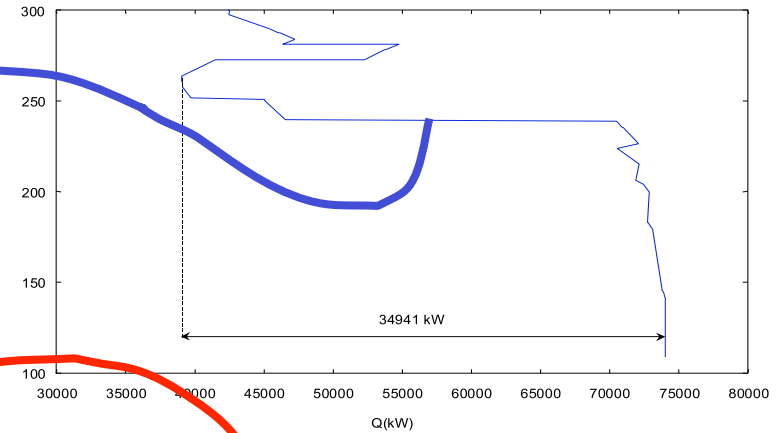
$$\text{Electricity production : } \sum_{w=1}^{n_w} f_w * w_w - \sum_{r=1}^{n_r} WC_r + EL_i - EL_o = 0$$

$$\text{consumption : } \sum_{w=1}^{n_w} f_w * w_w - \sum_{r=1}^{n_r} WC_r + EL_i \geq 0$$

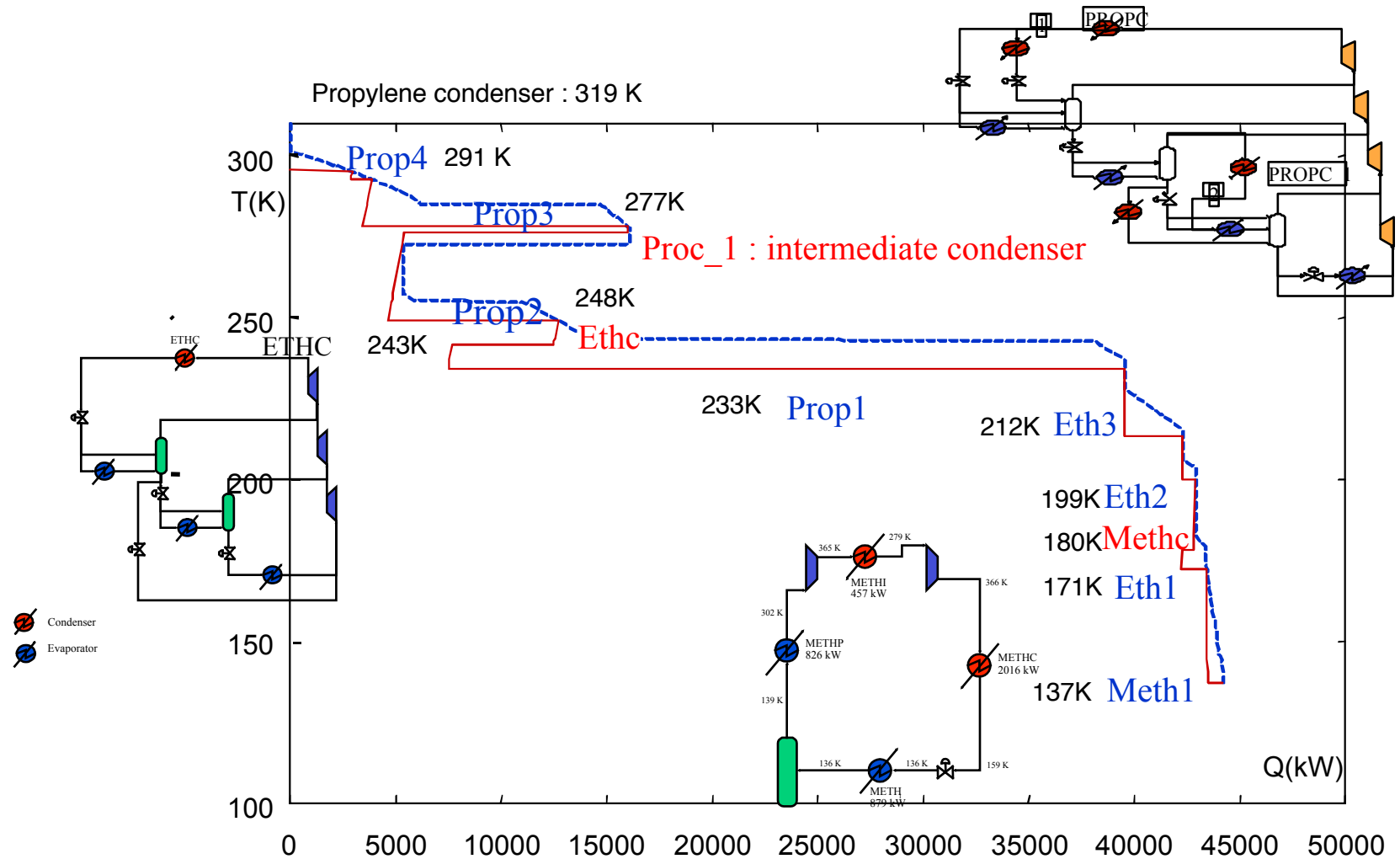
$$f \min_w y_w \leq f_w \leq f \max_w y_w, y_w \in \{0,1\}$$

$$R_k \geq 0 \quad \forall k = 1, \dots, n_k + 1$$

$$R_1 = 0, R_{n_k+1} = 0$$



Integrated process and refrigeration system



After process modifications

	Actual	Optimized	Simulation	Off streams
	Wmec	Wmec	Wmec	Wmec
	kW	kW	kW	kW
PROP1	17297	12685	-	10924
PROP2	5314	4281	-	4114
PROP3	6598	4759	-	4589
PROP4	1489	377	-	357
PROP1_1	1520	2029	-	2061
PROP2_1	0	0	-	0
Total propylene	32218	24131	26284	22045
ETH1	919	705	-	281
ETH2	312	463	-	248
ETH3	1378	655	-	343
Total ethylene	2609	1823	2001	872
METH1	746	655	655	0
Off streams contribution	0	0	-	-827
TOTAL	35573	26609	28940	22090

17%

38%