Solving strategies for the optimisation of energy systems

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IPESE
Industrial Process and Energy Systems Engineering
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Key messages

- What is an optimization strategy ?
- How can we use a model to state an optimization problem ?
  - Define of solving strategy to use an optimization algorithm
- What are the advantages and drawbacks of the solving strategies ?

Thermo-economic Model

System model

$$F(X_{state}, \pi) = 0 \quad \text{Model equations}$$

$$S(X_{state}, \pi) = 0 \quad \text{Context specifications}$$

$$with \quad \pi \quad \text{Model parameters}$$

$$X_{state} \quad \text{State variables}$$

DOF : Degree of freedom of the system

DOF of the system = N_{state variables} - N_{Model equations}

Decision variables

N_{Decision variables} = DOF - N_{Context specifications}

$$X_{d,min} \leq X_d \leq X_{d,max}$$

- Degree of freedom for which I do not have any rationale to fix the value
- I only know the possible range

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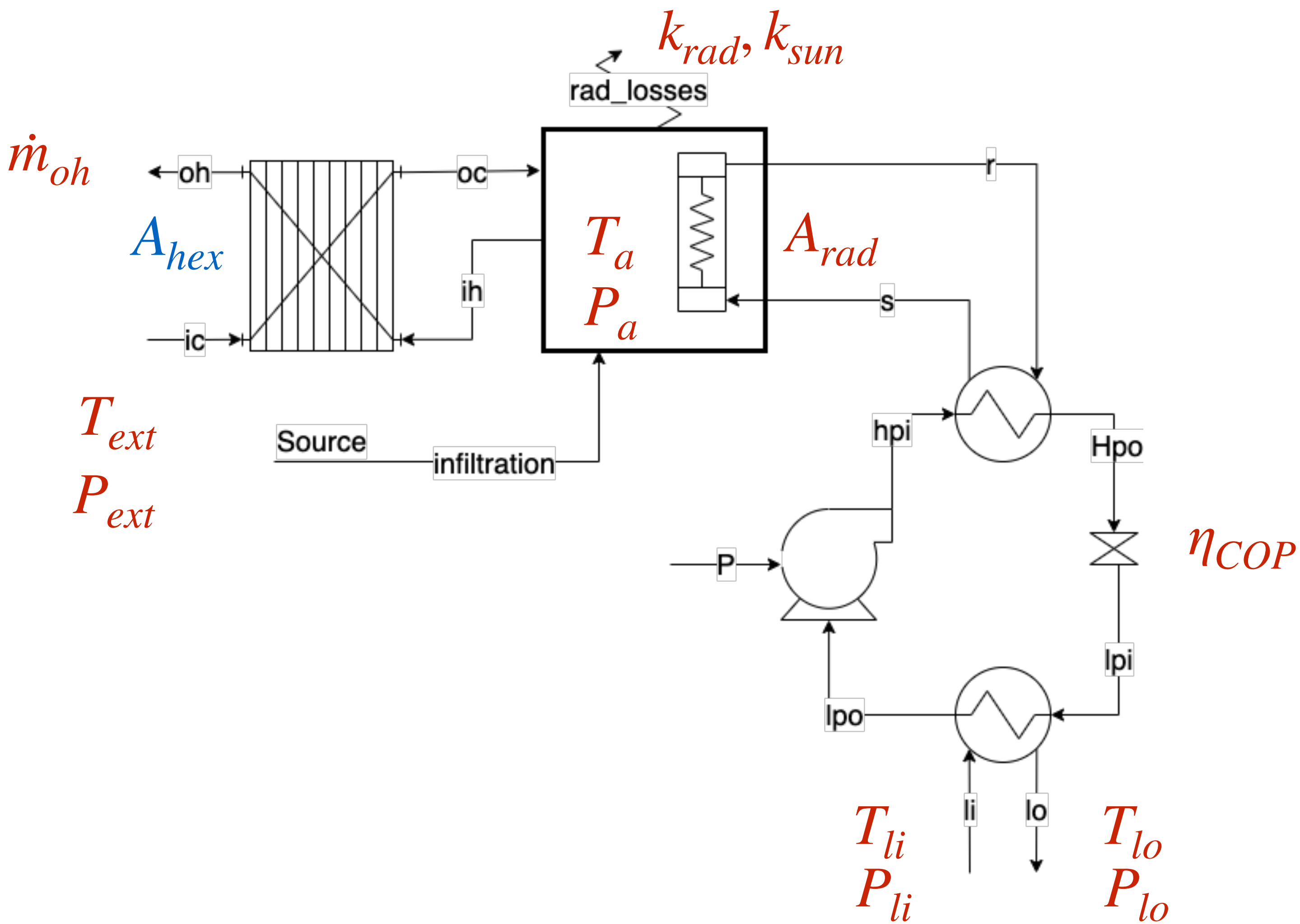
Heat recovery by a double flux heat exchanger

- Degrees of freedom ?
  - Specifications
  - Decisions



Example of objective function

$$TotalCost[CHF/year] = OPEX + CAPEX + Tax$$

$$OPEX = \sum_{p=1}^{n_p}(\sum_{r=1}^{n_r} \dot{m}^+_{r,p} c^+_{r,p} + \dot{E}^+_p c^+_{e,p} - \dot{E}^-_p c^-_{e,p} + \sum_{u=1}^{n_u} f_{u,p} cm_u) d_p$$

$$CAPEX = \sum_{u=1}^{n_u} \frac{1}{\tau(n_{y,u}, i)} (I1_u y_u + I2_u f_u^{max})$$

$$Tax = CO_2^+ \gamma^{CO_2^+}$$

$$CO_2^+ = \sum_{p=1}^{n_p}(\sum_{r=1}^{n_r} \dot{m}^+_{r,p} \epsilon_r^{CO_2} + \dot{E}^+_p \epsilon_{e,p}^{CO_2^+} - \dot{E}^-_p \epsilon_{e,p}^{CO_2^-}) d_p$$

$$Impact = \zeta^{CO_2^+} (CO_2^+ + \sum_{u=1}^{n_u} \frac{1}{n_{y,u}} (\xi_{c_u}^{CO_2} + \xi_{d_u}^{CO_2}) f_u^{max})$$

$$RES = \sum_{p=1}^{n_p}(\sum_{r_{res}=1}^{n_{r_{res}}} \dot{m}^+_{r_{res},p} + \sum_{u=1}^{n_u} f_{u,p} e_{u,p}^{res^+}) d_p$$

$$\dot{E}^+_p + \dot{E}^-_p + \sum_{u=1}^{n_u} f_{u,p}(e_{u,p}^{res^+} - e^-_{u,p}) = 0 \qquad \forall p = 1..n_p$$

Thermo-economic optimisation

Fixing the value of the decision variables

$$X_{d,min} \leq X_d \leq X_{d,max}$$

- Degree of freedom for which I do not have any rationale to fix the value
- I only know the possible range
- I would like to fix the value of X_d so that it minimise an objective function

$$\min_{X_{state}} \quad TotalCost(X_{state}, \pi)$$

Objective function

$$\begin{aligned}
s.t. \quad & F(X_{state}, \pi) = 0 \Rightarrow \text{equipment model} \\
& S(X_{state}, \pi) = 0 \Rightarrow \text{Specification equations} \\
& G(X_{state}, \pi) \leq 0 \quad \text{Inequality constraints} \\
& X_{state}^{min} \leq X_{state} \leq X_{state}^{max} \quad \text{Bounds}
\end{aligned}$$

$where$

$$X_{state} = \{x_{statevariables}, x_{UnitParameters}, y_{decision} \epsilon \{0,1\}\}$$

X_d is a subset of X_{state}

Inequality constraints

Constraints limit the search space for the values of X_d

- **Operating limits**
  - safety, equipment limits,
- **Regulation**
  - Emission : environment
- **Market limits => products quality**
- **Technology limits and heuristics**
  - Materials
- **Models limits**
  - Correlations
  - Structures and models
- **Numerical safety ;Two types**
  - Hard : can not be violated otherwise crash
    - E.g. flow inversion
    - try to place the hard constraints on the variables
  - Soft : may be violated during resolution

Different types of problems

- **Optimal operation**
  - Decision : Operation set point
  - Objective : Operating cost (CHF/s)
- **Optimal operation stratgey**
  - Decision : Set point strategy + start/stop
  - Objective : Operating cost (CHF/period)
- **Scheduling**
  - Decision : Set points strategy, start/stop, when ?, in which equipment ?
  - Objective : Operating cost (CHF/period)

Different types of problems

- **Optimal sizing**
  - Decision : operating set points, unit sizes, investment
  - Objective : Total cost (CHF, CHF/an)
- **Optimal design**
  - Decision : operating set points (Operating strategy), unit sizes, investment, configuration
  - Objective : Total cost (CHF, CHF/an)
- **Retrofit (reuse of existing equipment/configuration)**
  - Decision : operating set points (Operating strategy), unit sizes, configuration, reuse, reconfiguration, investments
  - Objective : total cost

Black box method

Optimisation : min $OBJ(X^*_{decision})$

Subject to $G_{inequality}(X^*_{decision}) \geq 0$

$X^*_{decision}$

$OBJ(X^*_{decision})$

$G(X^*_{decision})$ inequality

Status

Model : Solve

$F(X_{dependent}, X_{specification}, X_{decision})=0$

$S(X_{dependent}, X_{specification}, X_{decision})=0 \Rightarrow X(X^*_{decision})$

$X_{decision} - X^*_{decision} = 0$

then calculate $OBJ(X(X^*_{decision}))$

$G(X(X^*_{decision}))$

Black Box strategy

$$\min_{X^*_{decision}} \quad TotalCost(X^*_{decision}, X(X^*_{decision}), \pi)$$

$$s.t. \quad G(X^*_{decision}, X(X^*_{decision}), \pi) \leq 0 \quad \text{inequality constraints}$$

$$where$$

$$X^*_{decision} = \{x_{decision}, y_{decision} \epsilon \{0,1\}\}$$

$X(X^*_{decision})$ Calculated by solving:

NxN problem

$$\begin{cases} F(X_{state}, \pi) = 0 \Rightarrow \text{model equations} \\ S(X_{state}, \pi) = 0 \Rightarrow \text{Specification equations} \\ X_{state} - X^*_{decision} = 0 \Rightarrow \text{Specification of the value of decision variab} \end{cases}$$

$$where$$

$$X_{state} = \{x_{StateVariables}, x_{UnitParameters}, y_{decision} \epsilon \{0,1\}\}$$

Black Box approach

+ Can be used in any kind of optimisation method

Heuristic, direct et indirect (derivatives)

+ Solving method to be defined => problem analysis
+ Selection of $X^*_{decision}$ by the user
+ Robustness
+ Non convergence analysis
+ Gives a list of system states

- Flexibility
- Computation time
- Inequality might be a problem when not on decision variables
- Derivative calculation (especially when done in iterative loops)

$$\frac{\partial f}{\partial x_i} = \frac{f(X + \Delta x_i) - f(X)}{\Delta x_i}$$

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Numerical noise in derivative calculations

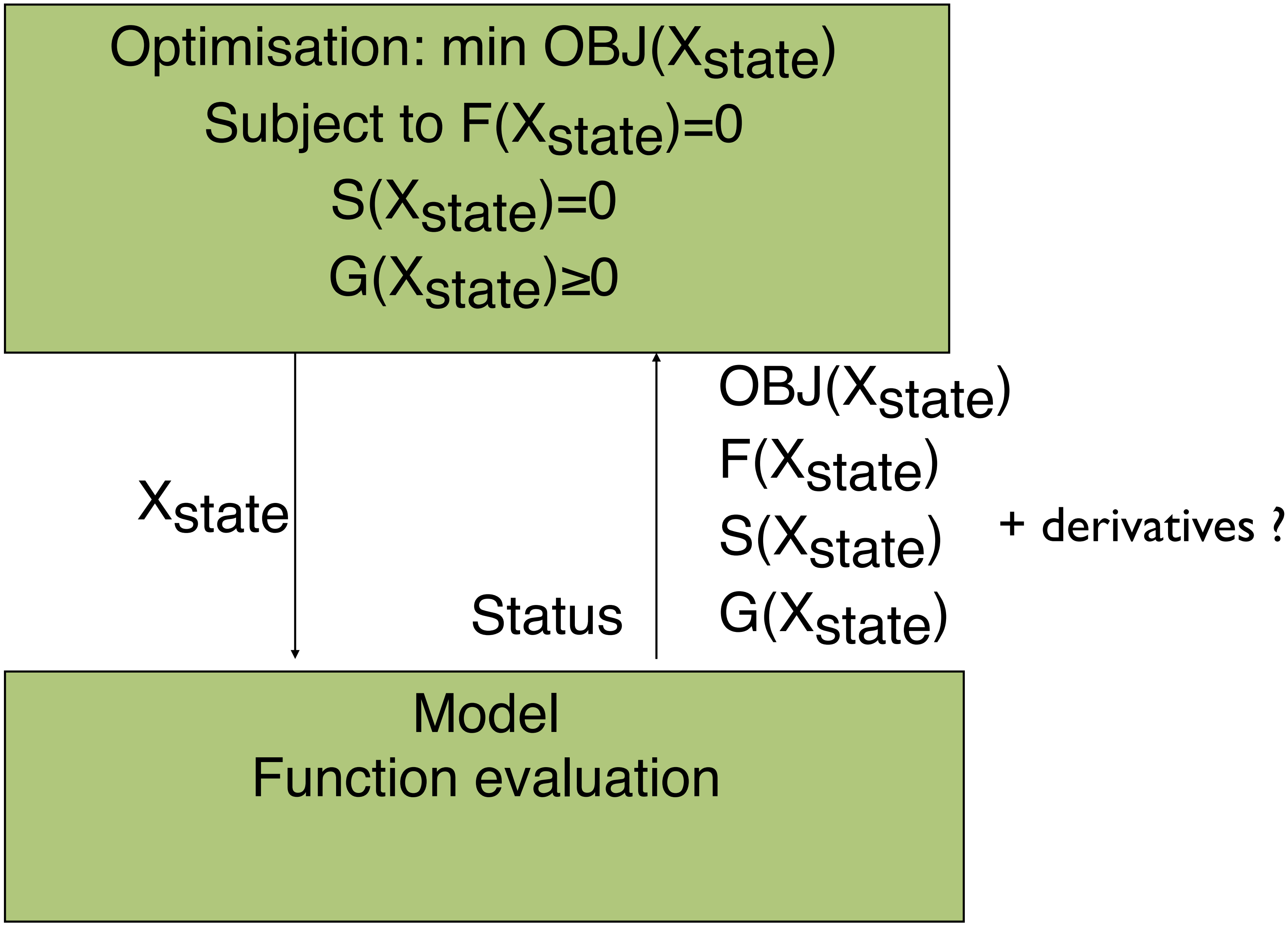
- Derivative

$$\frac{\partial f}{\partial x_i} = \frac{f(X + \Delta x_i) - f(X)}{\Delta x_i} \quad \forall i = 1, \ldots, n_X + 1$$

- Noice

$$\Delta f(X) = |F(X)_0 - F(X)_{n_X+1}|$$

Simultaneous method



Simultaneous strategy

$$
\begin{aligned}
\min_{X_{state}} \quad & TotalCost(X_{state}, \pi) \\
s.t. \quad & F(X_{state}, \pi) = 0 \Rightarrow \text{equipment model} \\
& S(X_{state}, \pi) = 0 \Rightarrow \text{Specification equations} \\
& G(X_{state}, \pi) \leq 0 \quad \text{Inequality constraints} \\
& X_{state}^{min} \leq X_{state} \leq X_{state}^{max} \quad \text{Bounds} \\
where & \\
& X_{state} = \{x_{statevariables}, x_{UnitParameters}, y_{decision} \epsilon \{0,1\}\}
\end{aligned}
$$

Requires : Non Linear Constrained Optimisation solving method

X_d is not defined (hidden in X_{state})

Simultaneous approach

+ Optimisation under constraints

  => indirect method with 2nd derivatives if possible

+ Flexibility : different pbm with the same model (changing specification set and not the model)

+ Efficient and robust for on-line optimisation systems

  Bounds definition !

+ Computing time

+ Automatic DOF analysis and sensitivity analysis

+ Hard and soft inequality constraints

- Initialisation !
- Derivatives calculation (during function evaluation, symbolic ?)
- No system state when no convergence
- Scaling !
- No explanations when the problem does not converge !
- Push button system?

Two levels approach

Optimisation:

$$\min OBJ(X^*_{decision}, X^*_{specification})$$

$$\text{Subject to } H(X^*_{decision}, X^*_{specification})=0$$

$$G(X^*_{decision}, X^*_{specification})\geq 0$$

Requires a constrained optimisation solver

$X^*_{decision}$,
$X^*_{specification}$

Status

$OBJ(X^*_{decision}, X^*_{specification})$

$H(X^*_{decision}, X^*_{specification})$
$G(X^*_{decision}, X^*_{specification})$
+ derivatives ?

Modèle

$$X_{dependent} = \Phi(X^*_{decision}, X^*_{specification})$$

Two level strategy

$$\min_{X^*_{decision}, X^*_{Specs}} \quad TotalCost(X^*_{decision}, X^*_{Specs}, X(X^*_{decision}, X^*_{Specs}), \pi)$$

$$s.t. \quad H(X^*_{decision}, X^*_{Specs}, X(X^*_{decision}, X^*_{Specs}), \pi) = 0 \quad \text{some equality constraints}$$

$$G(X^*_{decision}, X^*_{Specs}, X(X^*_{decision}, X^*_{Specs}), \pi) \leq 0 \quad \text{inequality constraints}$$

$$where$$

$$X^*_{decision} = \{x_{decision}, y_{decision} \epsilon \{0,1\}\}$$

$$(X^*_{decision}, X^*_{specs}) \quad \text{Calculated by solving :}$$

(N+Ns)x(N+Ns) problem

$$\left\{ \begin{array}{l} F(X_{state}, \pi) = 0 \Rightarrow \text{system model} \\ S(X_{state}, \pi) = 0 \Rightarrow \text{Specification equations} \\ X_{decision} - X^*_{decision} = 0 \Rightarrow \text{Specification of the value of decision variables} \\ X_{specs} - X^*_{specs} = 0 \Rightarrow \text{Specification of the value of specification variables} \end{array} \right.$$

$$where$$

$$X_{state} = \{x_{StateVariables}, x_{Unitparameters}, y_{decision} \epsilon \{0,1\}\}$$

Two levels strategy

+ Best of both world

+Robustness of unit models (single calculation mode)

+Code Maintenance

+Derivative chaining is possible

    Analytical calculation is possible

+ Soft inequality constraints

+ Sensitivity analysis

- Conditional simulation

- Computing time (solving the lower level iteratively)

- Hard bounds (except when they are at the decision variable level).

- Noise in derivatives evaluation

    Internal iterative calculations