ME-474 Numerical Flow Simulation

Comments on the exercise: convergence study

Fall 2021

For simplicity, the fluid properties are modified as follows: density 1 kg/m^3 and dynamic viscosity 0.005 kg/(m.s). The inlet profile is defined with the "expression" `-(y-1[m])/1[m]*(y-2[m])/1[m]*1[m/s]*4`. The maximum velocity is $U = 1$ m/s, therefore the Reynolds number is $Re = Uh/\nu = 1/0.005 = 200$.

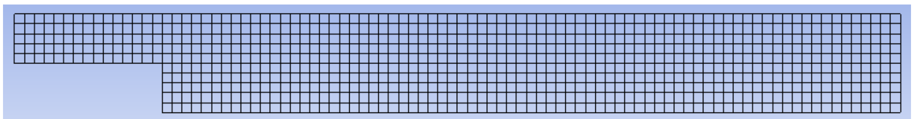

Figure 1: Initial (coarsest) mesh.

The initial mesh (mesh 1) is made of quadrilateral elements of size $\Delta x = 0.2$ m, yielding a total of $N = 825$ elements. Computations are performed with a second-order spatial discretization scheme for the momentum equation. The convergence criterion on the residuals is 10^{-5}. You can check that the default value 10^{-3} does not yield a fully converged solution, for instance by monitoring the velocity at some probes in the flow.

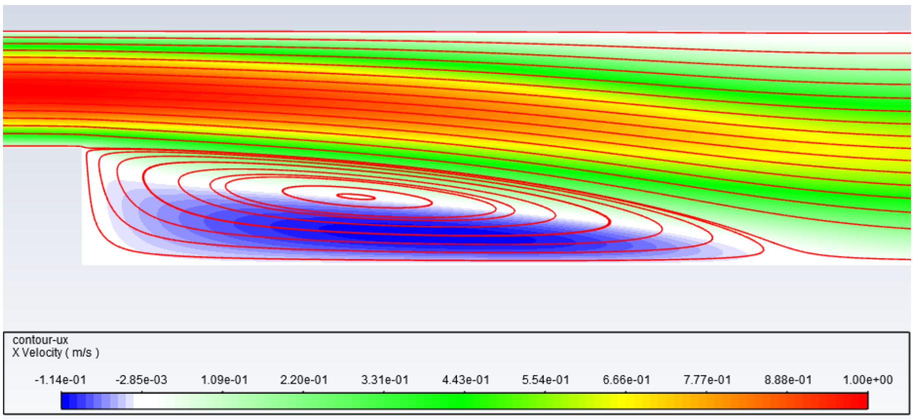

Figure 2: Streamlines (red solid lines) and contours of streamwise velocity near the recirculation region. The wall shear $\partial(u_x)/\partial y$ is negative in the recirculation region (u_x decreases from 0 at the wall to a negative value away from the wall) and positive downstream (u_x increases from 0 to a positive value).

In a steady flow, a stagnation point corresponds to a vanishing wall shear stress. Therefore, to find the location of the stagnation point, an "XY plot" is created for the normal shear (velocity derivative $\partial(u_x)/\partial y$) on the lower wall. You can adjust the range of each axis to zoom in and read the x value of the zero crossing (figure 3, left). On mesh 1, that value is approximately 5 m. To make the process easier and more accurate, one can write the data to a text file, and process that file with Matlab, Excel, etc (figure 3, right).

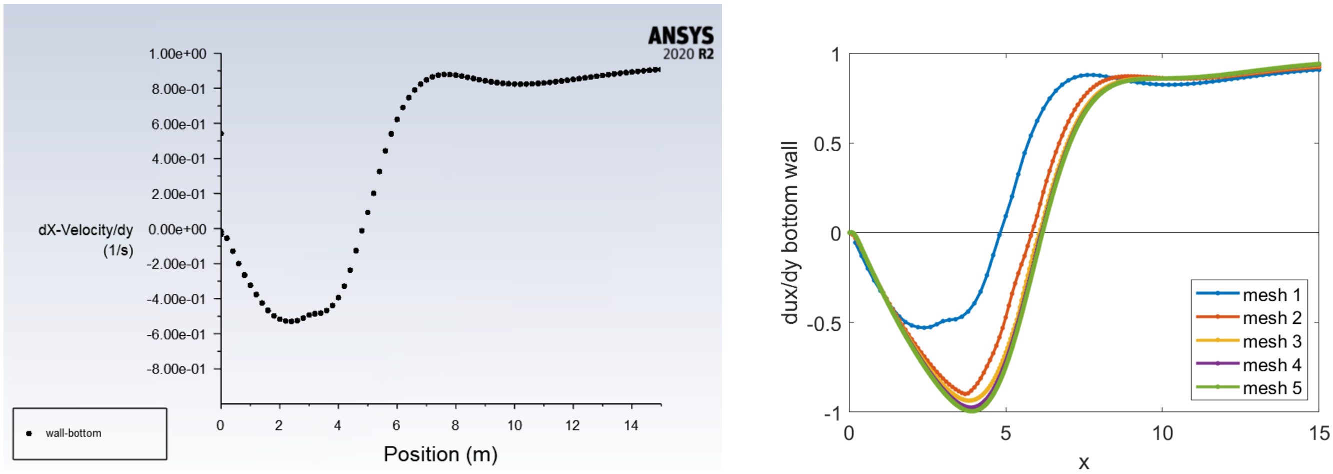

Figure 3: Normal derivative of the tangential velocity on the lower wall, $\partial(u_x)/\partial y|_{(x,0)}$.

Here, from one mesh to the next, the size Δx is simply divided by 2 uniformly (in the whole domain), which multiplies the number of elements N by 4. After 4 refinements, the finest mesh (mesh 5) corresponds to $\Delta x = 0.2/2^4 = 0.0125$ m and $N = 825 \times 4^4 = 211200$ elements. The results seem to converge well.

More quantitatively, figure 4 shows the variation of the length L of the recirculation region with Δx and N (top row), as well as the relative variation from one mesh to the next (bottom row). L converges toward a value close to 6.2. Initially the relative variation is large (17% from mesh 1 to mesh 2) but quickly drops (less than 5% from mesh 2 to mesh 3, and less than 1% from mesh 4 to mesh 5). Depending on the desired level of accuracy and the available computational resources, one can select an appropriate mesh.

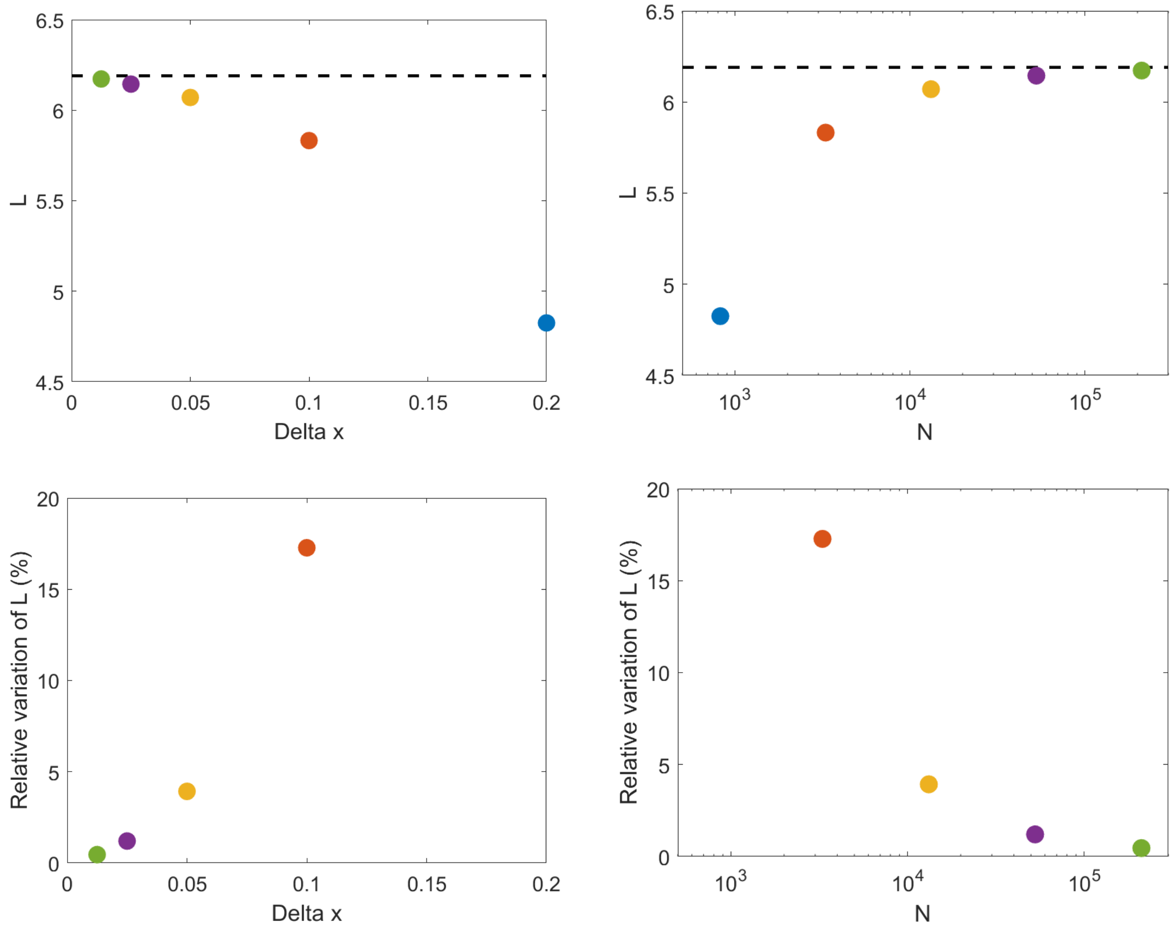

Figure 4: Top: length L of the recirculation region against mesh size Δx and number of elements N. Bottom: relative variation of L from mesh m to $m + 1$.

Using the finest meshes 3, 4 and 5, Richardson's extrapolation gives an estimated order of the scheme $p = 1.39$. With that value of p, the estimated discretization error decreases from 0.62 on mesh 2 to 0.017 on mesh 5 (see figure 5). Finally, the estimated exact value on mesh 5 is $L = 6.19$ (dashed lines in figure 4).

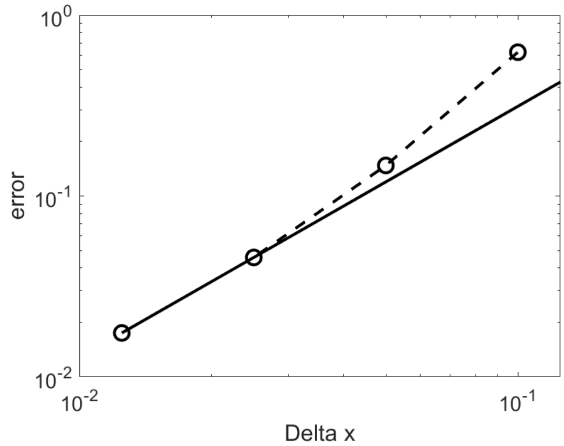

Figure 5: Symbols: estimated discretization error as a function of Δx. Solid line: slope $p = 1.39$.

As far as one can compare (visually), the value $L = 6.2$ m for $Re = 200$ is in good agreement with the results of Barkley *et al.* in figure 6. However, the agreement deteriorates for larger Reynolds numbers. This is partly due to the fact that, as Re increases, the lower recirculation region becomes longer, a secondary recirculation region appears on the upper wall, and the outlet (located at $x = 15$ m) is too close to capture the flow accurately. At $Re = 500$, the upper recirculation region already extends down to approximately $x = 18$ m, so the outlet is in a region of backward velocity and should definitely be moved farther downstream (see figure 7). (The results in figure 6 remain accurate because attention is paid to progressively move the outlet downstream as Re increases, for example at $x = 25$ or 35 m for $Re = 500$.)

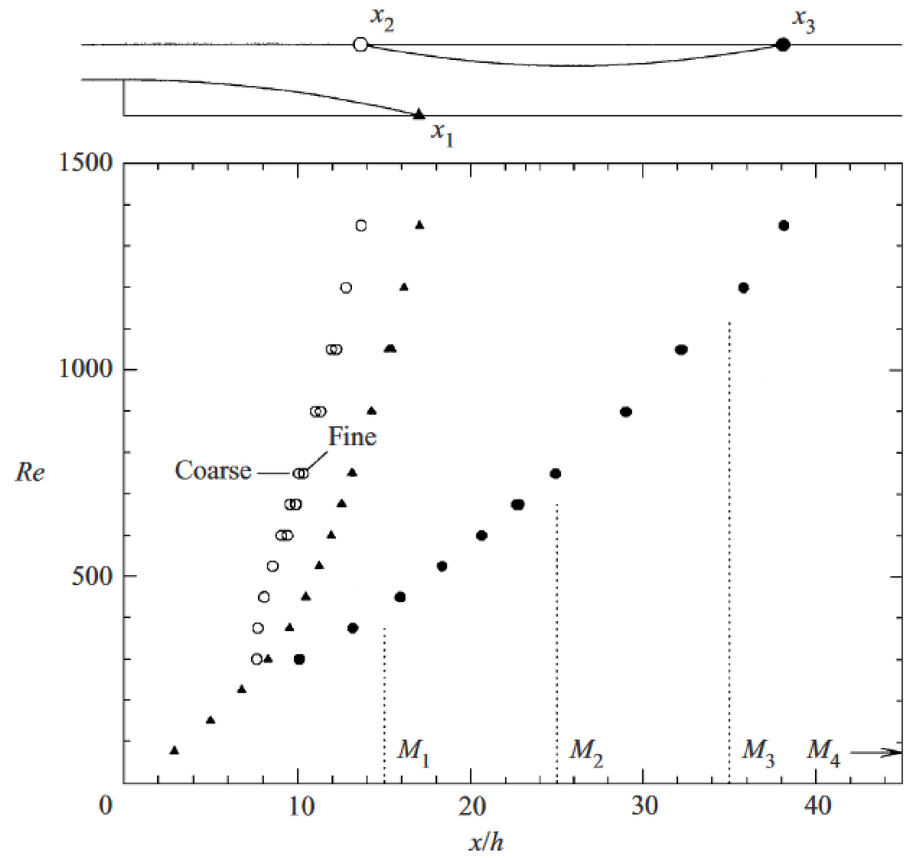

Figure 6: Location of the stagnation points in the backward-facing step flow (from Barkley, D., Gomes, M., & Henderson, R. (2002). Journal of Fluid Mechanics, 473, 167-190).

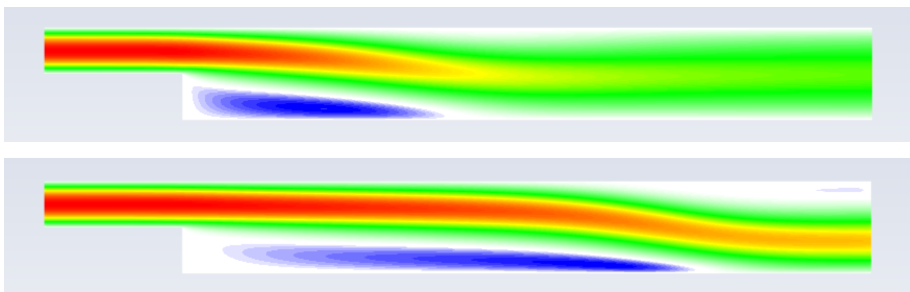

Figure 7: Streamwise velocity for $Re = 200$ and 600 (same colormap in both images).