ME470: Problem sheet 3 - 2D network structure

Solutions will be discussed in the exercise session on Thursday, November 14

(1) A first look at rubber A sample of soft rubber has a shear modulus $10,000$ times smaller than its bulk modulus.

(a) Calculate the Poisson ratio ν. What does this value means?

(b) Calculate the ratio of Young's modulus to bulk modulus. What is the physical meaning of this ratio?

(2) Pre-stressed triangular network Here, we will derive two-dimensional area stretch modulus, K_A and two-dimensional shear modulus μ_A for pre-stressed regular triangular lattice from considering an individual plaquette.

(a) Compute bulk modulus around the equilibrium point using an expansion of enthalpy functional. (Hint: consider a single plaquette that undergoes uniform extension)

$$K_A = \frac{\sqrt{3}k_{sp}}{2}\left(1 - \frac{\tau}{\sqrt{3}k_{sp}}\right) \tag{1}$$

(b) Compute shear modulus around the equilibrium point using an expansion of enthalpy functional. (Hint: consider a single plaquette under a simple shear)

$$\mu_A\left(\tau\right) = \frac{\sqrt{3}}{4}k_{sp}\left(1 + \frac{\sqrt{3}\tau}{k_{sp}}\right) \tag{2}$$

(3) Poisson ratio of triangular lattice Consider a triangular lattice under constant two-dimensional isotropic stress τ.

(a) Utilizing the relation between two-dimensional Poisson ratio ν_A, two-dimensional stretch modulus, K_A, and two-dimensional shear modulus, μ_A, derive an expression of ν_A in terms of τ and k_{sp}.

(b) Find the range of isotropic stress τ where $\nu_A < 0$

(c) Consider cytoskeleton of human blood cell, where the two dimensional shear modulus at zero isotropic stress is approximately equal to $\mu_A = 5 \times 10^{-6} J/m^2$. Calculate the minimum isotropic stress needed for $\nu_A < 0$.