EPFL

Optimising the investment of heat exchanger

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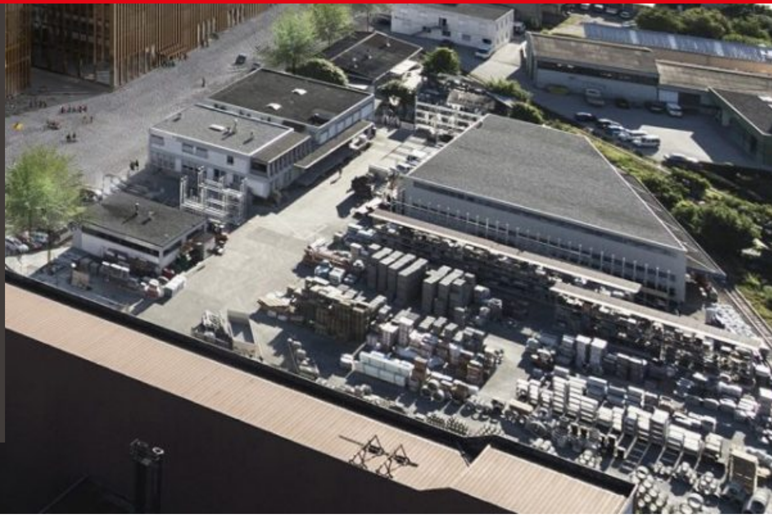
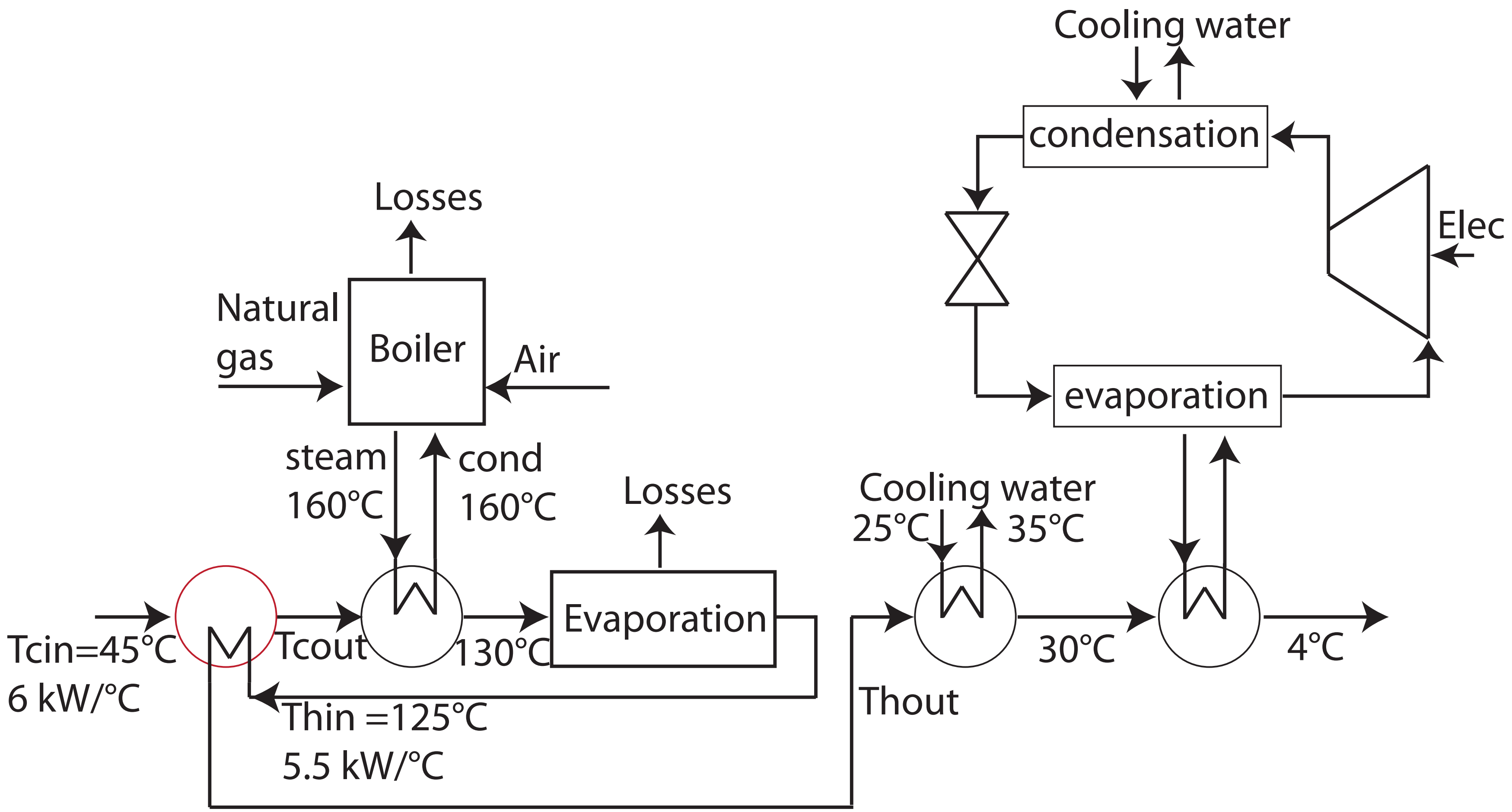


IPESE
Industrial Process
and Energy Systems
Engineering

Heat recovery by adding a heat exchanger



- Project evaluation
  - Buying a new heat exchanger

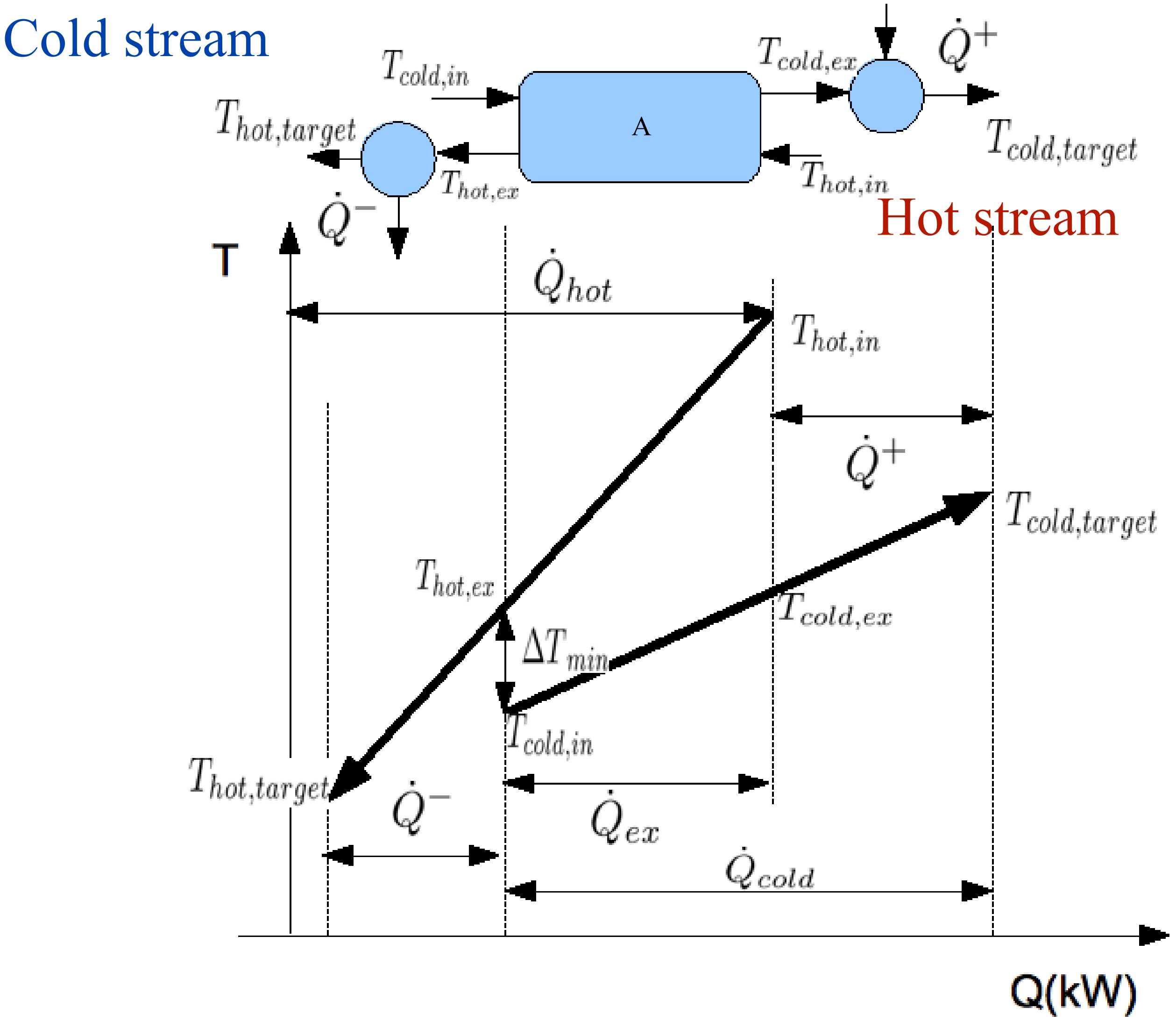
$$CAPEX[CHF/year] = \frac{I}{I_{ref}} k_1 (A)^{k_2} \frac{i(1+i)^n}{(1+i)^n - 1}$$

  - Energy savings : Operating cost reduction (OPEX)

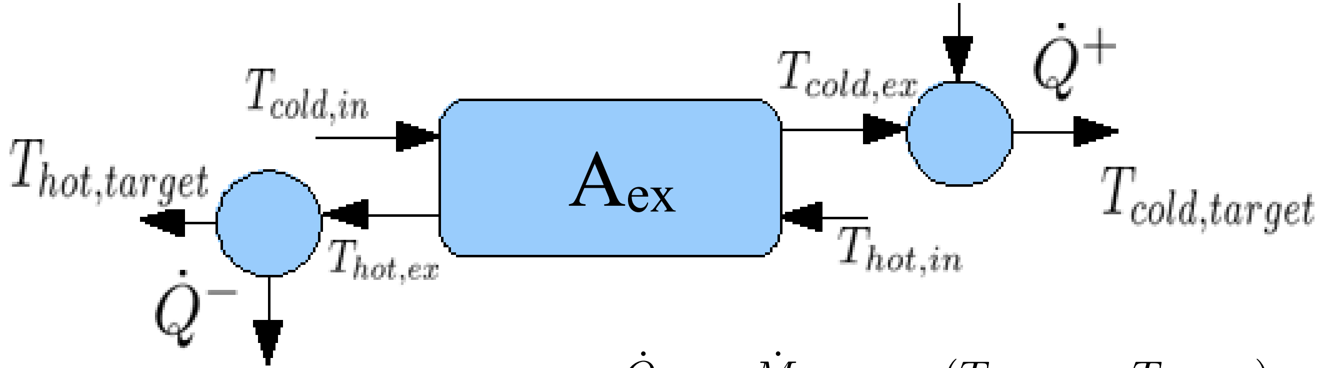
$$\Delta OPEX = \int_{years} \dot{m}^{0}_{energy}(t) c_{energy}(t) dt - \int_{years} \dot{m}^{new}_{energy}(t) c_{energy}(t) dt$$

- Compare ΔOPEX(+) and CAPEX(-)

Heat recovery : Heat exchanger



Heat exchanger model



$$\dot{Q}_{ex} = \dot{M}_{hot}\, c_{p\,hot}(T_{hot,in} - T_{hot,ex})$$

$$\dot{Q}_{ex} = \dot{M}_{cold}\, c_{p\,cold}(T_{cold,ex} - T_{cold,in})$$

$$\Delta T_{lm} = \frac{(T_{hot,in} - T_{cold,ex}) - (T_{hot,ex} - T_{cold,in})}{ln\left(\frac{(T_{hot,in} - T_{cold,ex})}{(T_{hot,ex} - T_{cold,in})}\right)}$$

$$A_{ex} = \frac{\dot{Q}_{ex}}{U_{ex}\Delta T_{lm}}$$

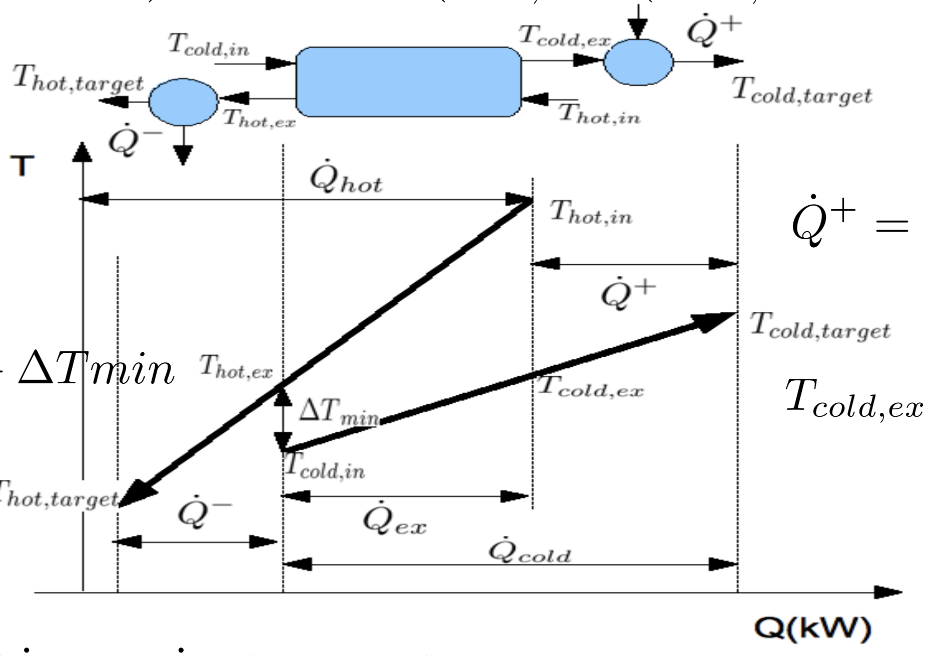
$$\frac{1}{U_{ex}} = \frac{1}{\alpha_{cold}} + \frac{e}{\lambda} + \frac{1}{\alpha_{hot}}$$

With:

U_{ex}	$[kW/m^2/K]$	the global heat transfer coefficient of the heat exchanger;
α_{cold}	$[kW/m^2/K]$	the film heat transfer coefficient of the cold stream;
α_{hot}	$[kW/m^2/K]$	the film heat transfer coefficient of the hot stream;
λ	$[kW/m/K]$	the thermal conductivity of the tubes;
e	[m]	the thickness of the tubes.

Heat exchanger : sequential resolution

2 $$\dot{Q}_{ex}(\Delta Tmin) = \dot{M}_{hot}cp_{hot}(T_{hot,in} - (T_{cold,in} + \Delta Tmin))$$



5 $$\dot{Q}^{+} = (\dot{Q}_{cold} - \dot{Q}_{ex}(\Delta T_{min}))$$

1 $$T_{hot,ex} = T_{cold,in} + \Delta Tmin$$

3 $$T_{cold,ex} = T_{cold,in} + \frac{\dot{Q}_{ex}}{\dot{M}_{cold}cp_{cold}}$$

4 $$A_{ex} = \frac{\dot{Q}_{ex}}{U_{ex}\Delta T_{lm}}$$

6 $$\dot{Q}^{-} = (\dot{Q}_{hot} - \dot{Q}_{ex}(\Delta T_{min}))$$

Calculation algorithm for each DTmin

$$T_{hot,ex} = T_{cold,in} + \Delta Tmin$$

$$\dot{Q}_{ex}(\Delta Tmin) = \dot{M}_{hot} cp_{hot}(T_{hot,in} - (T_{cold,in} + \Delta Tmin))$$

$$\dot{Q}^{+} = (\dot{Q}_{cold} - \dot{Q}_{ex}(\Delta T_{min}))$$

$$\dot{Q}^{-} = (\dot{Q}_{hot} - \dot{Q}_{ex}(\Delta T_{min}))$$

$$T_{cold,ex} = T_{cold,in} + \frac{\dot{Q}_{ex}}{\dot{M}_{cold} cp_{cold}}$$

$$\Delta T_{lm} = \frac{(T_{hot,in} - T_{cold,ex}) - (T_{hot,ex} - T_{cold,in})}{ln\left(\frac{(T_{hot,in} - T_{cold,ex})}{(T_{hot,ex} - T_{cold,in})}\right)}$$

$$A_{ex}(\Delta T_{min}) = \frac{\dot{Q}_{ex}(\Delta T_{min})}{U_{ex} \Delta T_{lm}(\Delta T_{min})}$$

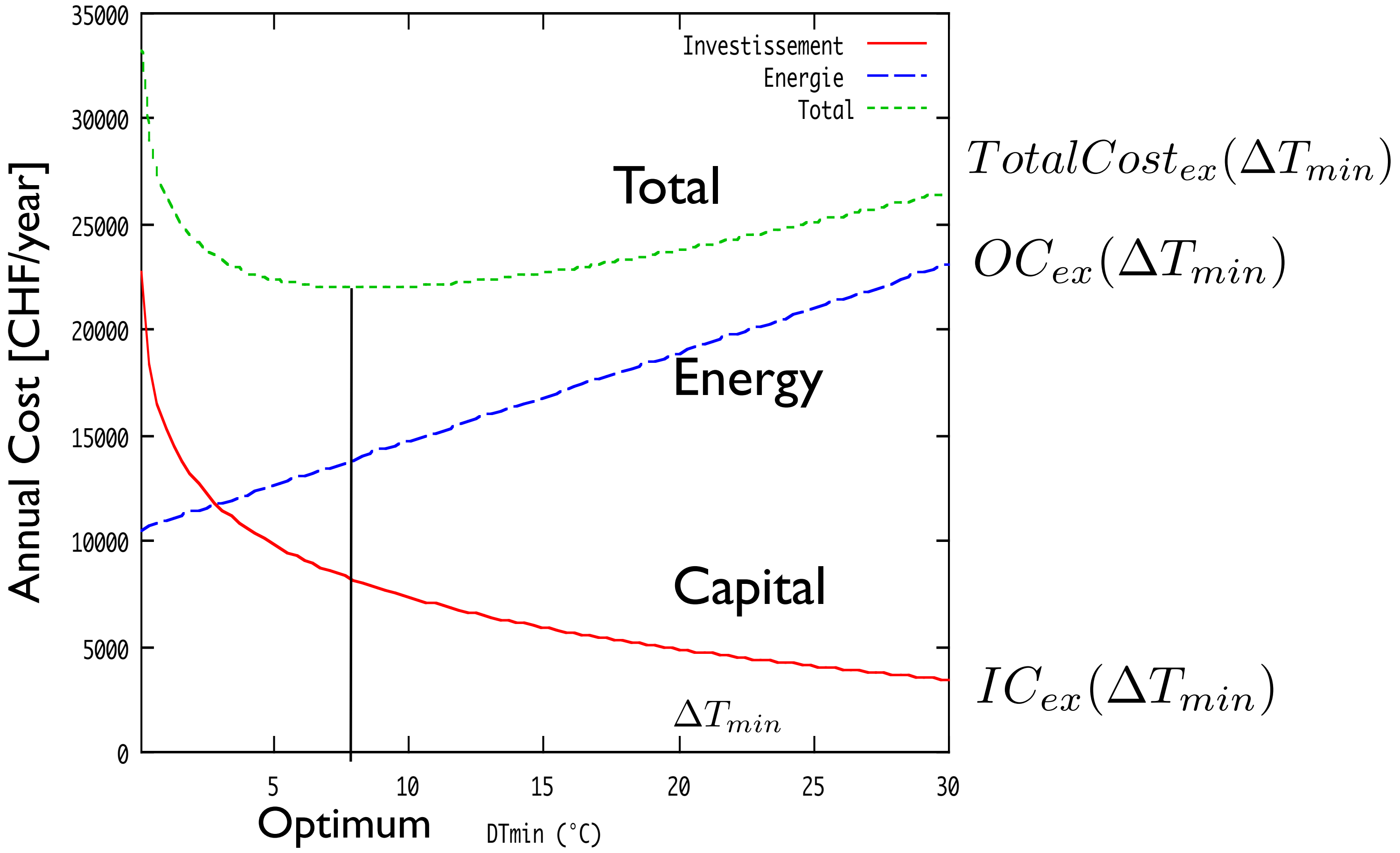
$$OC_{ex}(\Delta T_{min}) = (c_{cold} \cdot (\dot{Q}_{hot} - \dot{Q}_{ex}(\Delta T_{min})) + c_{hot} \cdot (\dot{Q}_{cold} - \dot{Q}_{ex}(\Delta T_{min}))) \cdot time_{year}$$

$$IC_{ex}(\Delta T_{min}) = (\frac{i(1+i)^{ny_{ex}}}{(1+i)^{ny_{ex}} - 1}) a_{ex}(A_{ex}(\Delta T_{min}))^{b_{ex}}$$

$$TotalCost_{ex}(\Delta T_{min}) = OC_{ex}(\Delta T_{min}) + IC_{ex}(\Delta T_{min})$$

ΔTmin value : optimization problem

Annual cost as a function of ΔTmin



$$\Delta OC_{ex}(\Delta T_{min}) = \Delta IC_{ex}(\Delta T_{min})$$

ΔTmin value : optimization problem

Decision variable

$$min_{\Delta T_{min}} TotalCost = OC_{ex}(\Delta T_{min}) + IC_{ex}(\Delta T_{min})$$

Objective function

$$OC_{ex}(\Delta T_{min}) = (c_{cold} \cdot (\dot{Q}_{hot} - \dot{Q}_{ex}(\Delta T_{min})) + c_{hot} \cdot (\dot{Q}_{cold} - \dot{Q}_{ex}(\Delta T_{min}))) \cdot time_{year}$$

$$IC_{ex}(\Delta T_{min}) = (\frac{i(1+i)^{ny_{ex}}}{(1+i)^{ny_{ex}} - 1}) a_{ex}(A_{ex}(\Delta T_{min}))^{b_{ex}}$$

Programming language : OCTAVE /MATLAB

Specifications of the problem

```
function g=init
% definition of the main parameters
%
% Cold stream
%
g.TinCold=45;
g.ToutCold=100;
g.McpCold=6;
%
% Hot Stream
%
g.TinHot=85;
g.ToutHot=30;
g.McpHot=4.5;
%
% Cost data
%
g.Interest=0.08;
g.Nyears=15;
g.OperatingTime=2000;
g.U=0.5/2;
g.HotUtilityCost=0.06/0.85;% chf/kWh
g.ColdUtilityCost_m3=0.01;% CHF/m3
g.ColdUtilityCost_kWh=g.ColdUtilityCost_m3/1000/41.86*3600;
```

OCTAVE/MATLAB model

```
function [Exchanger Cost ] = exchanger(DTmin,g)
Exchanger.ToutHex_h=g.TinCold+DTmin;
Exchanger.HeatLoad=g.McpHot*(g.TinHot-Exchanger.ToutHex_h);
Exchanger.ToutHex_c=g.TinCold+Exchanger.HeatLoad/g.McpCold;
Cost.Qhot=g.McpCold*(g.ToutCold-Exchanger.ToutHex_c);
Cost.Qcold=g.McpHot*(Exchanger.ToutHex_h-g.ToutHot);
dt1=Exchanger.ToutHex_h-g.TinCold;
dt2=g.TinHot-Exchanger.ToutHex_c;
Exchanger.DTlm=(dt1-dt2)/log(dt1/dt2);
Exchanger.Area=Exchanger.HeatLoad/g.U/Exchanger.DTlm;
Cost.Purchased=750*(Exchanger.Area)^0.7;
Cost.Capex=Cost.Purchased*4.54/400*510*annualisation(g.Nyears,g.Interest);
Cost.Opex=(Cost.Qhot*g.HotUtilityCost+Cost.Qcold*g.ColdUtilityCost_kWh)*g.OperatingTime;
Cost.TotalCost=Cost.Opex+Cost.Capex;
end
```

DOF => x = DTmin

Model

- sequential resolution
- Calculate CT(x)

```
function annualisation=annualisation(nyears,interest)
annualisation=interest*(1+interest)^nyears/((1+interest)^nyears-1);
endfunction
```

Octave 1 D method


```
% first call the initialisation function and fill the g structure
ProblemData=init
% solving procedure, we are just doing an enumeration
% initalising for the loop
MinTotalCost=10000000000;
DTmin0=1.0;
DTminOpt=DTmin0;
DTminMax=ProblemData.TinHot-ProblemData.TinCold;
Delta=1;
iopt=1;
nstep=200;
Delta=(DTminMax-DTmin0)/nstep;
printf('DTmin :    TotalCost :     OPEX :     CAPEX \n')
for i=1:nstep-1
  DTmin(i)=DTmin0+Delta*(i-1);
  [ExchangerResults(i) Cost(i)]=exchanger(DTmin(i),ProblemData);
  printf('%i : %f : %f : %f : %f\n',i,DTmin(i),Cost(i).TotalCost,
...
         Cost(i).Opex, Cost(i).Capex)
  if Cost(i).TotalCost < MinTotalCost
    DTminOpt=DTmin(i);
    MinTotalCost=Cost(i).TotalCost;
    iopt=i;
  end
end
print_solution(DTminOpt,Cost(iopt),ExchangerResults(iopt))
```

Black box

- Iteration on x
- Keep the min value of CT(x)
- CT(DTMIN)=Cost.TotalCost

OCTAVE/MATLAB SQP method

```
% first call the initialisation function and fill the g structure
ProblemData=init
% solving procedure, we are just doing a loop
% initalising for the loop
function obj=objective(x,ProblemData)
  [ExchangerResults Cost]=exchanger(x,ProblemData);
  obj=Cost.TotalCost;
% $$$   printf(' %f : %f : %f : %f\n',x,Cost.TotalCost, ...
% $$$          Cost.Opex, Cost.Capex)
  endfunction
DTmin0 = 4;
DTminMin=1;
DTminMax=ProblemData.TinHot-ProblemData.TinCold-1;
% solving with an sqp algorithm
         [DTmin, obj, info, iter, nf, lambda] = sqp (DTmin0, @(x)
objective(x,ProblemData), [], [],DTminMin,DTminMax,[]);

  [ExchangerResults Cost]=exchanger(DTmin,ProblemData);
print_solution(DTmin,Cost,ExchangerResults)
```

Calculation of CT(x)

Solving with SQP method :
min CT(x)

Programming Language : AMPL
(Advanced Mathematical Programming Language)

Simultaneous solving strategy

```
################################
# DTmin optimisation
################################

# parameters
param TinCold :=45 ;  # T cold in [C]
param ToutCold :=100 ;#[C]
param MCPc :=6 ;
param TinHot :=85 ; # T hot in [C]
param ToutHot :=30;#[C]
param MCPh :=4.5;
param Uex :=0.5/2 ;
param Ccold :=0.01/1000/41.86*3600; #(3600/4186)0.00086[CHF/kwh]
param Chot :=0.06;
param duration :=2000;#[h]
param i:=0.08 ;
param n:=15 ;
param ah:=750;
param bh:=0.7;
param Inew:= 510.0;
param Iref:=400.0;
param Fbm:=4.74;
param alfa:=1.18;
param eff_gas:=0.85;
```

Parameters = Specs

```
# variables
var Qex >=0 ;
var dt1 >=0.00001 ;
var dt2 >=0.00001 ;
#var Thex >=Thtar, =<Thin;
#var Tcex >=Tcin, =<Tctar;
var ToutHexHot >=ToutHot;  #[C]
var ToutHexCold
>=TinCold;    #[C]
var Aex >=0.000001 ;
var LNex >=0.00001 ;
var DTmin >=0.00001;
var Cop ;
var Cin ;
var objectiv ;
var Cinb;
var Cinref ;
var Cintot ;
```

Variables : state variables inc. decision variables : X

```
# constraints

subject to con1:
ToutHexHot>=TinCold+DTmin;

subject to con2:
TinHot>=ToutHexCold+DTmin;

subject to con44:
```

Constraints

$F(X)= 0$

$G(X)\geq 0$

AMPL model 2

```
Aex >=0.1 ;

subject to con6:
dt1=TinHot-ToutHexCold ;

subject to con7:
dt2=ToutHexHot-TinCold ;

subject to con8:
MCPc*(ToutHexCold-TinCold)=MCPh*(TinHot-ToutHexHot);

subject to con9:
Qex=MCPh*(TinHot-ToutHexHot);

subject to con10:
LNex=(dt1-dt2)/(log(dt1/dt2));

subject to con11:
Qex=Uex*Aex*LNex;


subject to con12:
#Cop=(Ccold*((MCPh*(Thin-Thtar))-Qex)+Chot*((MCPc*(Tctar-Tcin)-
Qex)/eff_gas))*duration;
Cop=(Ccold*((MCPh*(TinHot-ToutHot))-Qex)+Chot*((MCPc*(ToutCold-
TinCold)-Qex)/eff_gas))*duration ;

subject to con13:
#Cin=((i*(1+i)^n)/(((1+i)^n)-
1))*alfa*Fbm*(Inew/Iref)*10^(k1+k2*log(Aex))*1.02; #CHF
Cin=((i*(1+i)^n)/(((1+i)^n)-1))*Fbm*(Inew/Iref)*ah*(Aex)^bh; #CHF
/year


subject to con17:
#objectiv=Cop+Cin;
objectiv=Cop+Cin;


minimize obj : objectiv;
solve;
display DTmin;
display min((ToutHexHot-TinCold),(TinHot-ToutHexCold));
display Cop;
display Cin;
display Aex;
display Qex;
display LNex;
end;
```

CT(X)