ME470: Problem sheet 4 - Membrane, vesicles, and cells

Solutions will be discussed in the exercise session on Thursday, November 28

(1) Cytoskeleton and cell membrane A red blood cell (RBC) is surrounded by two layers: a lipid bilayer membrane and the cytoskeleton (the spectrin layer, a triangular network of entropic springs).

(a) Spectrin has a persistence length of $l_p = 15nm$ and forms roughly triangular networks of filaments with contour length $L_c \approx 200nm$. Determine (i) the area moduli μ_A and K_A of the cytoskeleton and (ii) the bending rigidity of the cytoskeleton, assuming that the spectrin layer can be modeled by a uniform shell of thickness $h = 25nm$. (Assume a room temperature, $T = 300K$, and the $k_BT \approx 4 \times 10^{-21}$)

(b) Compare the result to the bending modulus κ_b of a typical lipid bilayer membrane, which is on the order of $10k_BT$ (assuming room temperature). How important is the cytoskeleton for the response of a red blood cell to bending?

(c) RBC "ghosts" only have the cytoskeleton, but no membrane. How large would an osmotic pressure difference $\Delta\Pi$ between the inside and the outside of the RBC have to be in order to "blow up" the spectrin cytoskeleton, and thus the cell would have swollen into a sphere of $6\mu m$ diameter? Assume that the spectrin cytoskeleton is monolayer, with repulsive interaction potential as Coulomb potential, $k = 1$.

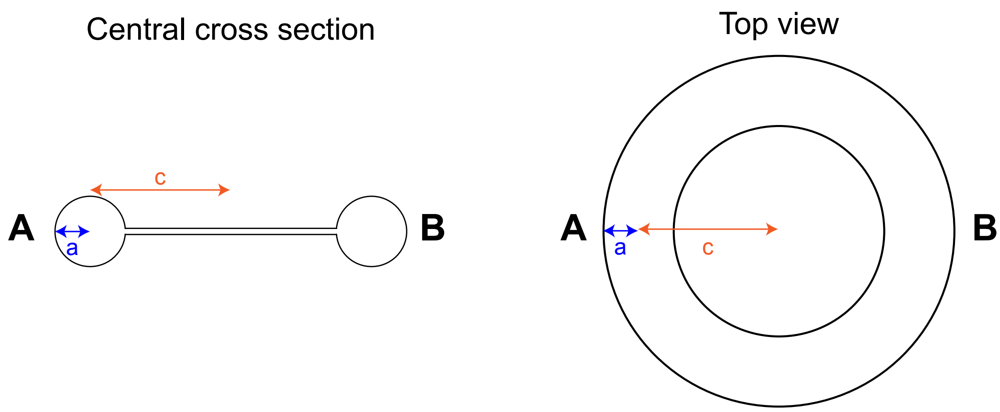

Figure 1: Schematics of red blood cell geometry

(2) Red blood cell shape energy We model a red blood cell as a torus with large radius c and small radius a (see Figure. 1), where the hole of the torus is filled by a thin planar disk at the center (of negligible thickness). The torus surface is then parametrized by $u, v \in [0, 2\pi[$ in the following way:

$$x = (c + a\cos v)\cos u \tag{1}$$

$$y = (c + a\cos v)\sin u \tag{2}$$

$$z = a\sin v \tag{3}$$

Here and in the following, we neglect any explicit modeling of the edge connecting the central disk and the torus.

(a) Show that the mean curvature is $H = -\frac{c+2a\cos v}{2a(c+a\cos v)}$ (Hint: use general differential geometry equations using first and second fundamental forms).

(b) Find the bending energy of the whole RBC assuming $\kappa_b \approx -\kappa_G \approx 10k_bT$ and $a = 1.5\mu m$, $c = 2.5\mu m$. (Hint; use the integral, $\int_0^{2\pi} \frac{(c+2a\cos v)^2}{c+a\cos v} dv = \frac{2\pi c^2}{\sqrt{c^2-a^2}}$)

(c) When a red blood cell is exposed to osmotic pressure, it swells, increasing the inner volume while -at first - keeping the area constant, By how much has the bending energy changed from the situation in (a) to the point where the RBC has swollen into a sphere?

(d) By how much would the bending energy change if the RBC were indeed toroidal (without the thin disk at the center)?

(e) The RBC torus, without the central disk, can also be swollen to a compact shape where $a = c$ (the horn torus). Again assuming constant area, by how much does the bending energy change?

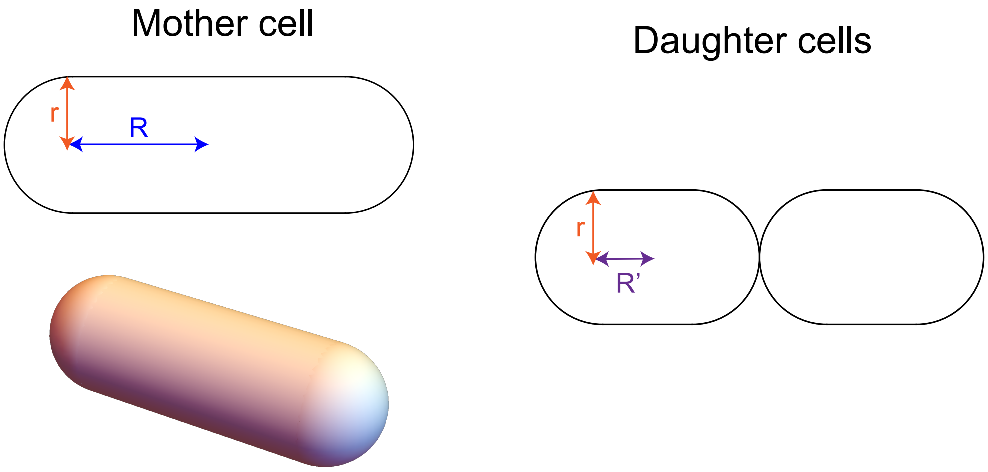

Figure 2: Schematics of dividing bacteria cells with spherocylinder shape

(3) Energy of dividing cells Suppose that the plasma membrane of a bacterium were constructed from an initially flat fluid membrane (see Figure. 2).

(a) How much energy (in units of $k_B T$) is required to bend this membrane into the shape of a spherocylinder $2R = 4\mu m$ long and $r = 1\mu m$ in radius? Assume $\kappa_b \approx -\kappa_G \approx 10k_BT$

(b) Now let this cell divide, keeping the volume and diameter, r, the same. What is the total bending energy of the daughter cells?

(4) Sliding bilayers Using the result $\kappa_b = K_A h^2/12$ for the single uniform bilayer, show that $\kappa_b = K_A h^2/48$ if two monolayers are freely sliding along the neutral surface. Assume that each monolayer has a thickness of $h/2$, respectively.